INTRODUCTION TO ONE-DIMENSIONAL COLLISIONS

Elastic and Inelastic Collisions

The following two experiments deal with two different types of one-dimensional collisions. Below is a discussion of the principles and equations which will be used in analyzing these collisions.

For a single particle, **momentum** is defined as the product of the mass and the velocity of the particle:

$$\boldsymbol{p} = m\boldsymbol{v}. \tag{1}$$

Momentum is a **vector** quantity, making its direction a necessary part of the data. For example for the one-dimensional case the momentum would have a direction in either the +x direction or the -x direction. For a system of more than one particle, the **total momentum** is the vector sum of the individual momenta:

$$\boldsymbol{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2 + \ldots = m\boldsymbol{v}_1 + m\boldsymbol{v}_2 + \ldots \tag{2}$$

So in other words you just add the momentum of each particle together.

One of the most fundamental laws of physics is that the **total momentum** of any system of particles is **conserved**, or constant, as long as the net external force on the system is zero. Assume we have two particles with masses m_1 and m_2 and speeds v_1 and v_2 which collide with each other without any external force acting. Suppose the resulting speeds are v_{1f} and v_{2f} after the collision. **Conservation of momentum** then states that the <u>total</u> momentum before the collision ($\mathbf{p}_{initial} = \mathbf{p}_i$) is equal to the <u>total</u> momentum after the collision ($\mathbf{p}_{final} = \mathbf{p}_f$):

$$\boldsymbol{p}_i = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2,\ \boldsymbol{p}_f = m_1\boldsymbol{v}_{1f} + m_2\boldsymbol{v}_{2f} \text{ and } \boldsymbol{p}_i = \boldsymbol{p}_f \tag{3}$$

In a given system, the **total energy** is generally the sum of several different forms of energy. **Kinetic energy** is the form associated with motion, and for a single particle

$$KE = \frac{mv^2}{2}. \tag{4}$$

In contrast to momentum, kinetic energy is <u>NOT</u> a vector; for a system of more than one particle the total kinetic energy is simply the sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + ¼ \tag{5}$$

Another fundamental law of physics is that the **total energy** of a system is always **conserved**. However within a given system one form of energy may be converted to another, such as in the freely-falling body lab where potential energy was converted to kinetic energy. *Therefore, kinetic energy <u>alone</u> is not often conserved.*

There are two basic kinds of collisions, elastic and inelastic:

In an **elastic collision**, two or more bodies come together, collide, and then move apart again with NO LOSS IN KINETIC ENERGY. An example would be two identical "superballs," colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad \text{or} \quad KE_{\text{initial}} = KE_{\text{final}} \tag{6}$$

In an **inelastic** collision, the bodies collide and come apart again, but SOME KINETIC ENERGY IS LOST. That is, some kinetic energy is converted to some other form of energy. An example would be the collision between a baseball and a bat.

If the bodies collide and stick together, the collision is called **completely inelastic**. In this case, MUCH OF THE KINETIC ENERGY IS LOST in the collision. That is, much of the kinetic energy is converted to other forms of energy.

In the following two experiments you will be dealing with a completely inelastic collision in which all of the kinetic energy of the objects is lost, and with a nearly elastic collision in which kinetic energy is conserved. Remember, in both of these collisions total momentum should always be conserved.

COLLISION EXPERIMENT OBJECTIVES

PRIMARY OBJECTIVES

- to measure the momentum and kinetic energy of two objects before and after a one-dimensional collision
- to observe that the concept of **conservation of momentum** is independent of **conservation of energy**, that is, that the total momentum remains constant in both elastic and inelastic collisions while kinetic energy does not remain constant in inelastic collisions

SECONDARY OBJECTIVE

- to try to account for any change in KE in the nearly elastic collision

ADVANCED OBJECTIVE

- to calculate the percentage of KE which will be lost (converted to other forms of energy, notably heat) in a completely inelastic collision between an initially stationary mass and an initially moving mass; and to compare this calculation with the result of the elastic collision

EXPERIMENT: INELASTIC ONE-DIMENSIONAL COLLISIONS

OBJECTIVES

- See previous page.

APPARATUS

A one-dimensional air track, a photogate timing circuit and an analytical balance will be used.

PROCEDURE

In this and the next experiment we will analyze collisions between two carts of varying masses in the case where one of the carts is initially at rest. The carts move on an air track. Therefore, in order to insure that no energy due to the gravitational potential is created, you should first make sure that your track is level.

In this lab we study the totally inelastic collision, by arranging that when two carts collide they will stick together and move with some final velocity common to both masses. Thus, we have only to measure the velocity of Cart 1 before the collision and the common velocity of the carts after the collision, since of course the velocity of Cart 2 is zero before the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows us to measure the time it takes for the cart or carts to go through it. The photogates record times by sensing the fins attached to the tops of the carts while they are moving through the photogates' light beams. Therefore the velocities for these labs are calculated by dividing the length of the fin (NOT the length of the cart) by the time measured by the photogate (speed = length/time).

