Chapter 6: Gravity and the body

A. The unit mass

In objects with mass, such as our bodies, tension or compression forces are not necessarily constant throughout as they are in massless ideal springs. Humans tend to use our bodies as detectors for forces, and it is the perception, or detection, of forces that lies at the heart of most force misconceptions. Before discussing the structure of our bodies, a simple model of an object with mass is constructed that closely reproduces most effects of gravity and elastic forces on them.

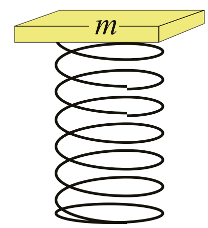

Figure 6.1 The unit mass

A unit mass, combining a thin mass attached to a massless ideal spring, is shown in Figure 6.1. This unit separates elasticity and mass, the two important properties of matter that govern its reaction to forces.

Objects having mass can be modeled as a sequence of unit masses with the spring of one unit attached to the mass of the next as shown in Figure 6.2. The behavior of this

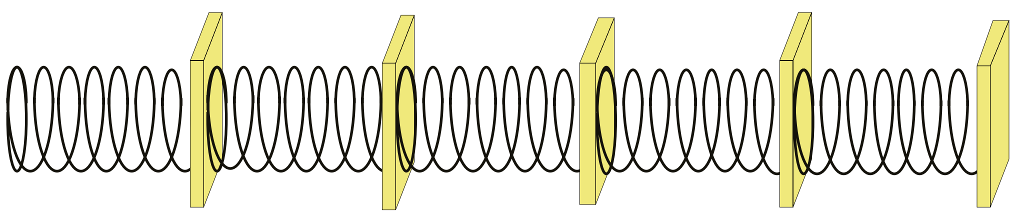

Figure 6.2 A series of units modeling an object with mass

attachment springs and masses is first described in a horizontal orientation, where gravity can be ignored, in preparation for the more difficult case where gravity also affects the stack of masses.

An object subjected to a pair of compressing forces, F, applied slowly, generates compression forces, C, in each spring, as shown in Figure 6.3. The compression forces

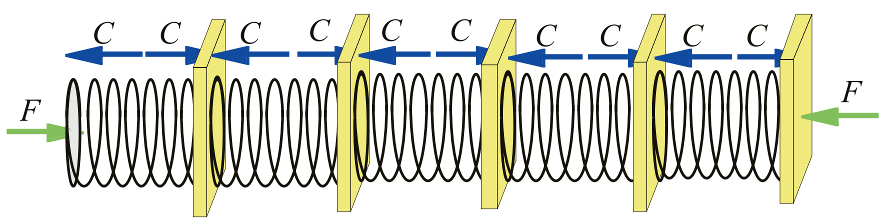

Figure 6.3 The reaction of the series of units to being compressed by a force F .

have the same magnitude as the applied force, $C = F$. Two forces with equal magnitude and opposite direction act on any one of the masses. Thus balanced forces act on each mass, and cause no part of the object to change its motion. In this case, the masses start and remain at rest. Each mass is compressed by a pair of compression forces, generating reaction forces of the same magnitude (the mass is too thin to show these forces) within the mass.

Before considering the effects of gravity on this stack of masses, consider a single spring compressed by the weight of one mass, and an identical spring compressed by the weight of five of these masses, are shown in Figure 6.4. Between the earth and the single mass, *m*, a gravitational force $F_1 = mg$, pulls the masses together to compress the spring. Between the earth and larger mass, 5*m*, there is a gravitational force, $F_5 = 5mg$, compressing its ideal spring a distance that is five times larger. Since the masses are at rest, each compressed spring must be applying a balancing compression force, on the earth and on the supported mass, with a magnitude equal to the corresponding gravitational force, $C_1 = F_1$, and $C_5 = F_5$.

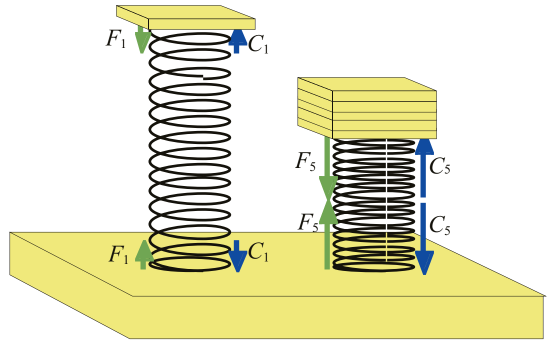

Figure 6.4 One mass and five masses compressing the same spring between it and the Earth

B. A quiz

Now consider the stack of 5 masses, *m*, and 5 springs. Use your common sense in solving a quiz regarding the gravitational force. Consider the identical mass & spring units arranged, as shown in Figure 6.5, in three vertical stacks with:

a) all springs having their natural length,

b) each spring compressed the same amount, and

c) the bottom spring compressed five times as much as the first.

