Chapter 9: Transfers of energy between *PE* and *KE*

A Transfer of *PE* to *KE*.

A mass, m, held against a ideal spring is released just as the top picture of Figure 9.1 is taken, showing only the spring's unbalanced compression force, C_0, acting on the mass. Between the times shown at the top and bottom of Figure 9.1, the compression force causes the potential energy stored by the spring to be transferred to the motion (kinetic) energy of the mass.

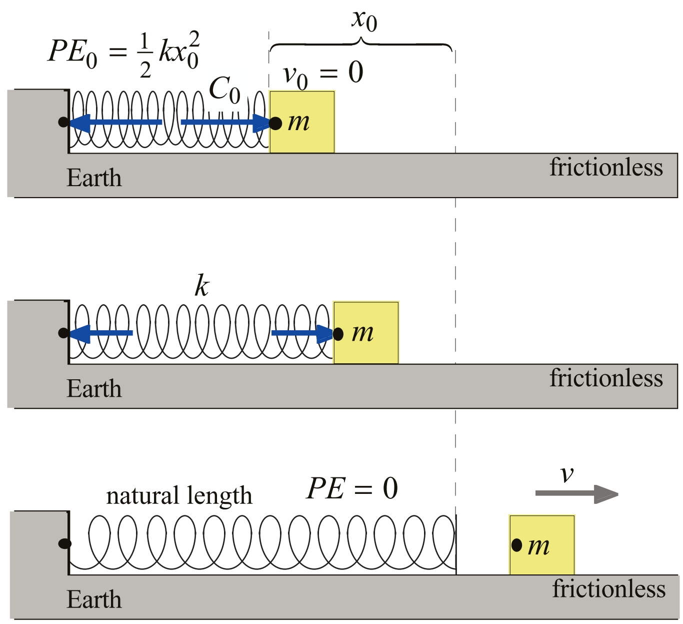

Figure 9.1 A compressed spring expands against a stationary mass and propels it to a speed v, giving the mass *KE*.

The action of the spring does not move the very massive earth, and therefore the spring does no work on the earth and no energy is transferred to it. When reaching its natural length, the spring looses contact with the mass, m, and *all* potential energy stored by the spring has been transferred to the *kinetic energy* (*KE*) of the mass. The expansion details, such as the time it takes to expand, does not affect the amount of energy transferred.

The dependence of the kinetic energy on the mass and its speed can be inferred from an earlier discussion. In Chapter 3, section B, it was stated that a spring constant, k, can be calibrated by oscillating a known mass, m (in kg), on the spring and measuring the oscillation period, t (in seconds): $k\,(\text{in N/m}) = m/(t/2\pi)^2$ (in kg/s^2). A spring constant, k = 1 N/m, expressed in mass and time units becomes, $k = 1\ \text{kg/s}^2$. A spring with a spring constant, k = 1 N/m, stretched by $x = 1$ m, will store a potential energy:

$$PE = \tfrac{1}{2}kx^2 = 0.5\ \text{N}\cdot\text{m} \quad (\text{using } k = 1\ \text{N/m, and } x = 1\ \text{m})$$

$$= 0.5\ \text{kg}\cdot\text{m}^2/\text{s}^2 \quad (\text{using } k = 1\ \text{kg/s}^2\text{, and } x = 1\ \text{m}).$$

The quantity, $0.5\ \text{kg}\cdot\text{m}^2/\text{s}^2$, has the units of a mass (kg) times a speed squared (m^2/s^2). The potential energy of this spring and the kinetic energy of a mass, m = 1 kg, traveling at a speed, v = 1 m/s, are equal if the expression for the kinetic energy is:

$$KE = \tfrac{1}{2}mv^2. \tag{9.1}$$

Work, potential energy, and kinetic energy have equivalent units,

$1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 1\ \text{J}$ (joule), and usually each is specified in joules. Note for later reference that this equivalence of units means that the newton unit of force can be expressed as, $1\,\text{N} = 1\,\text{kg}\cdot\text{m/s}^2$, a mass times its acceleration.