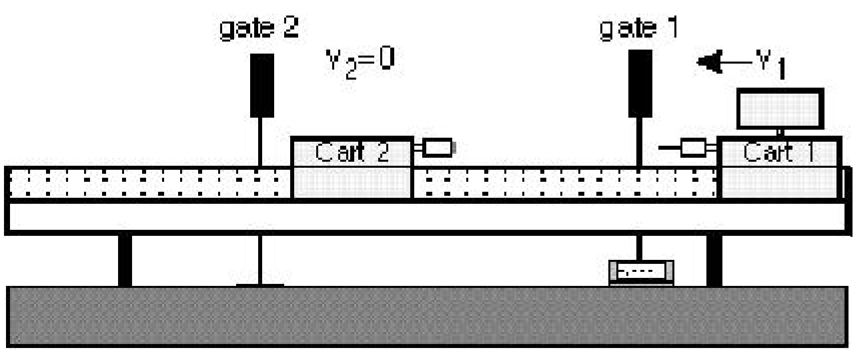

Figure 1: The Initial State of the Carts Before the Collision

In these instructions Gate 1 has the controllers on it and Gate 2 does not. Cart 1 has the fin on it and Cart 2 does not. Cart 2 is initially stationary. Position Cart 2 close to Gate 2. Set the photogate timer to the "GATE" mode and the memory switch to the “ON” position. In this mode the photogate display unit will ALWAYS display the FIRST time interval measured. Subsequent measurements will NOT be immediately displayed, but the times are added in the memory. By pushing the “READ” switch you can display the memory contents, which is the SUM OF ALL THE MEASUREMENTS.

Example: The initial reading for Cart 1 (the time that it took to pass through Gate 1) is 0.300 seconds. This time will be displayed on the photogate. Cart 1 collides with Cart 2 and they go together through Gate 2 (Figure 2). Suppose it takes them 0.513 seconds to go through Gate 2 together. The display will remain at "0.300," but the memory will contain 0.300 + 0.513 = 0.813 seconds. To find the second time, you have to subtract the first time from the contents of the memory that you can obtain by pushing the "READ" switch. Try this out by moving the cart through the gate by hand a few times.

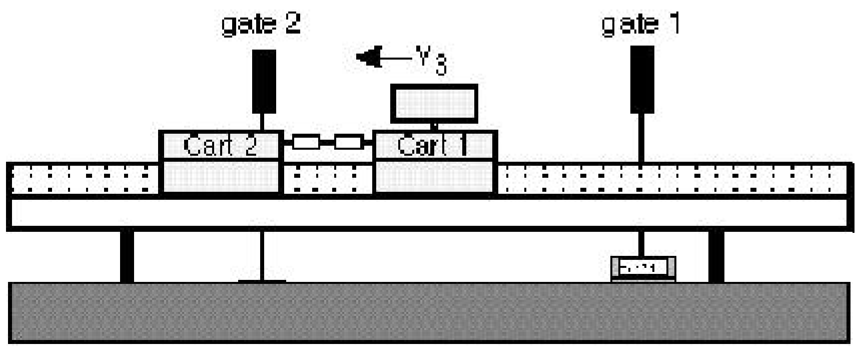

Figure 2: The Final State of the Carts After the Collision

The masses of the carts can be varied by adding weights (which in this case are small metal disks) to them. Remember that the experiment is done with Cart 2 initially at rest. So in the initial state we have

Initial state: Cart1: has a mass m_1 and initial velocity v_1
Cart 2: has mass m_2 and is at rest, so v = 0 cm/s

After the collision (final state):
the two carts will stick together
the total mass of the carts is (m_1+m_2)
the velocity is v_3

You will do 6 trials with the following m_1 and m_2 choices:

Trials 1 & 2: no mass disks on Cart 1, 4 mass disks on Cart 2

Trials 3 & 4: 2 mass disks on Cart 1, 2 mass disk on Cart 2

Trails 5 & 6: 2 mass disks on Cart 1, no mass disks on Cart 2

Enter the data into the spreadsheet and calculate the momenta and kinetic energies before and after the collisions, as well as the percent change in the total momentum and in the kinetic energy.

It is possible to calculate the percentage of the kinetic energy lost in a completely inelastic collision; you will find that this percentage depends only on the masses of the carts used in the collision, if one of the carts starts from rest.

The initial KE is given by:

$$KE_i = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \qquad \text{but since } v_2 = 0$$

$$KE_i = \frac{m_1 v_1^2}{2} \tag{5}$$

The final KE is given by:

$$KE_f = v_3^2 \, \frac{m_1 + m_2}{2} \tag{6}$$

From conservation of momentum

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v_3 \qquad \text{or, since } v_2 = 0$$

$$m_1 v_1 = (m_1 + m_2) v_3 \tag{7}$$

Combine equations (5), (6), and (7) to obtain an expression for KE_f in terms of KE_i. Compare this result with the results of the experiment. Do they agree (see checklist #3)?

CHECKLIST

Your lab report should include the following four items:

1) sample calculations of the momentum and of the kinetic energy of the system before and after the collision. Sample calculations for the errors in v, P, KE, P_{diff} and KE_{diff}
2) calculations for the percent changes in the momentum and kinetic energy through the collision
3) a calculation of the ideal KE_f in terms of KE_i in a totally inelastic collision. Compare at least one of your trials to this ideal expression.
4) comments on the uncertainties of the measurement of the total momenta and kinetic energies. HINT: If the errors in the measurements of P_{diff} and KE_{diff}. are large compared to P_{diff} and KE_{diff} themselves, that would mean that our measurement instruments (the timer, the ruler) are not precise enough to pinpoint the change in the total momentum and kinetic energy. Is this the situation with YOUR measurements?