Chapter 7: Static friction, torque and static equilibrium

A. Review of force vectors

Between the earth and a small mass, gravitational forces of equal magnitude and opposite direction act on the masses. On a stationary mass, or on a mass maintaining a constant speed and direction near the earth, additional forces must act to balance the earth's gravitational force. A balanced force condition can be very useful in determining the individual forces acting on a mass. A formal procedure has been presented (see Equation 2.1) to determine if the force vectors acting on an object are balanced. This procedure is reviewed here to prepare for a discussion of static friction and torque.

Force vectors are constructed by taking the force magnitude (always a positive number) and attaching a sign: plus for one direction or minus for the opposite direction. The vector quantity, containing both magnitude and direction, is written symbolically as a bold roman letter, such as **F**. The forces, $\mathbf{F}_1 = +F$ and $\mathbf{F}_2 = -F$, are vectors with a common magnitude *F*, but point in opposite directions. It is entirely your own decision which direction you would like to choose as positive, but once you choose a convention you must stick with it.

If the forces acting on a mass are in "balance", it means that the vector sum of the forces (the net force) is zero. Balanced forces do not affect the *motion* of an object: the object will remain stationary or will continue with the same speed and direction. Balanced forces, however, will compress or stretch the object. If there are only two forces then the balance condition demands that the forces have equal magnitudes and opposite directions.

The net force on an object is the result of the sum given in Equation 2.1:

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n. \tag{7.1}$$

This sum of the external force vectors acting on an object is called the "net force vector" $\mathbf{F}_{\text{net}}$. If there are only two force vectors (n = 2) with equal magnitudes and opposite directions, $\mathbf{F}_1 = +F$ and $\mathbf{F}_2 = -F$, then the forces are "in balance" and the net force acting on the object, determined by substitution into Equation 7.1:

$$\begin{aligned}\mathbf{F}_{\text{net}} &= \mathbf{F}_1 + \mathbf{F}_2 = (+F) + (-F)\\ &= (+F) - (+F) = 0.\end{aligned}$$

As expected, $\mathbf{F}_{\text{net}}$ is zero.

When $\mathbf{F}_{\text{net}} = \pm F_{\text{net}}$, where F_{net} is the net force magnitude (non- - zero) and the sign (±) is the net force direction, the motion of the object will be affected (its speed or direction will change). If we know, through some means (such as, the object is stationary or coasting along with a constant speed and direction), that forces acting on a mass are in balance, then the net force, given by Equation 7.1, can be set equal to zero, $\mathbf{F}_{\text{net}} = 0$. If, in addition, all but one of the force vectors acting on the mass are known, the value of the missing force vector can be determined by solving Equation 7.1, with $\mathbf{F}_{net} = 0$.

If the speed or direction of an object changes (not stationary and not moving with a constant speed and direction) then the net force acting on the object (Equation 7.1) will be non-zero. The sign of the net force vector, $\mathbf{F}_{net}$, is the direction of the net force. The discussion of how to determine, $\mathbf{F}_{net}$, for objects that are accelerating (changing speed or direction) will be covered in a later chapter. For now, to hone your skills in applying Equation 7.1, the examples provided in lecture and in homework will have $\mathbf{F}_{net} = 0$.

B. Static Friction

To move a mass that is secured to the ground with screws, glue, Velcro (locking surfaces), or other constraint, requires a large force. Until the securing agents yield or break, they react elastically, distorting sufficiently to generate compression or tension forces that balance the applied force. Even without a constraint securing it to the ground, a mass will not begin to move until a horizontal force above a critical value is applied. If the mass does not begin to move in response to a horizontal force, a balancing force (or forces) must also act on the mass. Static friction is the common name given to this additional force. Electromagnetic forces between the atoms of two objects in contact cause frictional forces as they attempt to slide on each other.

Static friction behaves like a securing agent: the magnitude of the static frictional force acting on an object will be equal in magnitude and opposite in direction to the applied force, and will therefore prevent the motion of the object. The details of how this force is generated are complex; however, the general character of the frictional force can be seen with a simple model reproducing its elastic behavior.

