OPTICAL MICROMETER

OBJECTIVES:

1) Describe qualitatively the operation of an optical micrometer.
2) Make measurements to verify the relationship between angle of rotation of a piece of parallel-surfaced refracting material and the displacement of a light beam passing through it.
3) Make measurements to verify that the light beam is displaced in a parallel manner.
4) Observe total internal reflection and calculate the critical angle.
5) Know the basis of how optical fibers work.

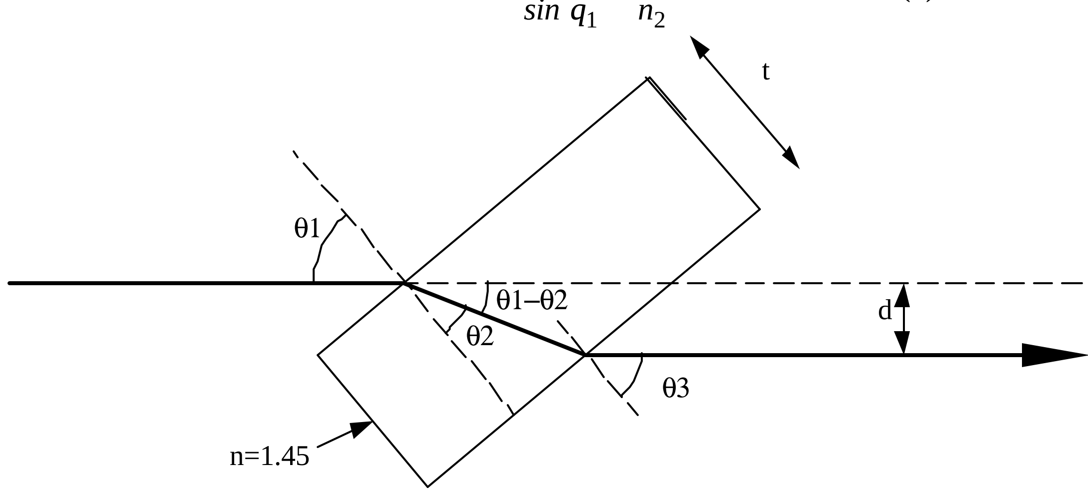
CAUTION!

The laser is a device that can produce an intense, narrow beam of light at one wavelength. NEVER look directly into the laser beam or its reflection from a mirror, etc.

INTRODUCTION

A beam of light traveling in a medium of index of refraction which is incident at an angle q_1 on the surface of another medium with index of refraction n_2 will be bent to a new angle q_2 according to Snell's Law,

$$\frac{\sin q_2}{\sin q_1} = \frac{n_1}{n_2} \qquad (1)$$



A beam of light passing through an object with parallel surfaces
Figure 1

By application of Snell’s Law, one can show that a beam of light will be displaced in a parallel manner when it passes through a piece of transparent material with parallel surfaces as shown in figure 1, that is $q_1 = q_3$. This is the basis of a device called the “Optical Micrometer”. This device can be put on the end of a transit or telescope and allows the beam to be shifted by a small controlled distance without changing its angle. By geometric construction, one can show that the displacement, d, is given by

$$d = \frac{t \sin\left(q_1 - q_2\right)}{\cos q_2} \qquad (2)$$

Here t is the thickness of the transparent material, and q_2 is given by Snell’s Law.

For our situation, the initial index of refraction, n_1, is 1.00, the value for air.

Total internal reflection is another application of Snell's law. It's called "total" internal reflection because normally, part of the light is reflected and part transmitted through refraction. In total internal reflection, no light is transmitted and all is reflected and remains internal to the material.

