Chapter 12: Velocity, acceleration, and forces

A. States of motion

For motion on or near the surface of the earth, it is natural to measure motion with respect to objects fixed to the earth. The 24 hr. rotation of the earth has a measurable but very small effect over short times or distances, while the effects of the motion of the earth around the sun can safely be ignored. A mass is said to be "at rest", "stationary", or "not moving", *only* if its position with respect to the earth remains the same over a time interval. A mass is said to be "in motion", or "is moving" if its position changes over a time interval. The measurement of the position of the object at a single time cannot determine if the object is at rest or moving. For an object to be at rest between two times, the position must be the same at all intermediate times. Newton's 1st law states that under the action of balanced forces, a mass will remain at rest or move with a constant speed and direction. Newton's 3rd law, describing the existance of an action – reaction pair of forces at a point of contact between two masses, is true in all states of motion: at rest, moving with constant speed and direction, or moving with a changing speed or direction.

B. Characteristics of motion

Kinetic energy and momentum are measures of the motion of a mass and use in their definitions the "instantaneous" velocity, $\mathbf{v}$, with a speed, v, and direction (+ or –). Everyone is familiar with speed, and knows that a constant speed can be measured by taking the traveled distance (a positive scalar) and dividing by the time of travel, $v = s/t$. The velocity is obtained by adding a sign in front of the speed, v, following the same convention for direction as used for displacement.

Formally, for velocities that could be changing, the average velocity is defined as:

$$\langle \mathbf{v} \rangle = \frac{\mathbf{s}}{t}, \qquad (12.1)$$

where the displacement vector, $\mathbf{s}$, is the same as the one used in the definition of work ($w = \mathbf{F} \cdot \mathbf{s}$).

A constant velocity is not very interesting; the net external force acting on the mass is zero. When the velocity changes over time, however, we are observing the effects of a non zero net external force. Measuring the value of the average velocity, as given by Equation 12.1, over a long time, t, averages over everything that happened to the object, and does not give a good picture of what effect the force is having on the object at any specific time. The motion at any specific time (or position), is called the "instantaneous" velocity, $\mathbf{v}$, and can be determined using Equation 12.1 in the limit of very small t. A statement that $\mathbf{v} = 0$ at a single time (or position), however, gives no evidence for, or against, the action of a net external force. Measurements of local changes in $\mathbf{v}$ determine the presence of a net external force on an object.

C. Effects of an net external force.

The internal tension or compression forces on the masses at the ends of an ideal spring, the gravitational forces between two masses, and the frictional forces between masses in contact, are examples of action - reaction force pairs. The definitions work, kinetic and potential energy, momentum and the conservation laws that govern their behavior were obtained by considering the action of the internal force pairs of an ideal spring on objects with mass. When one mass was small and the other was very large (the mass of the earth) the predictable effects of the elastic force on the large mass allowed us to infer a law for the conservation of energy. The effects of internal forces on finite masses at both ends of a spring allowed us to infer a law for the conservation of momentum.

A force is called an "external force" if the other member of the action - reaction force pair is not considered. Considered in isolation, the gravitational force on a small mass m near the surface of the earth is an external force with a constant value $F_G = mg$ (its weight). A force of this magnitude also acts on the earth in the opposite direction, but the effects of this small force on the motion of the earth can safely be ignored. This allows us to use the earth as a point of reference for the motion of the small mass.

D. Newton's 2nd law of motion: effects of a constant external force on a mass.

Newton discovered that there were two properties of motion affected by a net external force: kinetic energy, and momentum. An external force changes the kinetic energy of a mass traveling a distance, and an external force changes the momentum of a mass traveling for a period of time. The momentum change is equal to the product of the average force and the action time, $\mathbf{p} = \mathbf{p}_0 + \langle \mathbf{F} \rangle t$, where momentum, $\mathbf{p} = m\mathbf{v}$. Therefore, the relationship between the average force, the mass, and the change in velocity is

$$\langle \mathbf{F} \rangle t = \mathbf{p} - \mathbf{p}_0 = m(\mathbf{v} - \mathbf{v}_0)$$

$$\langle \mathbf{F} \rangle = m\frac{(\mathbf{v} - \mathbf{v}_0)}{t}.$$

The observed change in the velocity of a mass, divided by the time of the motion, is called the average acceleration, $\langle \mathbf{a} \rangle = (\mathbf{v} - \mathbf{v}_0)/t$. If the *net* force acting on a mass doesn't change, then $\langle \mathbf{F} \rangle = \mathbf{F}$, and $\langle \mathbf{a} \rangle = \mathbf{a}$, resulting in the most famous of Newton's discoveries,

$$\mathbf{F} = m\mathbf{a}. \qquad (12.2)$$

If there is more than one external force acting on a mass then its acceleration will be proportional to the *net force* obtained as usual by a vector sum.

E. Acceleration due to gravity near the surface of the earth

The gravitational force on a mass m near the surface of the earth is $\mathbf{F}_G = -mg$ (+ is upward) and is constant. If the only force acting on a mass is the gravitational force near the surface of the earth, Newton's second law, Equation 12.2 above, yields:

$$\mathbf{F} = \mathbf{F}_G$$

$$m\mathbf{a} = -mg$$

and with cancellation of the common factor of m one finds that the acceleration of a mass near the surface of the earth is

$$\mathbf{a} = -g\,. \qquad (12.3)$$

This is a statement that the acceleration of a mass near the surface of the earth is always downward and always has the magnitude, $g = 9.81\,\text{m/s}^2$, with the units of an acceleration.

