**Chapter 10: Gravitational potential energy**

A. Gravitational potential energy near the surface of the earth.

The forces of gravity and an ideal spring are conservative forces. With only the forces of an ideal spring and gravity acting on a mass, energy will be exchanged between the potential kinetic forms of energy. The sum of the potential energies (spring and mass) and kinetic energy (mass) will remain the same during transfers of energy between the potential and kinetic forms.

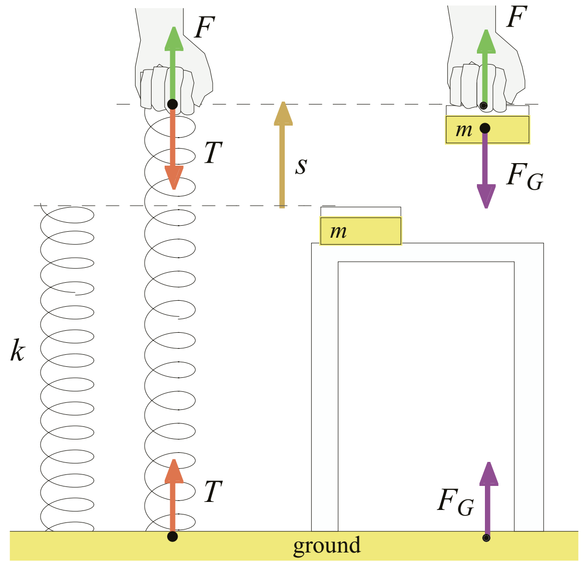

*Figure 10.1* Potential energy introduced into a spring and a mass. Forces acting downward on the earth are not shown.

Human action is *not* a conservative force. The work done by the human body, whether positive (adding energy) or negative (removing energy), will increase or decrease the total energy available, respectively. The human body generates forces by releasing chemical energy (stored by the electromagnetic forces in molecules) to contract muscles. These chemical reactions cannot be reversed to store mechanical energy; e.g., the kinetic energy of a baseball when caught cannot be stored as potential energy by the human body for later use.

Human action can, however, increase or decrease the potential energy stored by a spring or stored by a mass (due to gravity), as shown in Figure 10.1. As described in chapter 9, the work done by the human stretching the spring a distance, $s = x$, from its natural length, stores a spring potential energy,

$$PE_S = \frac{1}{2}kx^2.$$

Slowly lifting a mass, $m$, a distance, $s$, as shown on the right in Figure 10.1, an external human force, $F = mg$, does an amount of work, $w = Fs = mgs$, and the gravitational force stores an equal amount of potential energy in the mass. Defining the gravitational potential energy of the mass to be zero at the height of the tabletop, after being lifted to the height, $s = h$, above the table, the potential energy of the mass (due to gravity) is,

$$PE_G = mgh \qquad \text{Eq. 10.1}$$

Gravitational potential energy and kinetic energy, are both properties of a mass. The gravitational force can transfer the potential energy of the mass to kinetic energy of the same mass. For gravity, the potential energy does not reside in an intermediate object, like it did for the potential energy stored by a spring being transferred to a mass.

B. Transfers of gravitational potential energy, $PE_G$, to $KE$, near the surface of the earth.

Once a mass, raised to an initial height $h_0$, is released, as shown in Figure 10.2, the gravitational force acting on the mass makes it fall. In the process gravity does work on the mass transferring its potential energy to its kinetic energy. As the potential energy decreases the kinetic energy of the mass increases.

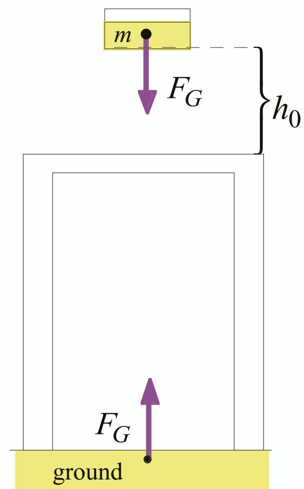


*Figure 10.2* A mass just released at a height, $h_0$ , from a table.

The conservative gravitational force makes no change in the total energy,

$$E = KE + PE,$$

where, $PE = PE_G$, if gravity is the only force involved (subscript, $G$, removed to simplify notation). The total energy will remain constant, but the energy in the two components, the potential energy, $PE$, and the kinetic energy, $KE$, changes as the mass falls.

