EXPERIMENT: THE SPRING II
Time Dependence of Oscillations

OBJECTIVE

- To determine how the position, speed, and acceleration of an oscillating spring change as a function of time.

APPARATUS

A pre-recorded videoclip of a simple spring with a mass attached to it and VideoPoint software will be used.

THEORY

*Oscillation

When a mass is attached to a spring, the spring is stretched by a distance given by Hooke's Law. The position where the mass is at rest is called the equilibrium position ($x = x_0$). The downward force due to gravity $F_g = mg$ and the upward force due to the spring pulling upward $F_s = kx$ cancel each other. This is shown in the first part of Figure 1. If the mass is kept the same and the string is stretched by pulling it down and then releasing it, the spring will accelerate upwards ($a > 0$), because the force due to the spring pulling upwards is then larger than the force due to gravity pulling downwards. After release the spring will pass through the equilibrium point and continue to move upwards. Once above the equilibrium position gravity will start to exceed the force pulling upwards due to the spring and acceleration will then be directed downwards. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a) and velocities (v) are illustrated in Figure 1 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period T, which is determined by the spring constant and the total attached mass. This period is the time it takes to complete one oscillation, that is, the time necessary to return to the point where the cycle starts repeating (the points where x, v, and a are the same). One complete cycle is shown in Figure 1 and the time of each position is indicated in terms of the period T. The period is given by

$$T = 2p\ (m/k)^{1/2}$$

During this lab, the position of the mass attached to the spring will be measured as a function of time. From these data the speed and the acceleration will be determined as a function of time. The curves obtained will be compared to Figure 1.

HINT: To be prepared for class, be sure that you are familiar with the functions sine and cosine. Before you start measuring or even before class make a graph of sin(x) and cos(x) for values of x between 0 and 3π. Do this is steps of $x = 0.1$. You will need this graph in order to interpret the data you will record.

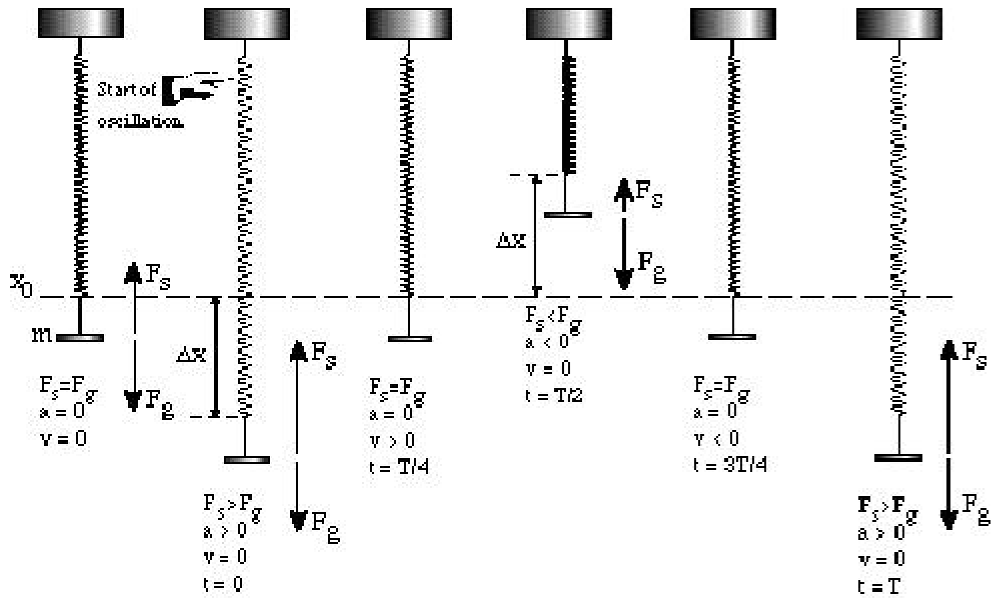

Figure 1: One Cycle of an Oscillation of the Spring

PROCEDURE

Make a graph of sin(x) and cos(x) for values of x between 0 and 3π, if you did not prepare it before class. Do this is steps of x = 0.1. Do this by having Excel fill a column with the numbers from 0 to 3p (9.42) in steps of 0.1 and then have it calculate sin(x) and cos(x) for you in the next two columns. If you forget how to do this refer back to the very first lab Introduction to Computer Tools. You will need this graph in order to interpret the data you will record.

Start the application program *VideoPoint* and open the videoclip containing the motion of the spring. As an initial test simply play the videoclip to see how long it lasts and whether there are gaps in it (if you forget the basics of using *VideoPoint*, refer back to the lab Projectile Motion). Select a set of images where the mass attached to the spring can be measured over **two complete** cycles (two cycles correspond to a time equal to two periods). In principle it does not matter at which point in time you start measuring, as long as you end at the same position point that you started at, only two cycles later. Go through the following steps for the measurement:

- Determine the conversion of pixel units to meters. For this purpose the videoclip contains a scale and the length is indicated on it (typically the scale will be a square black object with sides of 10 cm). Click the icon for setting the scale of the coordinate system and follow the instructions. Be sure you use the correct units!!

- After you perform the units conversion make sure that in the “Coordinate Systems” window, the “Origin” box “Scale 1” is given in (pixels) / (m). In the “TableWindow” the units should have switched from “pixels” to “meters”.

- With the bull’s eye, start measuring the position of the mass attached to the spring in the first frame you selected and keep doing this until you have completed the two full cycles. View your measurements in the “TableWindow.” You will measure both the X and Y position of the mass as a function of time. Of course, in principle X will hardly vary with time, but Y will.

- Save your results, so they are not lost in case of computer trouble.

- When you are done with your measurements, make a graph in VideoPoint of your measurements of Y vs. time to make sure you did not make any blatant mistakes. Remeasure points that are wrong.

- Once you have checked that your points are correct, select the time, X and Y columns and copy them to the clipboard for use in the *Excel* spreadsheet.

- After you transfer the data into the empty spreadsheet you will be ready to calculate the speed and acceleration as a function of time. This will only be done for the Y direction!! The determination of the speed (v_y in spreadsheet) and acceleration (a_y in spreadsheet) are described in the following. At any given moment in time the speed is given by

$$v = \text{distance}/\ \text{time interval} = Dy\ /\ Dt$$

Let’s consider an example. Your measurements are at several points in time, which we will label $t_1,t_2,t_3,t_4,\ldots$ and the positions measured are $y_1,y_2,y_3,y_4\ldots$ In previous experiments we have used a technique to calculate the speed at the point t_3, for example, by using the position and time of the adjacent points t_2 and t_4. This technique is not precise enough for this experiment and we will use a similar but slightly different procedure. The procedure is explained in the following example where we use the time points t_2 and t_3. The distance the mass travels between these two points is $\Delta y = (y_3-y_2)$. The time it takes to do this is $\Delta t= (t_3-t_2)$. So the **average speed** between the points t_2 and t_3 is $v = \Delta y/\Delta t$. The mass had this speed at some point in time t_v which is somewhere between t_2 and t_3 and we will define it as the <u>mean</u> of the two: $t_v = (t_2+t_3)/2$. This time t_v is called time_v in the spreadsheet. So t_v will be in the middle between t_2 and t_3.

For the acceleration (a_y in the spreadsheet) we use the same technique. Here the basic equation used is

$$\text{acceleration} = a = \text{change in speed}\ /\ \text{time interval} = Dv\ /\ Dt$$

The same formulae as for the speed are used but now the <u>position</u> y is replaced by the <u>speed</u> at a certain point in time. **BEWARE:** <u>You will also have to calculate the time at which you measure the acceleration (time_a in the spreadsheet), and the times which should be used in actually calculating acceleration</u>.

After calculating these quantities in the spreadsheet make the following three graphs, using exactly the same scale for the time axis in all three graphs so that they can be easily compared:

I. Position (Y) of the mass as a function of time showing at least two cycles

II. Speed of the mass in the Y-direction as a function of time

III. Acceleration as a function of time

Make sure your graphs are titled correctly and that the axes are labeled and have proper units.

QUESTIONS

1) What is the amplitude of the oscillation in the Y direction? Read this from the graph of Y vs. time.
2) What is the period (T) of the oscillation, that is, after what time does the motion repeat ?
3) What function describes the dependence of displacement on time and what function describes the dependence of speed on time?
4) On each of your three graphs indicate the points corresponding to the equilibrium position and the two positions where the excursion from the equilibrium is maximum.
5) At these equilibrium and "maximum excursion" points, determine the speed and acceleration from your graphs and compare them to the values given in Figure 1.

CHECKLIST

Make sure that your lab report contains the following three items:

1) the spreadsheet with your Y, v, and a data
2) three properly labeled graphs with the equilibrium positions and "maximum excursion" points labeled
3) answers to the questions including analysis of the speed and acceleration of the equilibrium and "maximum excursion" points