

## EXPERIMENT: THE PENDULUM

### OBJECTIVES

- to investigate the functional dependence of the period (T) of a pendulum on its length (L), the mass of its bob (m), and the starting angle ( $\theta_0$ )
- to check for systematic errors by calculating g

### APPARATUS

Point masses and string, a digital timer, period gate, and meter stick will be used.

### THEORY

Where there exists a constant net force (F), Newton's Law  $F = ma$  tells us that the acceleration (a) is a constant and therefore the position of the object can be written as

$$x = x_0 + v_0t + (at^2/2).$$

This is the formula we have tested in the Free Fall labs.

In the analysis of the motion of a pendulum we should realize that

- 1) the motion is part of a circle so angular acceleration ( $\alpha$ ) is the most useful variable
- 2) the angular acceleration will not be a constant throughout the motion

Consider the pendulum shown on the next page. The acceleration of the bob **tangential** to the arc "drawn" by the pendulum as it swings,  $a_t$ , is determined by  $F_t$ , the force tangential to the arc. Since the tension in the string always acts along the **radius**, it doesn't contribute to  $F_t$ . Decomposing the gravitational force  $mg$  into components perpendicular and parallel to the string as shown in the diagram below, we find that

$$F_t = mg\sin\theta.$$

Therefore the acceleration tangent to the circle is given by

$$a_t = F_t / m = g\sin\theta.$$



The angular acceleration  $\alpha$  is then found by the relationship for circular motion

$$\mathbf{a} = -(a_t / r) = -(g / L)\sin\mathbf{q}.$$

Thus, as we have suggested, the angular acceleration  $\alpha$  is not a constant but varies as the sine of the displacement angle of the pendulum.

For small angles (about  $\theta < 0.5$  radian) angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle  $\theta$  by

$$\mathbf{q} = \mathbf{q}_0\cos(2\mathbf{p}t / \mathbf{t})$$

where  $\theta_0$  is the angle at time  $t = 0$  (when we release the pendulum), and  $\tau$  is the period of the motion.

This formula describes the motion of a pendulum. In this lab we will investigate the validity of this formula.

## **PROCEDURE**

The three parameters of the system are  $L$ , the length of the pendulum from the support to the center of mass of the "point mass;" the mass  $m$ ; and  $\theta_0$ , the angle through which you displace the mass as you pull the pendulum back to start it. The quantity you measure is the period  $\tau$ .

For a pendulum with starting angle  $\theta_0 = 0.5$  radians, make a graph showing the angle of the pendulum at  $t = 0, 0.1\tau, 0.2\tau, \dots, 1.0\tau$  (every  $0.1\tau$  - use the small angle approximation!) Have the computer fit a "smooth" curve through the points and comment on the values of  $t$  when  $\theta = 0$  (**this should be done before lab**). The displacement is zero when  $t = \tau / 4$  and  $t = 3\tau / 4$ .

**BEFORE starting the measurements write your introduction and write what YOU think the period of the pendulum depends on. Forget for the moment the formulae you have learned and simply use your intuition to estimate whether the period depends on the pendulum length, mass and/or the initial angle. Your measurements will verify which one is correct.**

Using the period gate measure the period several times for several different combinations of  $L$ ,  $m$  and  $\theta_0$ . The period gate uses light and a photodetector in the following way: when the light beam is interrupted the first time the timer is started, it keeps going when the beam is interrupted a second time but stops on the third interruption of the beam. This measures the time taken for one complete oscillation of the pendulum, or in other words the period.

You should keep two parameters fixed while you vary the third. When you vary  $L$ , do it over the largest range of values that you can with your apparatus. Always keep the starting angle  $\theta_0$  less than 0.5 radian.

Prepare the following graphs:

- I.  $\tau$  vs.  $m$  (for fixed  $\theta_0$  and  $L$ )
- II.  $\tau$  vs.  $\theta_0$  (for fixed  $m$  and  $L$ )
- III.  $\tau$  vs.  $\sqrt{L}$  (for fixed  $m$  and  $\theta_0$ )

Comment on how the period depends on each of the three parameters. If any of the graphs should be a straight line, have the computer fit it with a best-fit line and get the equation of the line.

## **QUESTIONS**

- 1) Use the slope of the graph of  $\tau$  vs.  $\sqrt{L}$  to calculate  $g$ . Compare your result with  $981 \text{ cm/sec}^2$ .
- 2) For one of your measurements calculate the angular acceleration ( $\alpha$ ) at the starting angle  $\theta_0$ , at  $\theta = 0$ , and at the far end of the swing.
- 3) Estimate the period of a pendulum, the length of which is that of the tallest a)tree b)building c)mountain in the world. You should not need or use the Guinness Book of World Records for this.

## **CHECKLIST**

Your lab report should include the following four items:

- 1) the spreadsheet with your data
- 2) three graphs with best-fit line and equation of best-fit line where appropriate
- 3) comments on each of the three graphs on how the period depends on each of the three parameters  $m$ ,  $\theta_0$ , and  $L$
- 4) answers to the questions