EXPERIMENT: ANALYSIS OF A FREELY-FALLING BODY

Part I: Dependence of Speed and Position on Time

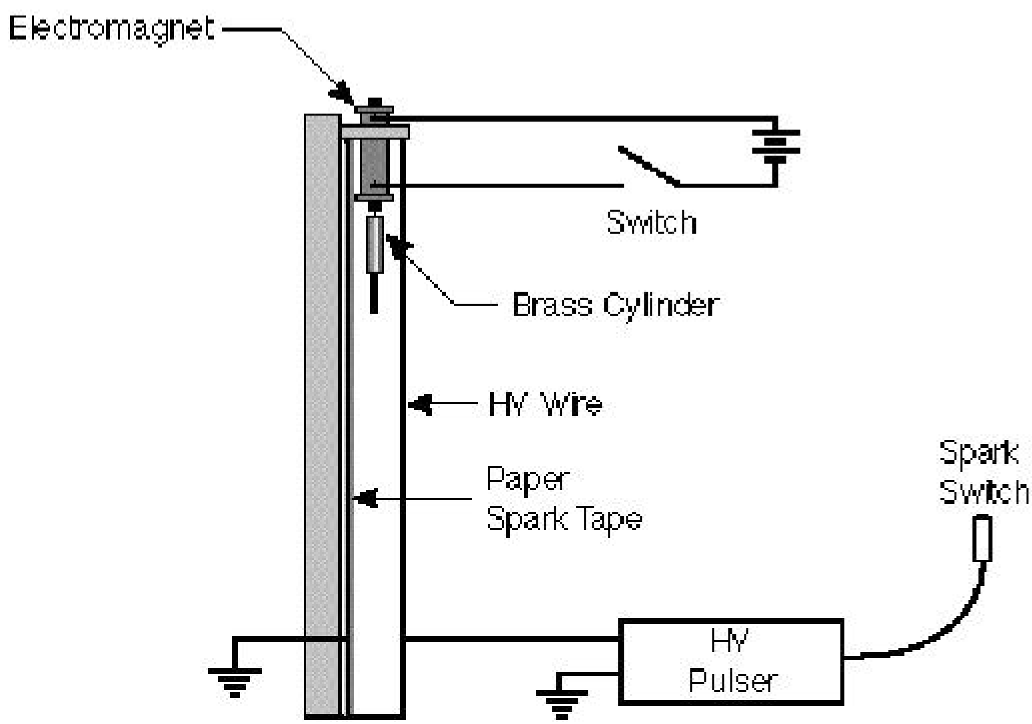
OBJECTIVES

- to verify how the distance of a freely-falling body varies with time
- to investigate whether the velocity of a freely-falling body increases linearly with time
- to calculate a value for g, the acceleration due to gravity

APPARATUS

A Behr Free-Fall Apparatus and Spark Timing System will be used.

BEHR FREE FALL SYSTEM



THEORY

In this experiment a brass cylinder is dropped and a record of its free fall is made. Before the measurement, the cylinder is suspended at the top of the stand with the help of an electromagnet. When the electromagnet is turned off, the cylinder is released and starts to fall. Simultaneously, the spark timer starts to send evenly-spaced, high-voltage pulses through two wires which are stretched along the cylinder's path.

At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the special paper tape that lies between the cylinder and one of the wires. The time interval between two adjacent sparks, τ, is 1/60 of a second.

Measuring the distances between any two marks, Δy, and knowing the times between the corresponding sparks, Δt, it is possible to calculate the average velocity during this interval using the formula

$$v = \frac{\Delta y}{\Delta t} \tag{1}$$

If Δt is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity.

In general, for the motion of a body with a constant acceleration a, the velocity v is given by the equation

$$v = at + v_o, \tag{2}$$

where v_o is the velocity of the cylinder at t = 0. Since in our case the brass cylinder is falling freely,

$$a = -g, \tag{3}$$

where $g=9.81\ m/s^2$ is the magnitude of the acceleration due to gravity. Therefore it follows from (2) that for a freely-falling body

$$v = v_0 - gt. \tag{4}$$

Thus g can be determined from a plot of v vs. t since the slope of any velocity versus time graph is just the acceleration. The obtained value of g can then be compared with the known value of the acceleration due to gravity. The position of the cylinder, y, as a function of time is given by the standard equation for an object that is undergoing constant acceleration. If at time = 0 the object has height y_o and velocity v_o, then this equation looks like

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2 \, . \tag{5}$$

PROCEDURE

Using the free fall apparatus, drop the brass cylinder and record on spark tape the location of the falling cylinder at a series of equally spaced time intervals, $\tau = 1/60$ s.

The quantities needed to analyze the motion are the position (y), velocity (v) and time (t) of the points on your spark tape. The choices of t = 0 and y = 0 are arbitrary and do not necessarily refer to your first or last data points. However, as your object falls the position *y* must decrease (becoming negative if necessary).

