DIFFRACTION AND INTERFERENCE

OBJECTIVES:

1) Observe Fraunhofer diffraction and interference from a single slit, double-slit and multiple-slit (a diffraction grating).

2) Calculate the slit width, which produces the single-slit diffraction pattern, and observe how the slit width affects the diffraction pattern.

3) Calculate the slit width and the slit spacing for double-slit, and compare the pattern to that given by a single-slit.

4) Calculate the slit spacing of a diffraction grating and thereby determine the ruling density.

5) Verify Babinet's Principle by observing the diffraction pattern from a thin wire.

CAUTION!

The laser is a device that can produce an intense, narrow beam of light at one wavelength. NEVER look directly into the laser beam or its reflection from a mirror, etc.

INTRODUCTION

Diffraction occurs when a portion of a wave passes through a slit. Interference occurs when two or more coherent waves overlap. (Coherent means that the waves have a fixed phase relationship.) Constructive interference takes place at certain locations where two waves are in phase (both waves have maximum). Destructive interference takes place where two waves are out of phase (one wave has maximum, the other has minimum).

In the case of a single-slit, diffraction is the only effect present. In the case of two or more slits, two effects are present: a) diffraction from each individual slit; b) if the incident light is coherent and the diffraction patterns of each slit overlap, then interference takes place in the region of the overlap, (i.e., inside the diffraction envelope).

The simplest diffraction and interference patterns involve plane waves (collimated light beams). Diffraction patterns associated with plane waves are called Fraunhofer patterns, named after the German scientist who first explained the effect. In this experiment, we will use a laser as our light source. A laser produces collimated and coherent light beams at one wavelength.
Single-slit diffraction

Figure 1 shows the pattern of light intensity as a function of position, with $x = 0$ at the position the beam would have taken without the slit. The top pane of the drawing shows the physical arrangement, with a sketch indicating the intensity at different points. The bottom part of the figure is a blowup of the intensity sketch.

The intensity curve shows how much light appears as a function of position; the original beam of light is redistributed by diffraction. Maxima (peaks) of light intensity will be seen as bright spots if you look at the screen. Minima (valleys) of light intensity will be seen as dark spots. The principal maximum position, $x=0$, is the center of the central bright spot, which is where you might have thought all the light would go. To measure the location of a maximum, use the center of a bright spot. To measure the location of the minimum, use the center of a dark spot.

The pattern in figure 1 is what happens for light of a single wavelength. Superimposing diffraction patterns from different wavelengths of light would blur the pattern, because maxima for some wavelengths would land on minima for other wavelengths. This is one reason we use a laser in this experiment.
For a single-slit, the condition for diffraction minimum to occur is:

\[ w \sin \theta_n = n\lambda. \]

Since,

\[ \theta_n = \tan^{-1} \left( \frac{x_n}{D} \right) \]

\[ \therefore \quad \sin \theta_n = \sin \left( \tan^{-1} \left( \frac{x_n}{D} \right) \right) \]

Therefore,

\[ w = \frac{n\lambda}{\sin \left( \tan^{-1} \left( \frac{x_n}{D} \right) \right)} \]  \quad (1)

Here, \( w \) = slit width
\( n \) = order number of diffraction minimum
\( \lambda \) = wavelength of light
\( D \) = distance from screen to slit
\( x_n \) = distance from the principal maximum to the \( n \)th diffraction minimum
\( \theta_n \) = angle from the center line to the \( n \)th diffraction minimum

Equation (1) is called the Diffraction Equation.

Note that \( x_n \) is a signed distance: it is positive for \( n \) positive.
Double-slit diffraction and interference

For two nearby slits (the double slit), the condition for interference maximum to occur is:

\[ d \sin \theta_m = m\lambda \]

Thus,

\[ d = \frac{m\lambda}{\sin \left( \tan^{-1}\left( \frac{x_m}{D} \right) \right)} \quad (2) \]

Here,
- \( d \) = slit spacing
- \( m \) = order number of interference maximum
- \( \lambda \) = wavelength of light
- \( D \) = distance from screen to slit
- \( x_m \) = distance from the principal maximum to the \( m \)th interference maximum
\[ \theta_m = \text{angle from the center line to the } m\text{th interference maximum} \]

Equation (2) is called the Interference Equation. The algebra looks the same as for the Diffraction equation except that \( x_m \) refers to the location of \textit{maxima}, not minima and \( d \) is the \textit{spacing} between the two slits, not the width of the individual slits.