The mass, when released at $t = 0$, has the gravitational potential energy, $PE_0 = mgh_0$ and a kinetic energy, $KE_0 = 0$ (the mass has the speed $v = 0$ when released). The total energy at the time of release,

$$\begin{aligned} E_0 &= KE_0 + PE_0 \\ &= 0 + mgh_0 \end{aligned}$$

will have this value at any other time during the fall. Just prior to hitting the tabletop, the mass has lost all of the gravitational potential energy (given to it by lifting), so that, $PE = 0$, while gaining the kinetic energy, $KE = \frac{1}{2}mv^2$. The total energy just prior to hitting the table is, therefore,

$$\begin{aligned} E &= KE + PE \\ &= \tfrac{1}{2}mv^2 + 0. \end{aligned}$$

The total energy at release and just before hitting the table must have the same value,

| if only conservative forces (gravity and springs) are involved<br>$E = E_0$, | Eq. 10.2 |
|---|---|

and allows a prediction for the speed at the time the mass hits the table:

$$E = E_0$$

$$\tfrac{1}{2} mv^2 = mgh_0$$

$$v^2 = 2gh_0, \qquad \underline{v = \sqrt{2gh_0}} \quad .$$

The units of $g$ in this expression must use the equivalent unit of force, $1\text{N} = 1\text{kg}\cdot \text{m/s}^2$ (see Chapter 9) to make the conversion: $g = 9.81$ N/kg, to $g = 9.81\ \text{m/s}^2$. It will be shown later that $g$ has an acceleration interpretation as well as one relating mass and weight, and a similar expression for the speed of an object moved a distance under a constant force and acceleration will appear in a later chapter.

This analysis seems to ignore the possibility that the mass could fall off the table. In contrast to a spring where a natural length spring is the clear choice for the zero of a spring's potential energy, gravity does not have a unique state that is logically defined as the zero of gravitational potential energy. The proper place for the gravitational potential energy to be zero must be chosen for each situation and some training is needed to make a wise choice.

C. The zero of potential energy.

Near the surface of the earth, the gravitational potential energy of a mass will be a minimum when the separation of the mass from the center of the planet is the smallest allowable value. If a mass could never fall off a table, the logical place for zero gravitational potential energy is at the surface of the table. If the mass can fall to the ground, a more logical choice for a state with zero potential energy is at the surface of the earth. If the mass can fall to the bottom of the basement (below ground level), the basement floor becomes the logical place to define as the location with zero potential energy.

The potential energy must decrease for a mass carried into a basement. The potential energy of the mass, taken as zero on the surface of the earth, is *negative* at any point lower than the surface, as shown in Figure 10.3.

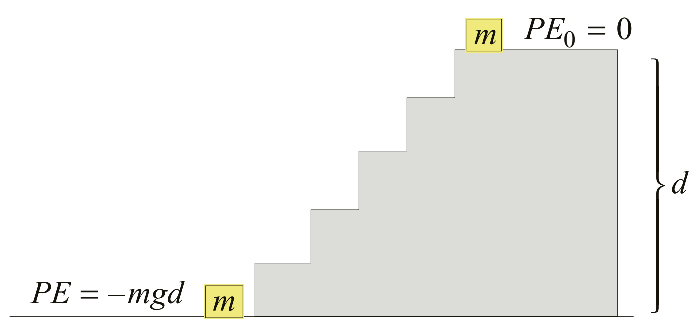


*Figure 10.3* A mass is carried into a basement a distance, *d*, below the surface, changing *PE* from 0 to –*mgd*.

Only changes in potential energy, $\Delta PE$, are constrained by the conservation of energy: $\Delta KE + \Delta PE = 0$, (see previous chapter). Changes in the potential energy of a mass are insensitive to the height at which the potential energy is defined to be zero. Brought up from the basement the potential energy starts with a negative value and is zero at the surface resulting in a potential energy change that is positive.