A typical application of the kinetic and potential energy concepts is the experiment shown in Figure 9.1, where the speed, v, of the mass when it leaves the spring can be predicted, given the mass, m, the spring constant, k, and the initial compression, x_0 (initial values, at time $t = 0$, are identified with a subscript of zero; final values are without subscript or with a subscript of a number other than zero)

When the spring has transferred all of its potential energy to the mass, it becomes free of the spring. The final speed of the mass is obtained by equating the *initial* potential energy of the compressed spring with the *final* kinetic energy of the mass. The solution for the *final* speed v of the mass is obtained with just a little algebra:

$$KE(\text{final}) = PE(\text{initial}); \quad \text{where } KE = \tfrac{1}{2}mv^2, \text{ and } PE = \tfrac{1}{2}kx^2$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_0^2$$

$$v^2 = \tfrac{k}{m}x_0^2 \quad \text{and} \quad \underline{v = \sqrt{\tfrac{k}{m}}\,x_0} \quad ,$$

where the equivalent units, kg/s^2, for k are used to obtain a speed in the units, m/s.

Measurements of the speed of the mass for physical springs (nearly ideal) agree closely with these predictions, confirming the assumption of complete energy transfer. A moving mass that hits a spring can transfer its *KE* to potential energy by compressing a spring. Transfers of kinetic to potential energy are investigated in the next section.

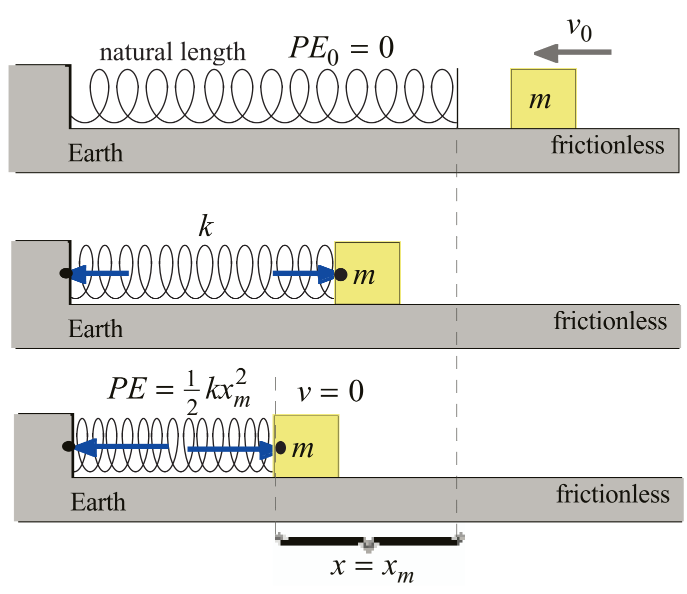

Figure 9.2 A mass with a speed, v, collides with and compresses a spring transferring its *KE* to the *PE* of the spring.

B. Transfer of *KE* to *PE*.

A mass, m, with a speed, v_0, and, therefore, a kinetic energy, $\tfrac{1}{2}mv_0^2$, moves toward an ideal spring, attached to the earth as shown in Figure 9.2. The spring begins to compress on contact with the mass and continues to compress until the spring reaches maximum compression, $x = x_m$, where the speed of the mass has been reduced to zero.

As the mass moves to the left $(-)$, the *unbalanced* compression force of the spring acting to the right(+) does work (negative) on the mass. The work done by the spring's force absorbs the *KE* of the mass transferring it into the spring's *PE*. When the spring reaches the maximum compression, x_m, the initial kinetic energy of the mass is completely transferred to the potential energy of the spring. The earth does not move so no energy is transferred to it. The maximum compression can be determined by equating the *initial* kinetic energy of the mass to the *final* potential energy of the spring:

$$PE(\text{final}) = KE(\text{initial}); \quad \text{where } PE = \tfrac{1}{2}kx_m^2 \text{ and } KE = \tfrac{1}{2}mv_0^2$$

$$\tfrac{1}{2}kx_m^2 = \tfrac{1}{2}mv_0^2$$

$$x_m^2 = \tfrac{m}{k}v_0^2 \quad \text{and} \quad \underline{x_m = \sqrt{\tfrac{m}{k}}\,v_0} \quad .$$

<u>C. Partial transfers of *KE* and *PE* and conservation of total energy.</u>

The entire energy in one object (a spring or mass) was transferred to the other object in the two cases given above. At any intermediate stage in the expansion or compression only a partial transfer of energy occurs between the two objects. After a partial transfer of energy, the speed, v, of the mass and the compression distance, x, of the spring have intermediate values and, therefore, both the kinetic energy of the mass and the potential energy stored in the spring will also have intermediate values. Subscripts are needed to distinguish the initial values from the final values of speed and compression.