**Multiple-slit interference and diffraction (a diffraction grating)**

![Intensity distribution of a diffraction grating](image)

The condition here for interference maximum is the same as for double-slits, but the pattern may be very different because \( d \) (the slit spacing) for gratings is very small.

\[ d \sin \theta_n = n\lambda \]

Thus

\[ d = \frac{n\lambda}{\sin \left( \tan^{-1} \left( \frac{x_n}{D} \right) \right)} \]  \( \text{(3)} \)
Babinet's Principle

Two screens are said to be complementary when the transparent regions on one exactly correspond to the opaque regions on the other and vice versa. Figure 4 shows two examples of complementary screens. Babinet’s Principle implies that the Fraunhofer diffraction patterns from complementary screens are nearly identical. Thus, if a thin wire were placed in the laser beam, one would expect to obtain a diffraction pattern similar to that of a single-slit.

![Examples of complementary screens](Figure 4)

**IN A NUTSHELL:** For all these phenomena, the equation relating object size, \( S \), and the location of maxima or minima is of the form

\[
S = \frac{n\lambda}{\sin\left(\tan^{-1}\left(\frac{X_n}{D}\right)\right)}
\]

To see what is going on, consider the often interesting case of consider small angles, that is \( X_n \ll D \). When angles are small, \( \sin \theta \approx \tan \theta \approx \theta \) (radians). Then the equation simplifies to

\[
S \approx \frac{n\lambda D}{X_n}
\]

This approximate equation can be rearranged to concentrate on \( \frac{X_n}{n} \), the spacing between peaks or valleys. We thus estimate the fineness of the diffraction pattern to be

\[
\frac{X_n}{n} \approx \frac{\lambda D}{S}
\]

The peak spacing is **smaller** for **larger** object size \( S \). Common objects are much larger than the wavelength of light, so any interference effects are also usually “too fine to notice”. Further, in every day life, we use light with a variety of wavelengths, so that peaks of one wavelength tend to land on valleys of another wavelength. Thus diffraction effects are not only too small for our eyes to resolve, but are also smeared out because the wavelength is not a single well-defined value. Using small objects with a single well-defined wavelength helps us see these effects in the lab.
PROCEDURE

1. Single-slit diffraction
   a) Place the panel that holds four single-slits in the laser (between the laser and the screen). The pattern is most easily seen with the slit near the laser and the screen far away (even perhaps on the wall). Try different slits and observe their diffraction patterns on the screen.

   Q: How does the slit width affect the diffraction pattern?

   b) Choose a slit that gives a diffraction pattern of reasonable size. Measure the distance from the slit to the screen (D) and record the labeled slit width.

   c) Tape a piece of paper across the screen where you see the diffraction pattern. Mark carefully the positions of the center at the principal maximum and the center of the diffraction minima of several orders on the paper. Remove the paper from the screen and attach it to your lab report.

   d) Measure and record the distance of each minimum from the principal maximum ($x_n$) in the table below. Calculate the slit width ($w$) using equation (1). Use negative $x_n$ and n for the positions to the left of the principal maximum.

   \[
   D \text{ (distance from the slit to the screen)} = \text{______________ cm}
   \]

   \[
   \text{Labeled slit width = ______________ mm}
   \]

   Write down the equation you will use for calculating the slit width ($w$):

   \[
   \text{Note: The wavelength (} \lambda \text{) of He-Ne laser is 632.8 nm = 6.328 x 10^{-4} mm}
   \]

<table>
<thead>
<tr>
<th>Order of Diffraction Minimum $n$</th>
<th>distance from principal maximum $x_n$ (cm)</th>
<th>calculated slit width $w$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average slit width:</td>
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</tbody>
</table>