A large mass, M, starts with no horizontal applied force, and has a compression force, C, at the interface between it and the ground, with a magnitude equal to the weight of the mass, $W = Mg$, as shown in Figure 7.1. The compression measures how hard the two surfaces are being pressed together. When pushed from the side, a larger mass (same physical size) it is more difficult to get moving because the compression forces are larger. Also, it is usually more difficult to get a mass moving on a rough surface than on a smooth one. The roughness of a surface is characterized by a number, μ (mu), that normally takes values between 0 and 1 depending on the "stickiness" of the surface; $\mu = 0$ for a frictionless surfaces and, $\mu \sim 1$, for rough surfaces.

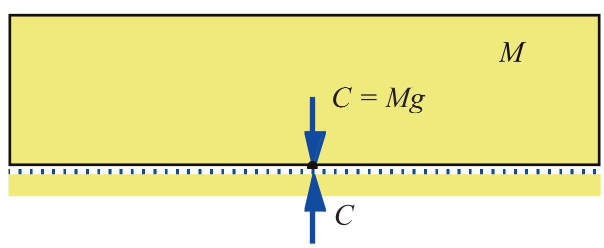

Figure 7.1 A mass, M, compresses the ground with a force, C, equal to the weight of the mass, Mg.

A large mass resting on the ground is pushed, as shown in Figure 7.2, to the right by a force, F, e.g., by a person lying between the mass and wall, (for clarity, the compression force vectors, C, shown in the previous figure, are not shown here). The

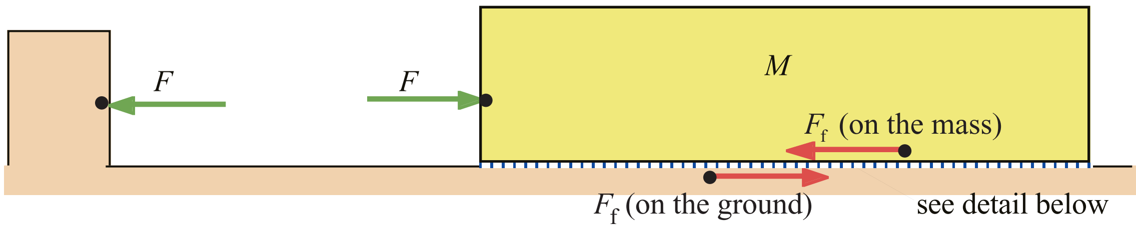

Figure 7.2 Two frictional forces, F_R, generated by pushing on the mass and ground with a force F.

mass remains stationary due to frictional forces, F_f, between the two surfaces. The frictional force acting *on the mass* is equal in magnitude and opposite in direction to that of the applied force, F, and will increase in proportion to the applied force until reaching a maximum value of, $F_{crit} = \mu C$. If nothing else touches the mass, $C = Mg$, and therefore, $F_{crit} = \mu Mg$. The mass begins to slide when F is greater than the critical frictional force,

$$F > F_{crit},\ F_{crit} = \mu Mg \tag{7.2}$$

The critical *horizontal* force, F_{crit}, depends on the magnitude of a *vertical* force, Mg. This may seem quite odd, but it does make sense in a model increasing the number of active atoms on more compressed surfaces. Generation of static frictional forces at the mating surfaces of the mass and the ground can be modeled using idealized elements, as shown in Figure 7.3.

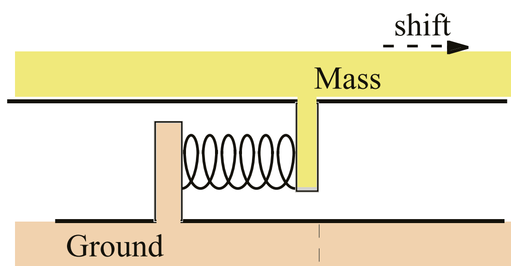

Figure 7.3 Detail of the mating surfaces of the Mass and the Ground.

Projections from the surface of the mass and ground represent roughness at the atomic scale. A natural length "friction" spring attached to these projections simulates the electromagnetic forces between the atoms.

When the mass is pushed to the right the mass will move very slightly to the right, thus stretching the microscopic friction springs, as shown in Figure 7.4 (pushing to the left compresses the spring and creates compression forces pushing back on both objects). The tension forces (T) in the stretched spring attempt to pull the top mass toward the left and the ground to the right. The frictional forces on the mass and ground are simulated in this model by the tension forces of a large number of springs.