The partial transfer of an *initial* kinetic energy of the mass, KE_0, to the potential energy of a spring, with a zero initial value, $PE_0 = 0$, is shown in Figure 9.3. The mass hits the spring and from the no initial compression, $x_0 = 0$, it compresses a distance that reaches the value, x, where the speed of the mass has been reduced from the initial value, v_0, to the final value, v. The partial compression by a distance x, and the speed v, can be related using energy transfer considerations. The spring continues to be compressed while the mass is still in motion and neither the potential energy of the spring nor the kinetic energy of the mass are zero.

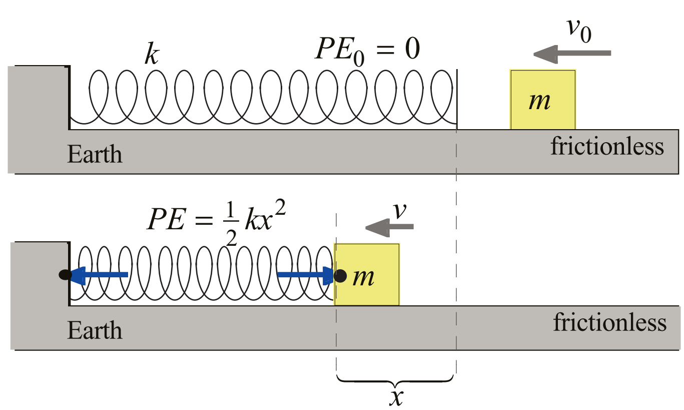

Figure 9.3 Partial transfer of kinetic energy to the potential energy of a spring

This and other similar situations are handled by observing that at any specified location of the mass the total energy, E_T, the *sum* of the potential energy in the spring and the kinetic energy of the mass, does not change. The only thing that happens in the *interaction* of the mass with the spring is the transfer of kinetic energy from the mass to the spring's potential energy or vise-versa. If the energy of one object (mass or spring) increases, the energy of the other object (spring or mass) must decrease by the same amount. The total energy E_T remains the same throughout the energy transfer.

This preservation of the total energy, E_T (at any specified time, the sum of all kinetic and potential energies), is a property of interactions that involve only conservative forces: the forces of an ideal spring and gravity, but not friction, human forces, or an explosion. This relationship is symbolically stated in Equation 9.2 and is referred to as the law of conservation (equivalence over time) of energy:

$$E_T = KE_0 + PE_0 = KE + PE \text{ (at any specific time)}, \qquad (9.2)$$

where *PE* is the energy stored by conservative forces (ideal spring or gravity).

We can now use conservation of energy to attack any situation that involves the kinetic energy of masses and the potential energy stored in springs. An example, shown above in Figure 9.3, is to determine the residual speed, v, of the mass when the spring (attached to the earth) has been compressed by an amount, x, that is less than the maximum. The solution is obtained by first noting that the initial values (before the mass collides with the spring) of the kinetic energy of the mass and the potential energy of the spring, using a subscript, 0, for initial values, are:

$$\text{(mass) } KE_0 = \tfrac{1}{2}mv_0^2; \quad \text{(spring) } PE_0 = 0 .$$

When the spring has been compressed by an amount, x, the final values (no subscript) of the kinetic energy of the mass, moving with the speed, v, and the potential energy of the spring, are:

$$KE = \tfrac{1}{2}mv^2; \quad PE = \tfrac{1}{2}kx^2.$$

Conservation of energy, Equation 9.2, equates the total energy (the sum of the potential and kinetic energies of the spring and mass, respectively) at the two times,

$$KE_0 + PE_0 = KE + PE$$
$$\tfrac{1}{2}mv_0^2 + 0 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2.$$

This can then be solved for the unknown final speed, v, for any final compression, x:

$$v_0^2 - v^2 = \frac{k}{m}x^2$$

$$v^2 = v_0^2 - \frac{k}{m}x^2 \quad \text{and} \quad \underline{v = \sqrt{v_0^2 - \frac{k}{m}x^2}}\ .$$

A similar expression relating the square of the initial and final speeds of a mass will reappear when the details of accelerated motion are discussed.

D. Adding a constant to all potential energies.

In an interaction of a mass with a spring, conservation of total energy requires that an increase in the kinetic energy of the mass be accompanied by a decrease in the potential energy of the spring by the same amount, and vise - versa. These changes in the kinetic and potential energies are calculated by subtracting the initial values from the final values and are written as, $\Delta KE = KE - KE_0$, and $\Delta PE = PE - PE_0$, respectively. The conservation of total energy, Equation 9.1, is written in terms of these changes as:

$$\Delta KE + \Delta PE = 0. \tag{9.3}$$

This very succinct expression shows that any kinetic energy change, ΔKE, must be accompanied by an opposite potential energy change, ΔPE, by the same size.

A natural place to take for the zero of potential energy for a spring is its natural length. It is not necessary, however, to use this definition and the zero may be shifted by any arbitrary amount without changing the predictions for the motion. When an arbitrary amount of energy, ε, is added to both the initial and final potential energies of the spring, the change in the potential energy is unaffected (the ε terms will cancel):

$$\Delta PE = PE - PE_0 = \left(\tfrac{1}{2}kx^2 + \varepsilon\right) - \left(\tfrac{1}{2}kx_0^2 + \varepsilon\right)$$
$$= \tfrac{1}{2}kx^2 - \tfrac{1}{2}kx_0^2\ .$$

If the zero of potential energy can be chosen arbitrarily then the value of the total energy , E'_T , is also arbitrary:

$$E'_T = KE + (PE + \varepsilon) = E_T + \varepsilon.$$

Conservation of energy, however, says nothing about the original amount of energy available, only that whatever total energy exists at the start of the motion is the same total energy that will be available throughout. The motion is determined solely by the transfer of energy between *KE* and *PE* while maintaining their sum a constant.

This is a general property of conservative forces and their associated potential energy. It allows the zero of potential energy to be moved to the most useful location. It is often useful, if gravity is involved in the problem, to shift the zero of gravitational potential energy to a more logical location. We will address this again when discussing gravitational potential energy.

E. Internal forces experienced by masses interacting with springs.

The energy considerations of the previous sections have shown that a mass traveling with an initial speed has a kinetic energy and that this energy can be transferred to the potential energy of a spring if the mass compresses a spring. This process seems, at first, to contradict a statement made earlier: it takes two forces to compress a spring. At any stage in the compression of a spring, however, the outward forces of the spring acting on the masses must be accompanied, as required by Newton's 3rd Law, by equal magnitude and opposite direction forces acting inward on the spring,.

Compression forces act at the boundary between each mass and the spring compressed between them, as shown in Figure 9.4. The compression force of the spring

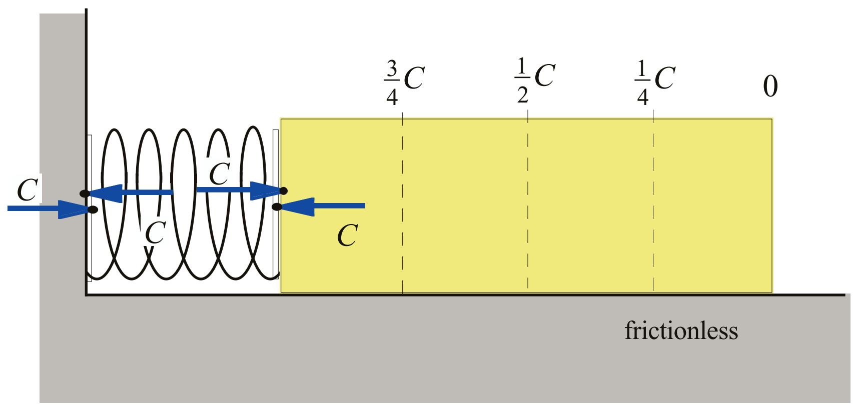

Figure 9.4 Forces acting at the boundary between a moving mass and a compressed spring.

acts on and affects the motion of the mass: this force will decrease the speed of the mass (remove kinetic energy) if the mass is moving toward the spring, and will increase the speed of the mass (add kinetic energy) if the mass is moving away from the spring. At the point of contact, the value compression force generated in the mass, by Newton's 3rd Law, is equal to the compression force of the spring acting on the mass.

The magnitude of the compression force at any particular location within the mass can be determined by noting the spring's compression from its natural length. The motion of the mass will quickly turn around if the spring is strong but if the spring is weak the spring must compress significantly to transfer the kinetic energy of the mass to the potential energy of the spring and then back to the mass as it moves away from the wall.

The compression forces generated at each point within the mass are also indicated in Figure 9.4. At the free end of the mass there is no external force and no internal compression force at that point, as required by Newton's 3rd law. The compression force within the mass must drop from the value of the spring compression, *C*, on the side touching the spring, to a value of *C*/2 at the midpoint, and reaches zero at the free end of the mass. The compression force at any point within the mass is only as large as is needed to remove (or add) kinetic energy from (to) the mass to the right of that point. (only a small compression force is needed to change the kinetic energy of the small amount of mass near the free end).

If the spring compresses far enough (the mass has sufficient initial kinetic energy) that the compression force on the mass has same magnitude as the weight of the mass, $W = Mg$, then the magnitude of the compression forces within the mass will be identical to the compression forces caused by gravity acting on the mass supported by the spring in the vertical direction, as shown in Figure 9.5, and discussed in Chapter 6. The equivalence of the internal forces acting within a mass in these two situations is the basis for the generation of "artificial gravity" in a spinning space station and other curious effects.

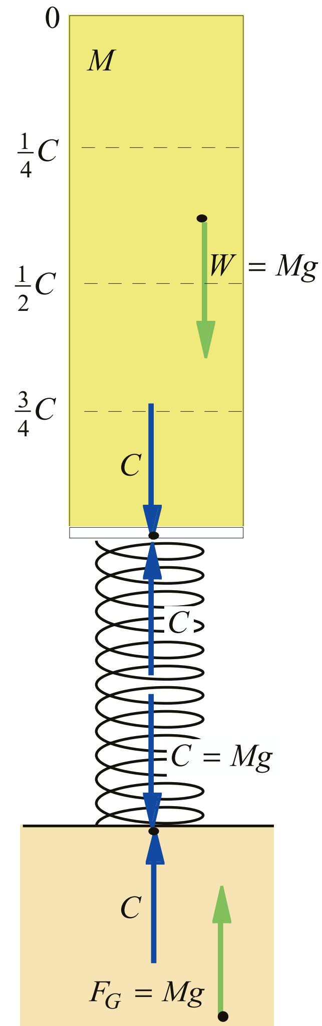

Figure 9.5 Compression forces within a mass supported by a spring.

The compression force of the spring is paired with an opposing compression force, also with the magnitude, $C = Mg$, generated by compression of the mass at the point where it touches the spring. At any other point within the mass the magnitude of the compression force is the weight of the mass above that point. Halfway up the compression force is, $C/2$, one half the weight, and at the top of the mass the compression force is zero.

The next chapter will introduce gravitational potential energy and apply energy techniques to describe the motion of objects near the surface of the earth. If only the motion of the mass is of interest then the transfer of its kinetic energy to the potential energy of a spring, or the reverse, is easier to follow than is the more complicated discussion of the forces acting within the spring and in the interior of the mass. The existence of these elastic action and reaction forces, however, cannot be avoided when explaining the perception of forces by human beings.

Chapter Summary:

- The energy of motion is kinetic energy (*KE*); $KE = \frac{1}{2}mv^2$ for a mass *m* with speed *v*.
- The *PE* in a spring (attached to the earth) can be completely transferred to the *KE* of a mass and vise-versa.
- The total energy, E_T, of a mass interacting with a spring (attached to the earth) is the sum of the *KE* of the mass and the *PE* of the spring, $E_T = KE + PE$, and is a conserved quantity.
- The conservation (equivalence over time) of the total energy in an interaction implies that the initial value, ($KE_0 + PE_0$), and the value at all other times, ($KE + PE$), must be the same: $KE_0 + PE_0 = KE + PE$.
- The changes in kinetic and potential energies, $\Delta KE = KE - KE_0$, $\Delta PE = PE - PE_0$, must sum to zero, $\Delta KE + \Delta PE = 0$, if the total energy is conserved.
- The conservation (equivalence over time) of the total energy is unaffected if a constant is added to a *PE* expression, however, the total energy will always exhibit this additional energy.