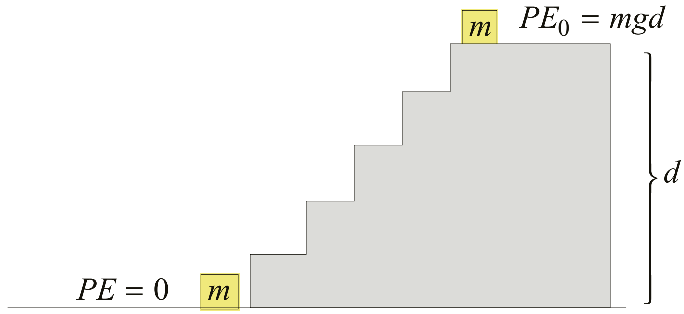


*Figure 10.4* Defining *PE* to be zero in the basement.

Defining the zero of potential energy the basement, as shown in Figure 10.4, the potential energy of the mass, raised from the basement to the earth's surface, starts at zero and becomes higher at the surface, resulting in a change in potential energy that is again positive.

D. Energy conservation in the action of springs and gravity

There are many phenomena that are the result of the action of just the two conservative forces, gravity and an ideal spring. A mass fired upward by a compressed spring, as shown in Figure 10.5, is an example of a process involving only conservative forces. The process can be thought of as taking the potential energy stored in a spring, transferring it to the kinetic energy of the mass. When the mass reaches its highest point, where the speed, *v*, is zero, the energy has been completely transferred to the gravitational potential energy of the mass.

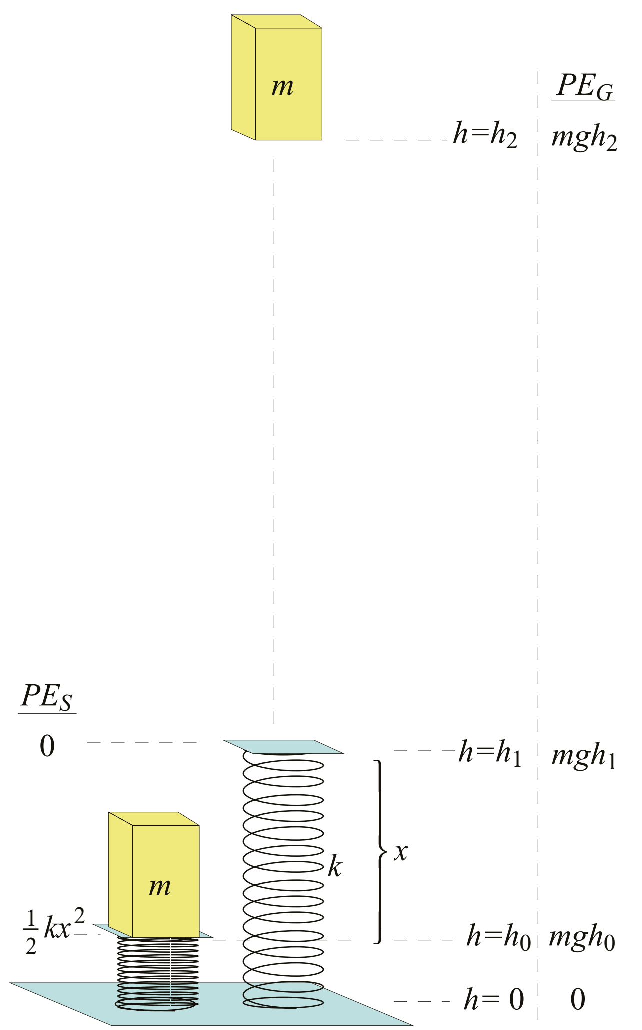


Figure 10.5 A compressed spring firing a mass upward.

A detailed analysis shows how the speed and height of the mass are related to the initial energy stored in the spring. The mass, *m*, is placed on the spring compressed a distance, *x*, and then released. The initial (the time of release) values of the kinetic energy and the two potential energies are:

$$KE = 0 \quad \text{(mass starts with } v_0 = 0\text{)},$$
$$PE_S = \tfrac{1}{2}kx_0^2 \quad \text{(spring } PE\text{), and,}$$
$$PE_G = mgh_0 \quad \text{(gravitational } PE\text{).}$$