   Q: Compare the average slit width with the labeled one. Comment:
2. Double-slit interference and diffraction
   a) Place the panel that holds four double-slits in the laser beam. Try different double-slits.
      Q: What is the main difference between the single-slit and double-slit pattern?
      Q: What causes the main difference from the single-slit pattern?
      Q: How does the slit width affect the pattern?
      Q: How does slit spacing affect the pattern?

   b) Choose a double-slit that gives a reasonable pattern. Measure the distance from the slit to
      the screen (D) and record the labeled slit width and slit spacing.
   c) Tape a piece of paper across the screen where you see the pattern. Mark carefully the
      positions of the principal maximum, the interference maxima, and the diffraction minima on
      the paper. (You may want to distinguish the marks for each kind.) Remove the paper from
      the screen and attach it to your lab report.
   d) Measure and record the distance of each interference maximum and each diffraction
      minimum from the principal maximum ($x_n$) in the following two tables respectively. Calculate
      and the slit width ($w$) using equation (1) and the slit spacing ($d$) using equation (2).

      \[
      D \text{ (distance from slit to screen)} = \text{________________________ cm}
      \]
      \[
      \text{Labeled slit width} = \text{________________ mm}
      \]
      \[
      \text{Labeled slit spacing} = \text{________________ mm}
      \]

   e) Write down the equation you will use for calculating slit width ($w$):

   | order of | distance from | calculated |
   |———|———|———|
   | diffraction | principal maximum | slit width |
   | minimum n | $x_n$ (cm) | $w$ (mm) |
   | 1 | | |
   | 2 | | |
   | 3 | | |
   | 4 | | |
   | 5 | | |
   | Average slit width: |

   Q: Compare the average slit width with the labeled one. Comment:

   f) Write down the equation you will use for calculating slit spacing ($d$):
<table>
<thead>
<tr>
<th>order of interference maximum</th>
<th>distance from principal maximum ( x_n ) (cm)</th>
<th>calculated slit spacing ( d ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>4</td>
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<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average slit spacing:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Q: Compare the average slit spacing with the labeled one. Comment:

3. Babinet’s principle:
   a) Place a thin wire in the laser beam.

   Q: Is the position right behind the wire light or dark? (If you aren’t surprised, you should be!)

   b) Measure the distance between the wire and the screen (D) and record the labeled wire diameter.
   c) Tape a piece of paper to the screen. Mark the positions of the principal maximum and the diffraction minima. Remove the paper from the screen and attach it to your lab report.
   d) Measure and record the distance of each diffraction minimum from the principal maximum. Calculate the diameter of the wire using equation (1).

\[
D \text{ (distance from the wire to the screen)} = \text{________________________ cm}
\]

Labeled wire diameter = ________________ mm

Write down the equation you will use for calculating the wire diameter:
Order of
minimum
n
Distance from principal maximum
$x_n$ (cm)
Calculated
diameter
$w$ (mm)

1
2
3
4
5

Average wire diameter:

Q: Compare the average wire diameter with the labeled one. Comment:

Q: Is your hair thicker or thinner than the wire? How did you find out?

4. Multiple-slit interference and diffraction: a diffraction grating
   a) Place the grating in the laser beam (close to the screen, not far away).
   b) Measure the distance from the plane of the grating to the screen (D) and record the labeled ruling density (grooves/mm).
   c) Tape a piece of paper across the screen. Mark carefully the positions of the principal maximum and the diffraction maxima. Remove the paper from the screen and attach it to your lab report.
   d) Measure and record the distance of each diffraction maximum from the principal maximum ($x_n$). Calculate the slit spacing (d) using equation (2).

D (distance from the grating to the screen) = __________ cm
Labeled ruling density = __________ grooves/mm

<table>
<thead>
<tr>
<th>Order of maximum</th>
<th>Distance from principal maximum $x_n$ (cm)</th>
<th>Calculated slit spacing $d$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
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<tr>
<td>+1</td>
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<tr>
<td>+2</td>
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</tbody>
</table>

Average slit spacing:

e) Measured ruling density $= \frac{1}{d} = __________$ grooves/mm

Compare the measured ruling density with the labeled one. Comment: