EXPERIMENT: MOMENT OF INERTIA

OBJECTIVES

- to familiarize yourself with the concept of moment of inertia, I, which plays the same role in the description of the rotation of a rigid body as mass plays in the description of steady motion
- to measure the moments of inertia of several objects by studying their accelerating rotation under the influence of unbalanced torque

APPARATUS

See Figure 3.

THEORY

If we apply a single unbalanced force, F, to an object, the object will undergo a linear acceleration, a, which is determined by the force and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. The mathematical relationship which expresses this is

$$F = ma..$$

If we consider rotational motion, we find that a single unbalanced torque

$$t = (Force)(lever\ arm)^{\#}$$

produces an <u>angular</u> acceleration, α, which depends not only on the mass of the object but on how that mass is <u>distributed</u>. The equation which is analogous to F = ma for an object that is rotationally accelerating is

$$t = I\ a. \tag{1}$$

where τ is the torque in meters, α is the rotational acceleration in radians/sec^{2}* and I is the **moment of inertia** in kg*m^2. The moment of inertia is a measure of the way the mass is distributed on the object and determines its resistance to rotational acceleration.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i.e., perpendicular to the radius (see your instructor for the case when the lever arm is not perpendicular to the force).

* A radian is an angle measure based upon the circumference of a circle C = $2\pi r$ where r is the radius of the circle. A complete circle (360°) is said to have 2π radians (or radiuses). Therefore, 90^{o} (1/4 circle) is $\pi/2$ radians, etc. Angular accelerations (α) are measured in units of radians/sec^2

One point mass m on a weightless rod of radius r (**$I = mr^2$**):

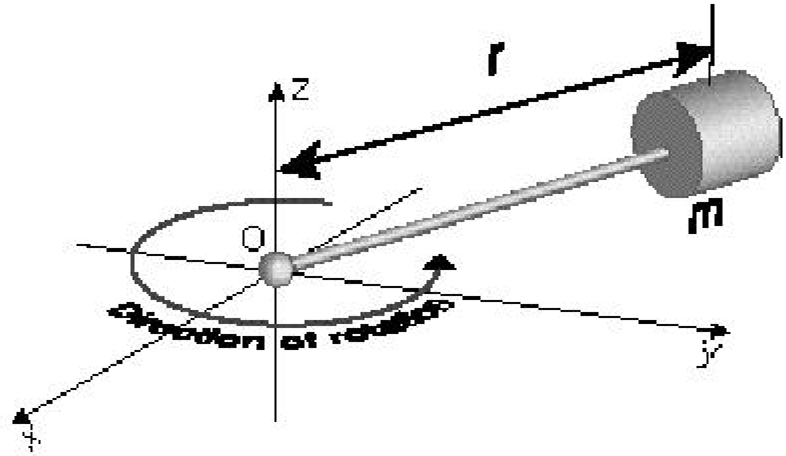

Figure 1

Two point masses on a weightless rod (**$I = m_1r_1^2 + m_2r_2^2$**):

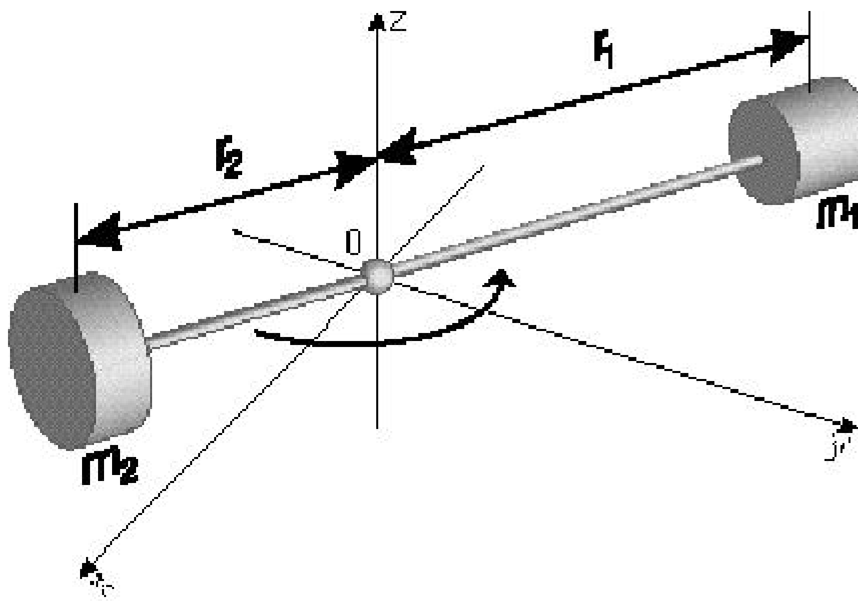

Figure 2

To illustrate we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (object #1 above):

$$I = mr^2 = (2\text{ kg})\,(2\text{ m})^2 = 8\text{ kg m}^2$$

If a force of 5 N were applied to the mass <u>perpendicular</u> to the rod (to make the lever arm equal to r) the torque is given by:

$$\tau = Fr = (5\text{ N})\,(2\text{ m}) = 10\text{ N m}$$

By equation 1 we can now calculate the angular acceleration:

$$a = \frac{t}{I} = \frac{10\ Nmeter}{8kg\,meter^2} = 1.25\left(\frac{radians}{\sec^2}\right)$$

IMPORTANT NOTE: We obtain the moment of inertia of a complicated object by <u>adding up the moments of each individual piece</u> (object #2 above is the sum of two object #1 components). We will use these concepts in this lab, where, by measuring the torque and angular acceleration of various objects, we will determine their moments of inertia.

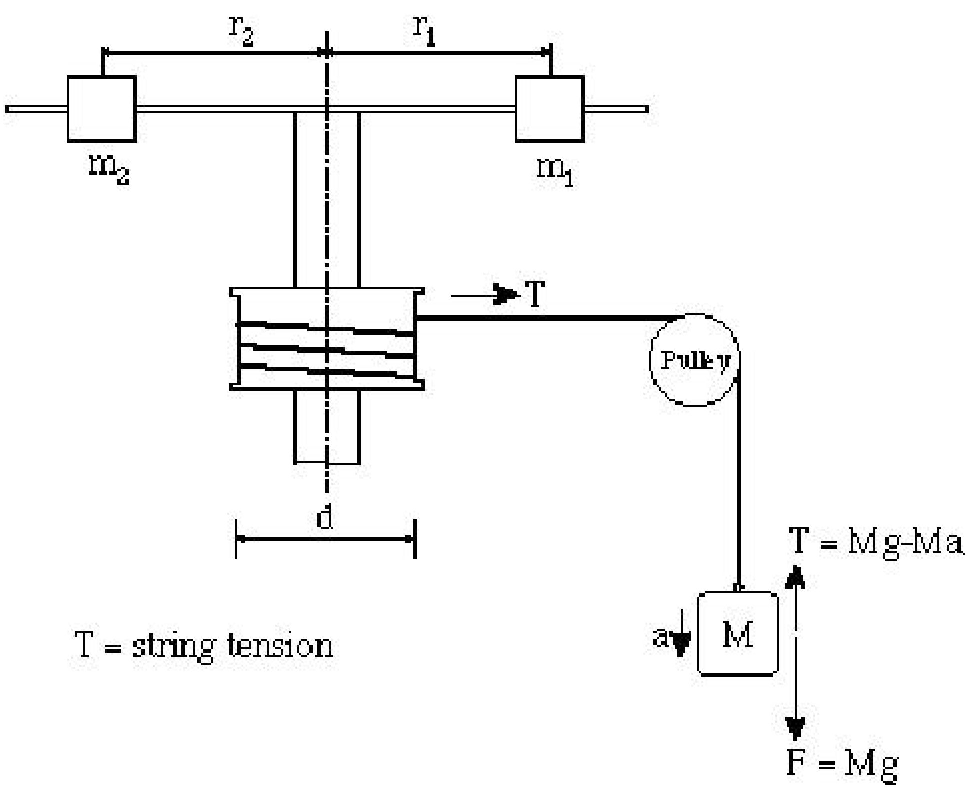

Figure 3

EXPERIMENTAL APPARATUS

In our case the rigid body consists of two cylinders, which are placed on the metallic rod at varying radii from the axis of rotation (Figure 3). We can not ignore the mass of the rod and the supporting structure in our measurements, so their moment of inertia is not equal to zero. However, we can remove the masses from the apparatus and measure the moment of inertia for the supporting structure + the rod alone, $I_{support}$. We can then place the masses back on the support structure and measure the moment of inertia of the entire system, $I_{measured}$. If we then wanted just the moment of inertia of the masses, I_{mass}, we would subtract I_{mass} from $I_{measured}$.

To set up your rigid body, wrap the string around the axle several times, run it over the pulley to a known weight as shown in Figure 3.

Consider the following series of steps:

If we release the weights from rest and measure how long it takes to fall a distance then from

$$y = \frac{at^2}{2}$$

we can solve for a, the linear acceleration of the weights, the string and a point on the side of the axle.

Using

$$\alpha \ = \ \text{angular acceleration} \ = \frac{\textit{Linear acceleration}}{\textit{radius of axle}} = \frac{a}{\left(\frac{d}{2}\right)}$$

we obtain the angular acceleration.

The torque acting on the axle is given by

$$t \ = (\textit{Force})(\textit{ lever arm})$$

which is

$$t \ = T\left(\frac{d}{2}\right) = (Mg - Ma)\frac{d}{2}$$

Since we now have α and τ, we can calculate I from equation (1). <u>BEFORE CLASS</u>, be sure you know how to use equation (1) and the above three steps to obtain the expression:

$$I = \frac{Mgd^2t^2}{8y} - \frac{Md^2}{4} \tag{2}$$

which allows calculation of I from measurements of time with no intermediate steps. In the measurements made in this experiment the constant term $\frac{Md^2}{4}$ is always quite small compared to the t^2 term. We will therefore <u>ignore the constant term</u> when calculating I.

$$\delta I = I*[(2*\delta d/d) + (2*\delta t/t) + (\delta y/y)]$$

PROCEDURE

Remove the masses m_1 and m_2 and measure the moment of inertia of the support structure alone. Do this five times and calculate a mean value, standard deviation, and standard deviation of the mean of your trials. Use the standard deviation of the mean value of t as the uncertainty (δt) for the measurements of t in this experiment. Calculate $I_{support}$ using the mean time you obtained.

Make a series of measurements of I, the moment of inertia of the entire rigid body, with the masses m_1 and m_2 placed an equal distance r ($r_1 = r_2$) from the axis of rotation. Do your best to make sure that the two masses are placed equidistant from the center of the rod. Take measurements for at least six different r values spanning the length of the rod. Make sure that you measure the radius r from the center of the cylinder to the axis of rotation, and not from either of the mass' edges. Again use the formula (2), neglecting the constant term, to calculate I_{meas}.

Since the moment of inertia is the sum of the moments of the individual pieces we may write

$$I_{meas} = I_{support} + I_{masses} = I_{support} + (m_1+m_2)r^2 \cdot$$

If you compare

$$I_{meas} = (m_1 + m_2)r^2 + I_{support}$$

to the form of the equation of a straight line

$$y = mx + b,$$

you can see that a plot of I_{meas} vs. r^2 should be a straight line. Make a plot of your measurements of I_{meas} vs. r^2. Compare the slope and intercept of this data with the values you previously measured for $m_1 + m_2$ and $I_{support}$. Do they agree?

QUESTIONS

1) In the plot I vs. r^2, why did we use r^2 and not r in the plot? What are the units of the slope of the plot I vs. r^2?

2) Explain why putting the masses at r = 0 (if we could) is the same or is not the same as removing them from apparatus.

CHECKLIST

Your lab report should include the following three items:

1) spreadsheet with your data for the two parts of the lab

2) graph of I vs. r^2 with best-fit line and equation of best-fit line

3) answers to the questions