The total energy at the initial time is the sum of these:

$$E_0 = (KE + PE_S + PE_G)_0$$
$$= 0 + \tfrac{1}{2}kx_0^2 + mgh_0$$

The only forces acting on the mass are the spring force and the gravitational force, both conservative forces, which will conserve energy through the flight of the mass. When the mass reaches the height, $h = h_1 = h_0 + x_0$, it will loose contact with the spring, have a speed, $v_1$, and the spring will have transferred all of its potential energy, $PE_S = 0$, to the kinetic and potential energy of the mass. The value of the total energy at this time is then:

$$E_1 = (KE + PE_S + PE_G)_1 = \tfrac{1}{2}mv_1^2 + mgh_1$$

and must have the same value as the initial value, $E_1 = E_0$. Equating the two expressions of the total energy yields a prediction for the speed, $v_1$:

$$E_1 = E_0$$
$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}kx_0^2 + mgh_0$$
$$v_1^2 = \frac{k}{m}x_0^2 - 2g(h_1 - h_0) \qquad (h_1 - h_0 = x_0)$$
$$v_1 = \sqrt{\frac{k}{m}x_0^2 - 2gx_0}\ .$$

When the mass reaches the highest point, $h = h_2$, the speed and kinetic energy of the mass will be zero (it would move higher if the speed were not zero). The gravitational potential energy, $PE_G = mgh_2$, will be a maximum, and the total energy will be

$$E_2 = (KE + PE_S + PE_G)_2 = 0 + 0 + mgh_2.$$

A prediction for the highest point reached by the mass is obtained by equating the initial and final values of the total energy:

$$E_2 = E_0$$
$$mgh_2 = \tfrac{1}{2}kx_0^2 + mgh_0$$
$$h_2 = \frac{kx_0^2}{2mg} + h_0\ .$$

The conservation of the total energy has lead to predictions of the speed and height of the mass at critical points in its flight in terms of the initial conditions.

E. The effects of work by non-conservative forces on a mass

In this section the energy conservation concept is modified to include the effects of non-conservative forces. Instead of a spring firing a mass, as shown in Figure 10.5, the mass is now lifted slowly with your hand (so that it never leaves your hand) from the initial height, $h_0$, up to the height, $h_2$, increasing the gravitational potential energy of the mass by,

$$\Delta PE_G = mgh_2 - mgh_0 = mg(h_2 - h_0).$$

This motion involves the introduction of energy by the non-conservative force of a human being.

The force of the hand is larger than the weight only for a short time at the start of the upward motion and, must be slightly less for a short time at the end of the motion. The magnitude of the force that your hand applies to the mass is equal to its weight as the mass moves upward with a constant speed, as shown in Figure 10.6.

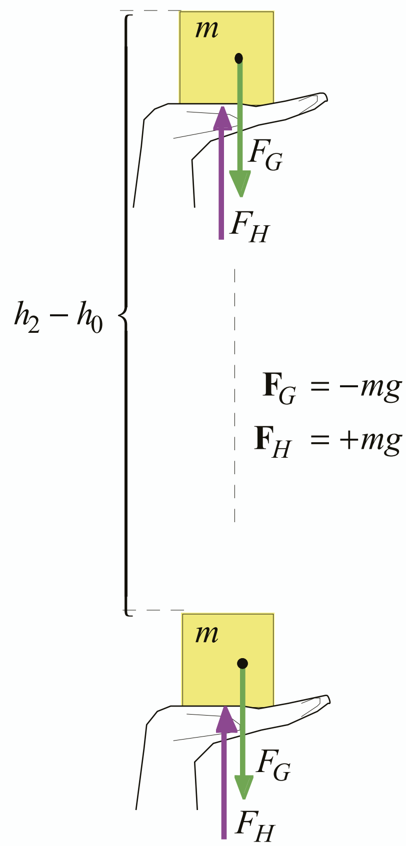


*Figure 10.6* Forces on a mass being raised by hand.

The total energy of the mass at the final height (the speed is zero) is entirely in potential energy. The amount of work done by the human ($w_H$) in lifting the mass from $h_0$ to the height, $h_2$, is $w_H = mg(h_2 - h_0)$.