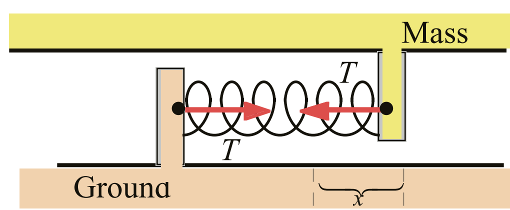

Figure 7.4 Detail of mating surfaces when the mass is pushed to the right.

The magnitude of the tension forces of the friction spring will be equal to the force applied to the mass by the pusher as shown in Figure 7.2. The force of the person against the wall and the frictional force acting on the ground are also equal in magnitude and opposite in direction). The net effect of the applied and frictional forces is to keep the mass in place, compressing it between the applied force and the frictional force on the mass. The person tried to push the mass, and instead, compresses the mass and stretches the ground.

It is often said that it is unnecessary to discuss the origin of frictional forces. Once there is a frictional force on the mass, there must be another force on the ground with equal magnitude and opposite direction by Newton's 3rd Law. Nevertheless should be comforting to know that an explanation of Newton's 3rd Law in each application is possible using the same model of elastic forces.

Extending the model can reproduce other properties of the frictional force. The mass will begin to slide if a force is applied to the mass that stretches (or compresses) the springs too far and they break, or the projections holding the springs break. Increasing the force that is squeezing the two surfaces together will increase the number of springs that are active, and increases the force necessary to make the mass start to slide, in agreement with the behavior of the real frictional force. Some aspects of the frictional force are not reproduced, but modeling static friction with the elastic forces of tension and compression, is reasonably accurate.

Once a mass is pushed with a force greater than F_{crit}, it begins to slide. While sliding a new force, called sliding friction, will begin to affect the motion. The force of sliding friction grows with speed, so that at some speed the sliding frictional force will balance the applied force, and the motion will then have a constant speed and direction.

C. Torque and Twisting (Sections C and D are optional reading)

In earlier chapters, examples were carefully chosen such that the forces were symmetrically placed: for every force on one side of a line through the center of the object there was another force on the other side with the same magnitude, direction, and distance from the line. Another symmetrical case is shown in Figure 7.5, where forces, labeled F, are applied to the upper and lower surface of a doorknob in the same direction (surely not the way to turn a doorknob). The two forces push the doorknob to the right and, not surprisingly, nothing moves or turns. The door (hinges on the right) is compressed and generates a compression force, $C = 2F$, acting on the doorknob, balancing the applied forces.

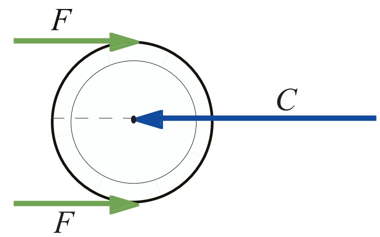

Figure 7.5 Pushing on a doorknob.

The example shown in Figure 7.6, however, differs from those considered previously. What makes this situation different is the twisting action, called "torque", that turns this doorknob clockwise. The doorknob will not move left or right because the two forces have equal magnitudes and opposite directions, and therefore, are in balance. The doorknob turns however, because the torques do not balance!

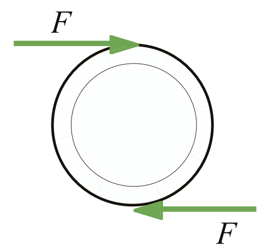

Figure 7.6 Forces that will turn a doorknob

Each of the forces acting on an object will generate an associated torque. The Greek letter Tau (τ) is the symbol used for torque (T is already used for tension). A force can be applied at a point on a doorknob, as shown in Figure 7.7, at angles, θ, from 0° to 90°, with respect to a line, of length r, from that point to the center of the doorknob. If a force is applied at $\theta = 90^\circ$, shown on the left, then the *magnitude* of the torque, τ, is a maximum given by:

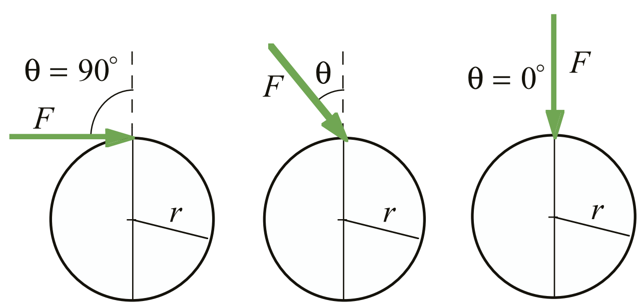

Figure 7.7 A force applied at different angles to a doorknob.

$$\tau = Fr, \tag{7.3}$$

the product of the magnitude of the force and the distance to the pivot. When the force is directed toward the pivot point, as shown on the right, the torque is zero. The torque is less than the maximum for intermediate angles shown in the center. In this text, to simply calculations, a force will be applied to an object only at the angle 0°, where the torque is zero, or at 90°, where the torque is given by Equation 7.3. Note that a force can be applied without causing a torque but a torque cannot be created without a force.

The *forces*, shown in Figure 7.8, are in opposite directions but they cause *torques* that are in the same direction (turn the doorknob clockwise). Forces applied in opposite directions, and on opposite sides of the doorknob, generate torques in the same direction. There is a net torque but at the same time the forces are balanced (applied in opposite directions) so that the object will not start to move to either side.

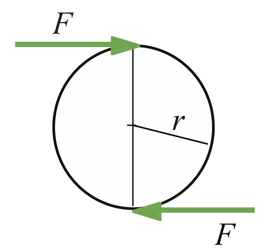

Figure 7.8 Geometry of a Doorknob

Torque is a vector quantity, represented as a bold letter Tau ($\boldsymbol{\tau}$), and the magnitude and direction must be specified with a procedure similar to the one used for forces. A positive torque will twist in the clockwise direction (most students prefer clockwise as the positive direction), while a negative torque will twist in the counter–clockwise direction. It is unusual to draw torque vectors, but I will use a curved arrow in the direction of the torque, labeled by the magnitude of the torque; a positive torque vector, shown in Figure 7.9, is on the right, while a negative torque vector is on the left.

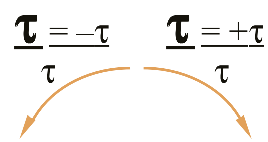

Figure 7.9 Torque vectors

Each of the forces acting on the doorknob will generate an associated torque. As shown on the left of Figure 7.10, the torque vectors, $\boldsymbol{\tau} = +Fr$ (upper), and, $\boldsymbol{\tau} = -Fr$ (lower) are generated by forces pointing to the right, yielding a net torque, $\boldsymbol{\tau}_{\text{net}} = 0$ (the torques balance). A pair of balanced torques does not turn the knob, however the unbalanced forces will push the doorknob to the right. Forces that cause torques in the same direction, as shown on the right in Figure 7.10, generate a net torque, $\boldsymbol{\tau}_{\text{net}} = +2Fr$, and will turn the doorknob clockwise. At the same time, the force vectors will balance so that the net force is zero. Therefore, as desired, the doorknob will turn and will not be pushed laterally in either direction.

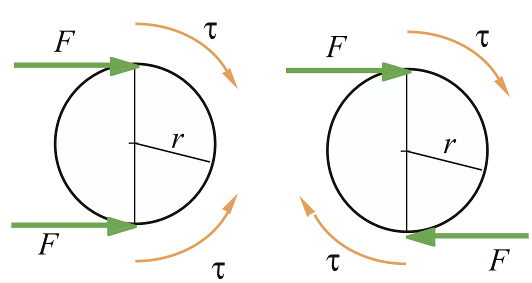

Figure 7.10 Torque vectors generated by forces acting on a door knob

<u>D. Generalized static equilibrium</u>

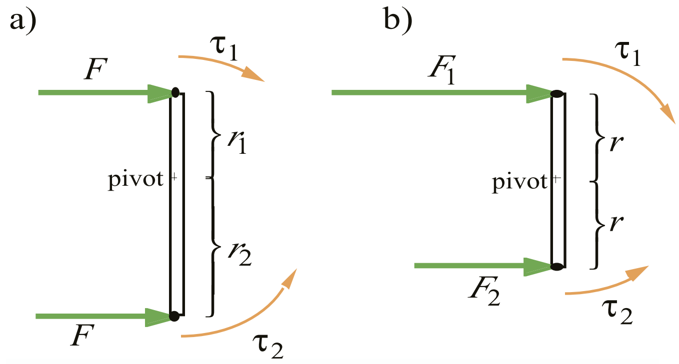

Figure 7.11 Forces causing a non-zero net torque on a bar.

