EXPERIMENT: THE SPRING I

Hooke's Law and Oscillations

OBJECTIVES:

- to investigate how a spring behaves if it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hook's Law.
- to verify that a stretched spring is also a good example of an oscillator with a characteristic period

APPARATUS:

A spring, photogate system, and masses will be used.

THEORY

* Hooke's Law

An ideal spring is remarkable in the sense that it is a system where the generated force is **linearly dependent** on how far it is stretched. This behavior is described by Hooke's law, and you would like to verify this in lab today. Hooke's Law states that to extend a string by an amount Δx from its previous position, one needs a **force** F which is determined by $F = k\Delta x$. Here k is the **spring constant** which is a quality particular to each spring. Therefore in order to verify Hooke's Law, you must verify that the force F and the distance the spring is stretched are proportional to each other (that just means linearly dependant on each other), and that the constant of proportionality is k.

In our case the external force is determined by attaching a mass m to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$. Regard Figure 1. Consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring ($k\Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point where the two forces are equal but pointing in opposite directions:

$$F_s - F_g = 0$$

or

$$kDx = mg \tag{1}$$

This point where the forces balance each other out is known as the **equilibrium point**. The spring + mass system can stay at the equilibrium point indefinitely as long as no additional external forces come to be exerted on it. This relationship in (1) allows us to determine the spring constant k when m, g, and Δx are known or can be measured. This is one way in which you will be determining k today.

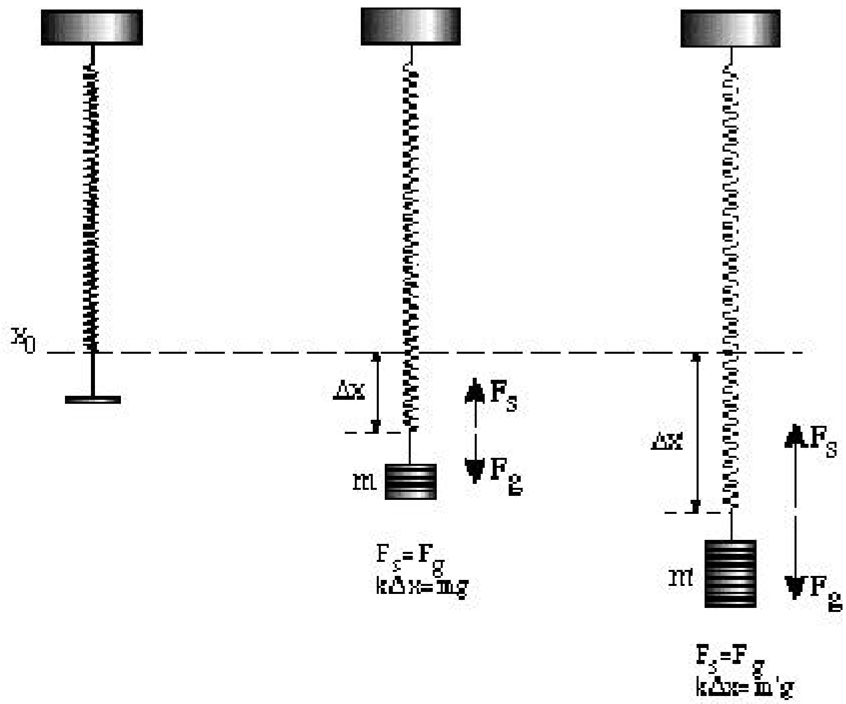

Figure 1: The Spring in Equilibrium

*** Oscillation**

The position where the mass is at rest is called the equilibrium position ($x = x_0$). As we now know, the downward force due to gravity $F_g = mg$ and the force due to the spring pulling upward $F_s = k\Delta x$ cancel each other. This is shown in the first part of Figure 2. However, if the string is <u>stretched beyond its equilibrium point</u> by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is <u>larger</u> than gravity pulling down. After release it will pass through the equilibrium point and continue to move upward. Once above the equilibrium position, gravity will start to exceed the force pulling upward due to the spring and acceleration will be directed downward. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a) and velocities (v) are illustrated in Figure 2 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period, T, which is determined by the spring constant and the total attached mass. This period is the time it takes for the spring to complete one oscillation, or the time necessary to return to the point where the cycle starts repeating (the points where x, v, a, are the same). One complete cycle is shown in Figure 2 and the time of each position is indicated in terms of the period T.

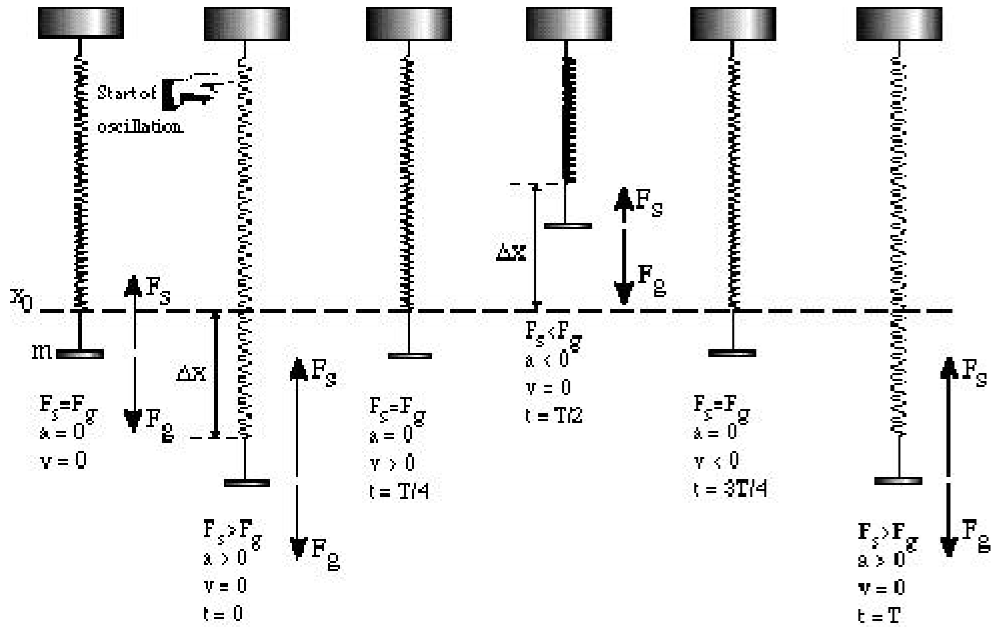

Figure 2: One Cycle of an Oscillation of the Spring

The period is given by

$$T = 2p\ (m/k)^{1/2}$$

By measuring the period for given masses the spring constant can be determined. This is the second way you will determine k today. You will use this value of k to verify that the proportionality constant you determined for Hooke's Law in the first part is indeed the correct k for the spring.

PROCEDURE

Part I: Hooke's Law

Determine the initial mass, m_0, by weighing the support table. Next attach the support table for the masses to the spring. Measure the position of the end of the spring after the table has been attached. This position is the initial position x_0.

Start measuring by increasing the mass attached to the spring to 120 grams. Then increase the mass by increments of 10 grams up to a total of 220 grams and measure the corresponding position of the spring for each mass. This results in a series of measurements m_i and x_i. To calculate the forces due to gravity and the spring calculate $\Delta x_i = x_i - x_0$ and $\Delta m_i = m_i - m_0$. The corresponding forces for gravity and the spring are $F^i_g = \Delta mg$ and $F^i_s = k\Delta x$. Right now you do not know k, so you will only have your spreadsheet calculate F_g for you. But remember, at equilibrium positions such as we are measuring, F_g equals F_s!

Graph F_g vs. Δx. If you have a straight line you have already verified the first part of Hook's Law, that force and distance the spring is stretched are linearly dependent. Now have the computer fit your plot with a best fit line, to determine the constant of proportionality, or the slope, which you have determined for Hooke's Law. You will verify this value of k by determining k a second way that is independent of Hooke's Law in Part II.

Part II: Period of Oscillation

Determine the period for attached masses varying from 120 to 220 g in steps of 10 g (the same masses as in Part I). When displacing the masses, DO NOT stretch the spring more than about 2 cm from its equilibrium position. Measure the time of the period by causing the masses to oscillate through the photogate while it is on the setting "PEND." This setting will start timing the first time the masses pass through it, continue timing through the second pass, and stop timing when it senses the masses a third time. Thus it measures the time for the masses (and thus the spring), to complete a whole period.

For each measurement of the period T, determine the spring constant k using $T = 2\pi\ (m/k)^{1/2}$. Note that in this equation m is the total mass attached to the spring. Average these 11 values for k together to get your spring constant value for Part II. Calculate the mean and the standard deviation of the mean for this k.

QUESTIONS

1) What are the units for the spring constant used in this lab?
2) You determined the spring constant in two independent ways. Do they agree?
3) Which is the more accurate measurement?
4) Have you proven Hooke's Law?

CHECKLIST

Your lab report should include the following three items:

1) spreadsheet with data from Parts I and II
2) graph of F vs. Δx with a best-fit line and equation of best-fit line
3) answers to the questions