Throwing a mass upward to a maximum height, $h_2$, and lifting the mass slowly to the same height, both involve a transfer of the same amount of energy from the human being to the mass. In throwing, energy is introduced rapidly into kinetic energy of the mass, while in lifting, energy is introduced slowly, however, when reaching the maximum height, the potential energy of the mass is the same for both motions

In free flight, only gravity (a conservative force) affects the motion of a mass, and therefore the total energy of the mass, the sum of its kinetic and potential energies, is conserved. When a human slowly raises a mass, the potential energy of the mass is increased with negligible changes in its kinetic energy. Work done by humans, $w_H$, and other non-conservative forces, such as sliding friction, require special treatment to incorporate their effects on total energy of a mass. Energy conservation, incorporating non-conservative forces can be written as,

$$KE_0 + PE_0 + w_{NC} = KE + PE, \tag{10.3}$$

where $w_{NC}$ represents the work done by humans, and also includes the effects of the forces of friction or an explosion on the available energy. Rearranging Equation 10.3 to show changes in potential and kinetic energies of the mass, yields,

$$w_{NC} = \Delta KE + \Delta PE. \tag{10.4}$$

The term labeled $\Delta PE$, includes changes in potential energies of springs, $PE_S = \frac{1}{2}kx^2$, and of a mass, $PE_G = mgh$, due to gravity. If the changes in kinetic and potential energies of a mass do not sum to zero, energy must have been added or removed by a non-conservative force. When a human being does a positive amount of work on a mass, as when lifting it, Equation 10.4 predicts an increase in the energy of the mass. No energy is lost or created in the process. Energy is transferred from the human to the mass.

The term $w_{NC}$ in Equation 10.4 corresponds to changes in the energy of the mass on which the non-conservative force acts. For friction, the energy change is nearly always negative and $w_{NC}$ corresponds to the energy that appears as heating the objects in contact. For explosions, the energy change is positive and $w_{NC}$ corresponds to the energy of the explosion added to the motion of the masses it has affected. For cases in which both humans and friction are involved, there are two $w_{NC}$ terms, one for each force. Consider a mass pushed at a constant speed by a human on a friction-generating table. As the mass moves, no changes occur to the kinetic energy (or potential energy) so that what ever energy the human adds by doing work, the frictional force removes by heating the surfaces: $\left(w_{NC}\right)_{human} = -\left(w_{NC}\right)_{friction}$.

The human body is very inefficient in converting stored food energy into work on masses, and much of the energy transfer in this process instead heats the body. Any change in the body heat is accompanied by an equal reduction in the food energy stored by the human. When a human slowly lowers a mass (work is negative), the potential energy of the mass decreases, while the body heat increases. Again, inefficiencies in the human body require additional food energy to be used that appears as heat in the body.

The interchange of heat and work is the subject of an entire course in thermodynamics. Here, it will suffice to say that once energy is transferred to heat, some additional heat must be generated in any attempt to utilize that energy to do work. This feature of nature, known as the second law of thermodynamics, prevents the construction of machines in perpetual motion with no outside energy source.

Chapter Summary

- The gravitational force, $F_G$, is a conservative force and any work it does on a mass changes the gravitational potential energy, $PE_G$, of the mass. For a mass, $m$, near the surface of the earth at a height $h$ above the ground, the mass has a gravitational potential energy, $PE_G = mgh$, in units, $\mathrm{N \cdot m}$, or equivalently, in joules (J).
- The height at which the gravitational potential energy, $PE_G$, is zero can be adjusted at the beginning of a problem, by defining the location of the height, $h = 0$, to fit the situation being addressed (usually the lowest point of the motion).
- If only conservative forces act on a mass, then the total energy of the mass will be conserved: $KE_0 + PE_0 = KE + PE$, or equivalently, $\Delta KE + \Delta PE = 0$.
- Near the surface of the earth, the work, $w_H$, done by a human being (or by other non-conservative forces), on a mass will result in a corresponding change in the sum of the kinetic and potential energies of the mass, $w_H = \Delta KE + \Delta PE$ .
- When a mass is lifted (or lowered) slowly ($\Delta KE = 0$) by a human, the work done on the mass is equal to the change in the gravitational potential energy of the mass, $w_H = \Delta PE_G$. When a mass is lowered slowly the change in the gravitational potential energy of the mass (and the work done by the human) will be negative, and will appear as an increase in the heat energy of the human body.