The sum of the torques acting on an object is called the *net* torque, and if non-zero, the torques are unbalanced, and a non-rotating object will *begin* to turn in the direction of the net torque. An example, shown in Figure 7.11a, where equal forces are applied at different distances, $r_2 > r_1$, from a fixed pivot, results in a negative net torque (non-zero), and that object *begins* to turn counterclockwise. Another example, shown in Figure 7.11b, where forces have different magnitudes, $F_1 > F_2$, but are applied at the same distance from the pivot, results in a positive net torque, and that object *begins* to turn clockwise.

With asymmetries in both force and distance, where the larger force is applied at a smaller distance from the pivot, as shown in Figure 7.12, a zero net torque can result. Derived below, is the condition on the two forces acting perpendicular to a bar and pivot that result in *rotational equilibrium*:

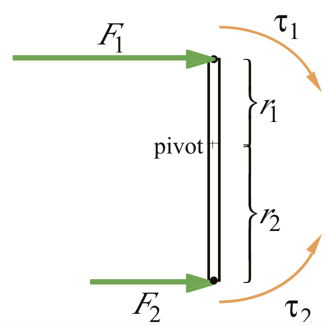

Figure 7.12 Forces causing a zero net torque on a bar.

$$\begin{aligned} \tau_1 + \tau_2 &= \left(+F_1 r_1\right) + \left(-F_2 r_2\right) = 0 \\ F_1 r_1 &= F_2 r_2 \end{aligned} \qquad \text{(Eq. 7.4)}$$

For an object to be in *static equilibrium* (not begin to move laterally or rotate), no net force and no net torque can act on the object. These two conditions must be met simultaneously and can be expressed as:

$$\boxed{\begin{aligned} \mathbf{F}_{\text{net}} &= \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n = 0, \\ \boldsymbol{\tau}_{\text{net}} &= \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots + \boldsymbol{\tau}_n = 0, \end{aligned}} \qquad \text{(Eq. 7.5)}$$

where the torque vectors, $\boldsymbol{\tau}_i$, have the magnitude, $\tau_i = F_i r_i$ for each force (i = 1, n) acting perpendicular to a line from the pivot to the point of action.

At the fixed (stationary) pivots of the bars shown in Figures 7.11 and 7.12, a force must act to the left to balance the forces shown acting to the right. The force acting on the pivot acts at zero distance from the pivot and does not generate any torque. In the solution of a problem involving static equilibrium, this feature of torque provides a (hidden) constraint, and the balanced condition on torque *should be applied first*.

For homework problems where a given set of masses is attached to a bar in static equilibrium, the application of Equations 7.5, can resolve all the forces and torques acting on the object. If one (or both) of the equations is not satisfied (not zero) then a stationary object must *begin* to move in the direction of the net force, or *begin* to turn in the direction of the net torque, or (if both are non-zero), *begin* to move and turn.

Chapter Summary:

- An stationary mass or one moving with a constant speed and direction will have no net force, $\mathbf{F}_{\text{net}} = 0$, acting on it.
- A mass with a changing speed or direction has a net force acting on it, $\mathbf{F}_{\text{net}} \neq 0$.
- On a stationary mass, a force applied parallel to the horizontal mating surfaces, generates a balancing static frictional force, F_{f}, pushing back on the mass.
- The static frictional force will immediately become zero and the object will begin to slide if the magnitude of the applied force is larger than a critical value, F_{crit}.
- The critical force to slide is given by, $F_{crit} = \mu C$, where μ is the coefficient of friction and C is the compression force between the surfaces in contact.
- Torque and force are separate quantities with different units. You cannot add a torque to a force.
- The *magnitude* of the torque, τ, caused by a force F applied at 90° to a line of length r drawn from the pivot to the point the force is applied is $\tau = Fr$.
- The *magnitude* of the torque is zero for a force applied at 0° to a line drawn from the pivot to the point the force is applied.
- The direction of a torque vector, $\boldsymbol{\tau}$, is positive if it tends to rotate an object clockwise and negative if it tends to rotate an object counter-clockwise.
- For static equilibrium, the general conditions on the forces applied to an object are:

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n = 0, \quad \text{and}$$
$$\boldsymbol{\tau}_{\text{net}} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots + \boldsymbol{\tau}_n = 0,$$

where $\tau_i = F_i r_i$ for forces applied at 90° to a line of length r drawn from the pivot to the point where the force is applied, and $\tau_i = 0$ for forces applied at 0° to the line.