Quiz - Decide which state represents:

1) the stack resting on the ground,

2) the stack in frictionless free fall, and

3) the masses of the stack affected by additional forces, other than gravity and the spring forces.

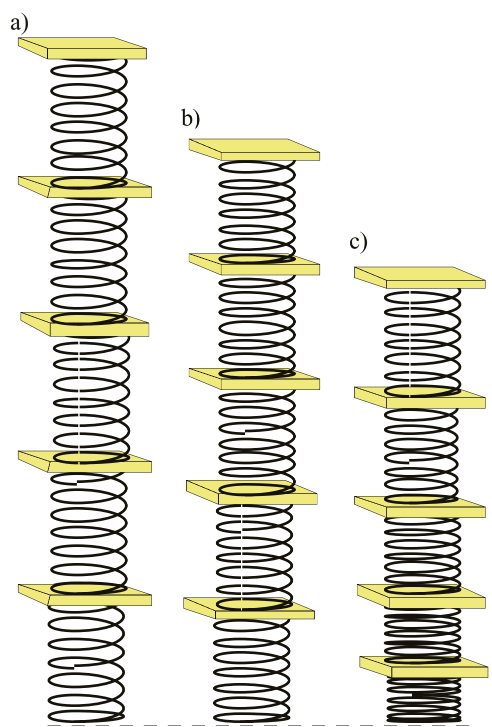

Figure 6.5 Three states of compression: a) uncompressed, b) equal compression, c) increasing compression.

How can you tell which of the states is the one sitting on the ground? Use a process of elimination. It can't be state (a) because the bottom spring must be compressed if the stack is sitting on the ground as it was in Figure 6.4. The choice is between state (b) and state(c) for the one sitting on the ground. Each spring in state (b) is identical and compressed an equal amount. The top spring must support only one mass, while the bottom spring must support all five masses above it, as if a massless box filled with 5 masses and 5 springs sat on the bottom spring. The bottom spring must compress five times that of the top. The compression of the springs in state(c) are consistent with the stack being placed on the ground.

For the free–falling object, the choice is between state (a) and state (b). State (b) can be eliminated because in free–fall there would be nothing pushing upward on its bottom spring yet it is compressed. This leaves only state (a). The lowest spring doesn't push upward on the lowest mass, and therefore the second spring cannot compress, and so on. None of the springs compress; all springs have the natural length. The gravitational force makes each mass fall but alone cannot compress the springs.

It follows that the equal compression seen in state (b) cannot occur without additional forces acting on the masses. Without masses between the springs, a stack of springs sitting on the ground, supporting only a top mass will result in equal spring compressions. If a mass is inserted between springs, as in state (b), the springs will no longer be compressed an equal amount by gravity.

C. Forces on and in a mass

Moving downward through stack (c), the compression forces produced by the springs increase in magnitude. At first this seems to contradict Newton's 3rd law. If compression forces can change from one side of a mass to the other, Newton's 3rd law will not be violated!

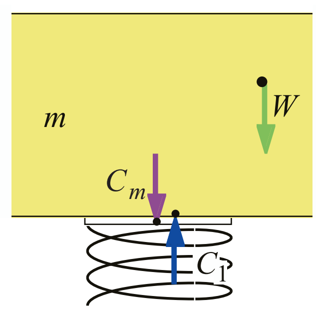

Figure 6.6 Forces W, C_1, acting on the mass and C_m acting on the spring.

Using a magnified view of the top mass in the stack as shown in Figure 6.6, we see that no forces act on the top surface leaving that surface uncompressed. The gravitational force, $W = mg$, pulls the entire mass downward compressing the bottom surface against the spring, causing two compression forces: C_m, produced by the mass acting downward on the spring, and C_1, produced by the spring acting upward onto the mass. Only two of the forces, shown in Figure 6.6, act on the mass (W and C_1) while the third force (C_m) acts only on the spring. The forces acting *on the mass* are balanced, therefore, $C_1 = W$. By Newton's 3rd law, where the mass and spring meet, the magnitude of the two compression forces must be equal, $C_m = C_1$. Therefore, all forces, shown in Figure 6.6, have a magnitude equal to the weight of the supported mass.

Massless objects have a uniform compression throughout, but within a mass the compression can change with position. At any level within the mass, shown in Figure 6.6, the magnitude of the compression force is equal to the weight of the portion above it. Any mass is itself a series of thin masses connected by microscopic and massless springs.

We now use these concepts on the second mass from the top of stack (c). A detail, as shown in Figure 6.7 (on the left), identifies forces that act on the mass, and on springs in contact with it. Also shown (on the right) are the compression forces, $C_1 = mg$, that would exist if an ideal spring were placed at this position instead of a mass. For the mass, in addition to the compression force C_1, its weight ($W = mg$) is a source of compression, $C_m = W$, acting on the lower spring.

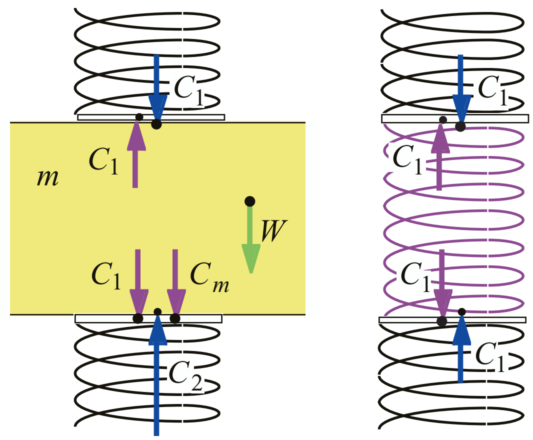

Figure 6.7 Forces acting across the second mass in the stack (left) and for an ideal spring in the same location

The mass does not distinguish between the two sources of compression, and creates a total force, $C_2 = C_1 + C_m = 2mg$ as shown in Figure 6.8. By Newton's 3rd law, the compression force in the lower spring must also have this value, $C_2 = 2mg$. The compression at the bottom of the second mass of stack(c), therefore, corresponds to the weight of the two masses supported above that point.

As an exercise, take the next lower mass within the stack shown in Figure 6.8, and identify those forces that act on the springs and those acting on the mass. You should find three forces acting on the mass, and one each on springs. The two compression forces generated by the mass each act on a spring.

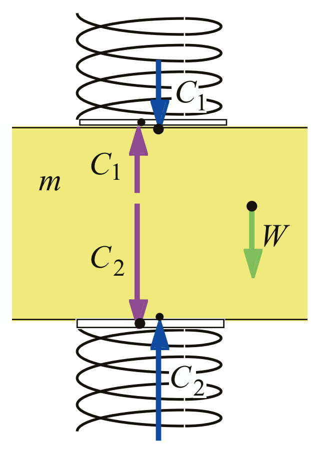

Figure 6.8 The forces across the second mass in the stack.

The forces acting on each mass in stack(c) are shown in Figure 6.9. The compression force in any spring is equal to the weight of all the masses above it. This pattern is repeated for each mass until the lowest spring, which is compressed by the full weight, $W_{\text{Total}} = 5mg$. By compressing, each spring and mass pass the weight of the masses above, on to the next mass. At a contact point between a spring and mass forces consistent with the 3rd law are generated.

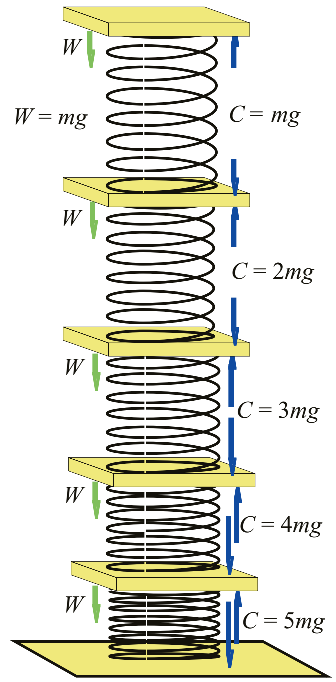

Figure 6.9 Forces on a stack of five masses

The force labeled, C_m, in Figure 6.7 is a compression force within the mass due to gravity acting independently on each of it's thin sections. The weight of these sections will compress the mass only if the bottom surface presses against another object. The removal of all spring forces results in a relaxation within the mass, removing the internal compression but the gravitational force remains, and the mass, instead of compressing, falls.

This discussion illustrates the one feature that distinguishes the behavior of an object with mass from a massless ideal spring:

Forces of different magnitude can act on opposite ends of a massive object!

Gravity and an elastic force acting on an object with mass, can create a compression in the object that varies from the top to the bottom; in Figure 6.7 the forces are, $C_1 = mg$, at the upper boundary and, $C_1 + C_m = 2mg$, at the lower boundary of the mass. The situation is more dramatic for the top mass where no force acts on the top surface while, $C_1 = mg$, acts from below (and gravity acts throughout).

External forces acting on opposite sides of a massless object, e.g., an ideal spring, must be equal in magnitude. However, external forces acting on opposite sides of a mass can differ. The most obvious example of this has just been described: near the surface of the earth, a stationary mass (it remains at the same place for a period of time) has a gravitational force pulling down and an upward force of equal magnitude, usually an elastic force, supporting from below. The upward force creates compression in the mass at the lower surface that equals its weight. The compression force at an intermediate point within the mass equals the weight of the mass above that point.

Gravity and an elastic force can create a state of varying compression or tension within a stationary mass. If balanced elastic forces act on a stationary mass (no gravity) a uniform compression or tension is created within it. However, generating an imbalance of elastic forces acting cause the mass to *begin* to move (does not remain stationary), and compression or tension will vary within the mass. There are two ways to generate a varying tension or compression within a mass: a gravitational force opposed by an elastic force, as seen in the stack of masses shown earlier, or in the absence of gravity, an imbalance of elastic forces acting on the surface of an object.

In Figure 6.9, the stack of five equal masses, m, separated by ideal springs can be considered as a single object with a mass of $5m$. The lower part of this object must support all of the mass that lies above it, and compression forces change linearly, in five steps, through the object. Any object with a uniform distribution of mass placed on the ground will have compression forces within it that change linearly from zero at the top to the full weight on the bottom.

Using a model of a massive elastic object, consisting of a series of small masses connected by massless ideal springs, an understanding of any "hidden" internal forces within objects can be obtained. If one is only interested only in the motion of an object, without regard to internal forces, then there are techniques that make it unnecessary to go into this detail. However, using our bodies to perceive the presence of forces, as we often do, internal elastic forces cannot be ignored.

D. The body paradigm

The concepts developed in the previous sections provide the tools needed to construct a model of the human body useful in understanding the human perception of forces. A highly schematic model of a human body is shown on the left in Figure 6.10. On the right, each body part is modeled either by a rigid mass, or by an ideal spring (massless) connecting the masses. The natural length of the springs (joints) in your body are small, but they are shown much larger exaggerate the changes in length used to measure the magnitude of the compression forces acting at that joint.

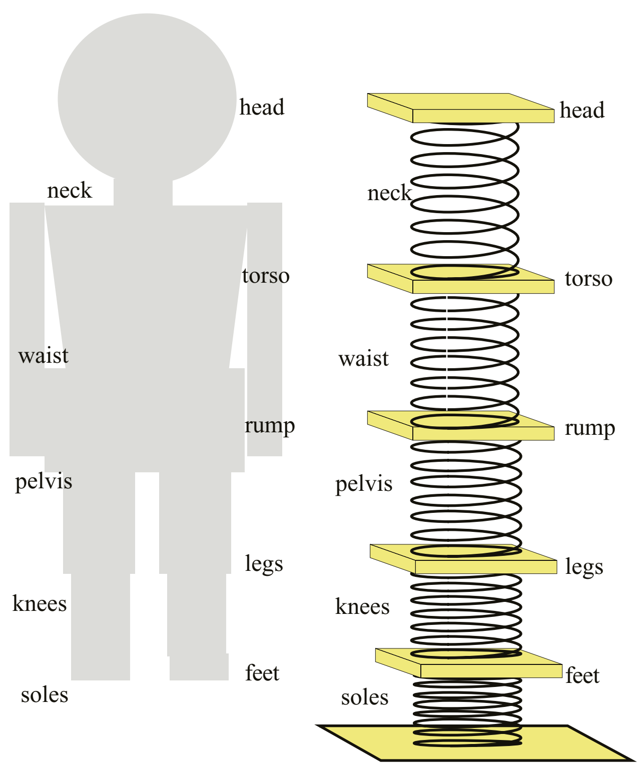

Figure 6.10 Model of the human body

The two legs and feet have been

merged in this version of the body but it is easy to divide those forces and spring constants by two, if desired. For simplicity, the compression is shown as if each body part had a similar mass. The compression of the neck, waist, pelvis, knees and soles exhibit a progressively increasing compression, indicating the increasing internal compression forces acting at these locations. The lower springs must support the weight of all the mass above it.