After you have performed the experiment, tape the paper strip to your lab table. Label the points on your tape, starting with the point AT THE BOTTOM OF THE OBJECT'S FALL as #31 and label them in DESCENDING numerical order. Make sure that point #31 is the last actual data point and not the point where the brass cylinder is stuck in the putty and simply sparking. NOTE: Point #1 will not necessarily correspond to your very first point, but the very first points are somewhat ambiguous anyway. It does not matter if you do not use some of the first points. Next, put a ruler on your tape such that height y=0.0 cm corresponds to point #31 and that height INCREASES as you move towards point #1. Measure the locations of each marker with this ruler and write the y position on the tape next to each marked point. Make sure you are labeling each point with its TOTAL distance from 0.0cm, NOT just with its distance from the previous point.

After completing this, transfer the y positions into the spreadsheet. Using the spreadsheet, calculate the velocity at all points during the fall except for the first and last points. Be sure you do at least one sample calculation to include in your lab report. As discussed in the theory section, the instantaneous velocities at these points are approximated by finding the average velocity at each of your points, $v = \frac{\Delta y}{\Delta t}$. In this equation Δy is the distance between the spark PRECEDING and the spark FOLLOWING the given spark for which you are trying to calculate a velocity. Similarly, Δt is the time interval between the PRECEEDING spark and the FOLLOWING spark. More details are given in the Appendix "Formulae, Definitions and Errors for the Free Fall experiment."

Transfer the columns of "Time," "y," and "Vy" from your spreadsheet to Kaleidagraph for graphing.

Prepare the following graphs:

I -- a graph showing Vy vs. t fitted with a best-fit line.* Make sure the equation of the best-fit line is on your graph. From the slope of the line determine the gravitational acceleration, g, in cm/s^2.

II -- a graph of y vs. time

*To fit a linear graph with a best-fit line, chose the menu item "Curve Fit," and then "Linear." Click the box beside the name of the column that has the data points you want to fit. Then click "okay." You should see your graph now fitted with a line. If the equation of the line has <u>not</u> automatically popped up on your graph, go back to "Curve Fit" and "Linear." Click the "view" box that has now appeared. A window will pop up with the general form of the equation of a line and the specific values of the slope and the y-intercept for YOUR line. Plug in the values and write the equation of your best-fit line on the graph after you print it out.

After your perfected graphs are printed out, write a 2 to 3 sentence "Graph Analysis" at the bottom of each graph. This should describe what is happening on the graph ("as the quantity along the x-axis does such and such, the quantity along the y-axis does this"), and how what is happening on the graph relates to what actually occurs with those quantities physically.

****NOTE: BE SURE TO SAVE YOUR SPREADSHEET!!! YOU WILL NEED TO USE YOUR DATA FOR NEXT WEEK'S EXPERIMENT: FREE-FALL PART II.**

QUESTIONS

1) When finding the velocities using $v = \frac{\Delta y}{\Delta t}$, why should you use sparks on <u>either side</u> of the point for which you are calculating the speed?

2) What is the y-intercept determined from your Graph I (or from the equation of its best-fit line)? What does it mean?

3) From Graph I, or the equation of its best-fit line, find the time at which v=0 cm/s.

4) What is your value of the gravitational acceleration in cm/s^2 determined from the slope from graph I?

5) What specific equation describes your Graph II?

6) When the initial velocity is zero, what would you plot to make graph II linear: y^2 vs. t, y^2 vs. t^2, or y vs. t^2?

CHECKLIST

Your lab report should include the following eight items:

1) your spreadsheet
2) sample calculations
3) plot of the height vs. time
4) plot of velocity vs. time with slope (in cm/s^2)
5) interpretation of the two plots
6) answers to questions
7) one member of each group should turn in your spark tape record of the free-fall
8) since you will use your results in the next lab, make sure you save the spreadsheet

Formulae, Definitions, and Errors for the Free Fall Experiment

DEFINITIONS

In this experiment you measure the position of a falling mass, m, at fixed time intervals. The fixed time interval is determined by a high-voltage spark source. Read off the time between sparks (τ) from the setting on the spark source.

You will measure the positions at each spark as y_1, y_2, y_3, y_4, etc. in centimeters [cm]. These positions will be referred to as y_i.

To measure the speed at point a particular point "i" first calculate

$$\Delta y_i = y_{i+1} - y_{i-1} .$$

For example for the sixth point

$$\Delta y_6 = y_7 - y_5 .$$

On your data sheet this is labeled as Δy(i). You are now ready to calculate the speed at point i by dividing the distance Δy_i by the time elapsed between the two points, or 2τ. So the speed V_{y_i} is given by

$$V_{y_i} = \frac{\Delta y_i}{\Delta t} = \frac{\Delta y_i}{2t} \quad \text{[cm/s].}$$

ERRORS

For each measured y_i you assign an error based on how accurately you can measure that point. This error is called dy. This error determines all other errors in this lab. For this lab and for the following formulae it is assumed that the error in τ and m are zero.

The error in Δy at each point i is the same and is given by $d(\Delta y) = 2\,dy$

The error in the speed at each point i is

$$d(V_y) = V_y \frac{d(\Delta y)}{\Delta y} = V_y \frac{2dy}{\Delta y} = \frac{V_y}{\Delta y} 2dy = \frac{2dy}{2t} = \frac{dy}{t} .$$