The gravitational force vector on a mass thrown upward near the surface of the earth is $\mathbf{F}_G = -mg$. Throughout the entire free flight, the acceleration vector is $\mathbf{a} = -g$, and therefore, the velocity is always changing ($\Delta\mathbf{v}$ is not zero). At the time the mass reaches its highest point, where the speed, $v = 0$, the mass is still moving: at any time earlier or later the speed is not zero, and the height of the mass will be lower than its maximum height.

F. The equations of motion for a mass under a constant acceleration

The definition of acceleration if constant, $\mathbf{a} = \left(\mathbf{v} - \mathbf{v}_0\right)/t$, and of average velocity, $\frac{1}{2}\left(\mathbf{v} + \mathbf{v}_0\right) = \mathbf{s}/t$, were discussed above. They are the first two equations of motion:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t\,, \qquad (12.4)$$

and $$\mathbf{s} = \tfrac{1}{2}\left(\mathbf{v} + \mathbf{v}_0\right)t\,. \qquad (12.5)$$

The kinetic energy change is equal to the work done on the mass by all forces (including conservative and non conservative), $\frac{1}{2}m(v^2 - v_0^2) = \mathbf{F}\cdot\mathbf{s}$. Replacing $\mathbf{F}$ by $m\mathbf{a}$ and dividing both sides by $m/2$, yields the third equation of motion:

$$v^2 - v_0^2 = 2\mathbf{a}\cdot\mathbf{s}. \qquad (12.6)$$

Eliminating the final velocity, $\mathbf{v}$, from Equation 12.6, yields the fourth equation of motion:

$$\mathbf{s} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \qquad (12.7)$$

The substitution, $\mathbf{a} = -g$, will yield the four equations of motion for a mass in free flight near the surface of the earth:

$$\mathbf{v} = \mathbf{v}_0 + (-g)t$$
$$\mathbf{s} = \tfrac{1}{2}(\mathbf{v} + \mathbf{v}_0)t$$
$$v^2 - v_0^2 = 2(-g)\cdot \mathbf{s}$$
$$\mathbf{s} = \mathbf{v}_0 t + \tfrac{1}{2}(-g)t^2$$

Eqs. 12.8 – 12.12

If there is no *net* force acting on the mass, the acceleration will be zero, $\mathbf{a} = 0$, the velocity will not change, $\mathbf{v} = \mathbf{v}_0$, and the four equations reduce to the one for a constant velocity: $\mathbf{s} = \mathbf{v}t$.

Table 12.1 Motion equations for a constant acceleration

Equation	$\mathbf{v}_0$	$\mathbf{v}$	$\mathbf{a}$	$\mathbf{s}$	t	missing	Free Flight Eq.
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$	*	*	*		*	$\mathbf{s}$	$\mathbf{v} = \mathbf{v}_0 + (-g)t$
$\mathbf{s} = \tfrac{1}{2}(\mathbf{v} + \mathbf{v}_0)t$	*	*		*	*	$\mathbf{a}$	$\mathbf{s} = \tfrac{1}{2}(\mathbf{v} + \mathbf{v}_0)t$
$v^2 - v_0^2 = 2\mathbf{a}\cdot\mathbf{s}$	*	*	*	*		t	$v^2 - v_0^2 = 2(-g)\cdot s$
$\mathbf{s} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2$	*		*	*	*	$\mathbf{v}$	$\mathbf{s} = \mathbf{v}_0 t + \tfrac{1}{2}(-g)t^2$

An asterisk (*) in the table identifies four of the five motion variables, $\mathbf{v}_0$, **v**, **a**, **s**, and *t*, used in each of the equations. In solving a motion problem, there will always be at least three equations that are potentially useful, because they contain the unknown motion variable. For problems where the initial velocity, $\mathbf{v}_0$, and another variable of interest are not given, the initial velocity can be determined from the equation above missing the variable of interest.

These relationships between $\mathbf{v}_0$, **v**, **a**, **s**, and *t*, have energy conservation hidden within them and there is much that these equations can explain about the motion of an object. They are quite useful in situations where the applied forces or the acceleration is known. They give very little information, however, on how accelerated motion will be perceived if the accelerated object is a human being. The next chapter addresses this deficiency.

Chapter Summary

- In the states, at rest, stationary, and not moving, an object remains at the same place for a given period of time. An object is MOVING (it is not at rest, and it is not stationary), if it has a zero speed at *one* time, but a non-zero speed any short time earlier or later.
- If an object is subjected to a constant acceleration and the velocity changes sign, there is a certain time and location where the velocity is zero. At this time the object is not at rest and not stationary, it is moving. It is "moving" because it does not stay at the position where the velocity is zero for even the shortest period of time.
- Newton's 2nd law of motion: "a constant *net* external force, **F**, the vector sum of all forces acting on a mass *m*, will cause a constant acceleration given by $\mathbf{F} = m\mathbf{a}$."
- The acceleration of an object in free fall near the surface of the earth is the constant, $\mathbf{a} = -g$.
- The four motion equations for a constant acceleration are shown in Table 12.1.