A person is generally unaware of these compressions. An injury to one of the lower springs (joints), however, will cause pain when standing. In many joint injuries, bone parts normally supported in the damaged area compress nerves, sending pain signals to your brain. The pain is a warning to you to be very careful with that part until the damage heals. If you ignore the pain, the two body parts not normally in contact can be further damaged, or they can interfere with the healing process.

When standing, the soles of your feet are compressed between you and the earth by a gravitational force; a force better known as the weight of your body. An average adult has about a mass $m = 70\,\text{kg}$ resulting in an approximate weight, $W = 700\,\text{N}$ (about 150 lb.), compressing the soles of the feet by an amount close to 5 mm. You may be unconvinced that soles of your feet are compressed by about this amount, but after viewing a picture of the bottom of a person's feet when standing on a piece of glass you would no longer have any doubts. The bottom of the foot looks mashed or crushed, showing the effects of the compression.

The spring constant (if the feet behave like an ideal spring) for combined effect of the soles of both your feet acting together is easily calculated using Hooke's law:

$$k = \frac{F}{x} = \frac{700\,\text{N}}{0.5\,\text{cm}} = \underline{1400\,\text{N}/\text{cm}}.$$

This is quite a strong spring. Its spring constant approaches values comparable to the spring in an automobile, although limited in its allowable compression, and it is 1400 times stronger than our standard spring with its spring constant of 1N/cm.

E. Altered states of the body

Any time the compression of the body is changed from that normally experienced standing, sitting or lying down, the body informs your brain that something unusual is occurring. If instead of standing on solid ground a person is holding onto and hanging from a bar, all of the lower body parts are in tension. The feeling we have when hanging from our hands is caused by tension in joints that are most often in compression.

On the other hand, if placed in a situation where your body is compressed in the same way as when standing on the earth, you would have no way of knowing you weren't on earth from the messages your body was sending to your brain. Your sight might give you a hint that something else was happening but your body would be saying that you were on the earth. A scene in the movie “2001” shows a crewmember trotting around the walls of a circular room. The appearance of “artificial gravity” is created by the rapid rotation of the room, as seen from outside of the ship. The walls of the room are pushing inward against the crew’s feet which allows the crew to rotate with the room. The invward force on the feet causes compression forces within the body that duplicate and simulate the effects of gravity when standing on the earth.

We will now consider the unusual situation where a person who is sky diving for the first time (hopefully with a parachute) has *just left the airplane and is in free fall,* feet first toward the earth. You may remember that we discussed a similar situation in the previous section. There, we determined that in a state of free–fall there is a complete loss of compression in the springs.

Here, the argument is the same: there is no force acting on the bottom of the skydiver’s feet to create the compression of the soles. Following this feature upward through the body we find that, although the gravitational force is acting on each body part, there is no compression of the skydiver’s body. You can imagine that in this state the body would inform the brain that “something unusual is happening here!” Experienced skydivers know and like this feeling, but incorrectly associate it with their bodies being “weightless”._The skydiver always has weight. The gravitational force on each body part is still there and each body part still has weight. The state of the body that the skydiver calls “weightless” is really a misnomer; this state should be called “compression-less”.

Thus far, most of our examples have been for objects that were at rest, however, the feeling of being weightless in the presence of a gravitational force on the body, can be understood as the lack of one of the two forces needed to compress the body. Falling motion will be discussed at length in upcoming chapters.

<u>Chapter Summary</u>

- A mass that is stationary (remains at the same place for a period of time) near the surface of the earth must have other forces, most often elastic forces, acting on it that balance its weight (the gravitational force on it).
- The internal behavior of objects with mass can be modeled as a sequence of smaller masses connected by massless ideal springs.
- In an mass placed on the ground, gravity and elastic forces will cause compression forces to be smaller in the upper portions than in the lower portions of the mass.
- In an mass suspended from the top, gravity will cause a tension forces to be larger in the upper portions than in the lower portions of the mass.
- The compression a person feels when standing on the earth can be simulated if other forces acting on the person produce an identical state of compression.
- There are no internal tension or compression forces acting in a mass in free fall. This state is incorrectly called "weightless" by human observers.