Consider Snell's law for light inside the lucite block, which strikes an outer surface, surrounded by air. Then $n_1 = 1.45$ and $n_2 = 1.00$, and θ_1 is angle from the normal at which the light strikes the surface. Snell's law indicates that $q_2 > q_1$. But as q_1 becomes larger, there will come a time when q_2 has already reached 90 degrees, and can't become any larger and $\sin(q_2) = 1$. Solving for q_1 which matches this condition, and using $n_2 = 1.0$, we find q_c, the critical angle, the largest value of for which there is refraction, or the smallest value of q_1 for which there is total internal reflection. Optical fibers use this phenomenon to efficiently transmit light directed along the axis of the fiber. Such light has q_2 near 90 degrees, which is above the critical angle, and it will be totally internally reflected, provided that the outer surface of the fiber is very smooth. The smoothness guarantees that there are no jagged corners where light can make a larger angle with the surface and escape through refraction.

Preparation for the lab: read from the introductory material of the lab book:

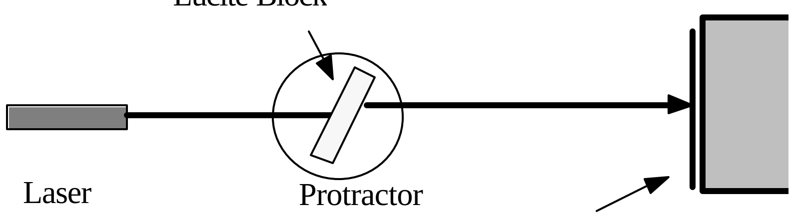
1. Uncertainty calculations for the general case of a function of a measured variable.
2. The Vernier Caliper (which you will use to measure the thickness of the lucite block).

Partner __________________________ Name _________________________

__________________________ Section ________________________

APPARATUS

The apparatus consists of a laser, a piece of Lucite (n= 1.45) mounted on a protractor, and a screen. The schematics of the set up are shown below.



Optical micrometer setup
Figure 2

PROCEDURE:

1. Align the Lucite block for normal incidence (Start with the screen fairly close to the block, about 0.5 meters)
   Turn on the laser. Rotate the protractor holding the Lucite block, and observe the movement of the beam reflected off the first surface of the Lucite block. When close to normal incidence, you can see the reflected beam hitting the surrounding area of the aperture at the face of the laser. Rotate the protractor to an angle so that the reflected beam hits the center of the aperture. (You can't really see the reflected beam when it hits exactly the center of the aperture. Just try to align it as best as you can.) You have now aligned the Lucite block for normal incidence.

   Write down the reading on the protractor: θ_0 = ___________________ degree
   Record the uncertainty of setting θ_0 . This includes the accuracy of adjusting the autocollimation, and the accuracy of reading the protractor angle. Measure the thickness of the Lucite block with the Vernier caliper. Take care not to scratch the Lucite with the calipers.

   $\delta\ \theta_0$ = ___________ degree t = _________ ± _________mm

2. Testing Equation (2) by measuring displacement as a function of incidence angle

a) Tape a piece of paper across the screen at an appropriate height where the transmitted beam hits the screen. Mark the position of the beam at normal incidence on the paper.

b) Set the protractor to $\theta_0\pm20°$, $\theta_0\pm40°$, $\theta_0\pm60°$ respectively.) Mark the position of the transmitted beam for each angle on the paper. Write the settings you used in the table labeled DATA. Mark the transmitted beam for each angle on the paper.

c) Remove the paper from the screen and attach it to your lab report. Measure the beam displacement from normal incidence, d_{meas}, for each angle, and record them in the DATA table, with positive or negative d for displacements corresponding to positive or negative θ_1. The directions in the following sections will help you fill out the rest of the tables. **Each column is marked with the section of the directions describing how to do the calculations**.

DATA

θ_1	b) $\theta_0 + \theta_1$	c) d_{meas} (mm)
20.0°		
-20.0°		
40.0°		
-40.0°		
60.0°		
-60.0°		

d) **For these steps keep "too many" significant figures**—these are theoretical numbers with high precision, and to calculate differences accurately you must avoid rounding until the very end. In the table labeled "CALCULATION of d_{calc} and its Uncertainty", calculate the values of θ_2 using equation (1) and d_{calc} using equation (2) with $n_{1(air)} = 1.0$ and $n_{2\ (Lucite)} = 1.45$. Below, write the equations you used for θ_2 and d_{calc}. Transfer d_{calc} to the "DATA" table and use $\Delta d = d_{meas} - d_{calc}$ to fill out the next column there.

e) Calculate the measurement uncertainties. Remember that θ_1 includes both the uncertainty of setting θ_0 and rotating the protractor by θ_1 degrees and making the reading. The d_{meas} values are obtained by transferring the light spots to pencil marks (δ_{mark}), then lining up the pencil marks with the ruler and finding the difference of position between 2 such marks. Therefor the value for δ_{meas} should included the error in measuring, the error in marking, and the error in setting you angle. It is the total error for measuring a given point.

$\delta\theta_1$ = _____ degree δ_{mark} = _____ mm δd_{meas} = _____ mm

f) Find the uncertainty of d_{calc} using the maximum uncertainty method:

$$\delta(d_{calc}) = |d'_{calc} - d_{calc}|$$

To do this, recalculate θ'_2 and d'_{calc} using $\theta'_1 = \theta_1 + \delta\theta_1$, an angle that differs from the angle you measured (θ_1), by the amount of its uncertainty ($\delta\theta_1$). This shows how much uncertainty in d_{calc} is induced by your uncertainty in θ_1. Enter this in the "CALCULATION" table.

Q: We neglected the uncertainty of the thickness in this calculation. Compare $\delta t / t$ with $\delta d / d$. Is it reasonable to have neglected $\delta t / t$? Why or why not?

Q: We only calculated d_{calc} and $\delta(d_{calc})$ for the positive θ_1. Explain how to find d_{calc} and the uncertainty of $\delta(d_{calc})$ for negative θ_1 and **justify your procedure**.

g) Use the uncertainties for d_{calc} and d_{meas} to calculate the uncertainty of $\delta(\Delta d)$. Show the formula you used.

CALCULATION of d_{calc} and its Uncertainty

θ_1	d) θ_2	d) d_{calc} (mm)	f) θ'_1	f) θ'_2	f) d'_{calc} (mm)	f) $\delta(d_{calc})$ (mm)	f) $\frac{d(d_{calc})}{d_{calc}}$	g) $\delta(\Delta d)$ (mm)
20.0°								
40.0°								
60.0°								

DATA and Comparison

θ_1	d) d_{calc} (mm)	d) Δd (mm)	g) $\delta(\Delta d)$ (mm)	g) compatible?
20.0°				
-20.0°				
40.0°				
-40.0°				
60.0°				
-60.0°				

Q: Are the calculated values of d compatible with the measurements?

3. Measure the angle of deviation to check for parallelism

a) Repeat steps 2 (a) through (c) with the screen as far away as possible. Measure L, the distance you moved the screen.

L = _____ mm

b) Measure and record d_{far} in the table below. Again, attach a piece of paper to your lab report, this time labeled "Deviation Measurement". Copy the values of d_{meas} from your previous table as d_{near}.

Q: What is the uncertainty in the quantity $|d_{far} - d_{near}|$? _____ mm

Deviation Measurement

θ_1	d_{far} (mm)	d_{near} (mm)	$d_{far} - d_{near}$ (mm)	Compatible with 0? (Use above error)	θ_{dev} (milliradians)
20					
-20					
40					
-40					
60					
-60					

c) The angle of deviation can be calculated by:

$$\theta_{dev}\text{ (milliradians)} = \left(\frac{d_{far} - d_{near}}{L}\right) \times 1000$$

Q: Explain the factor of 1000 in the equation above.

Q: If the displaced beam is perfectly parallel with respect to the incident beam, the displacements do not change, and the deviation is zero. Are your measurements consistent with the beam being perfectly parallel?

4. Observe total reflection.

   When a beam of light traveling in a medium with index of refraction n_1 incidents on the surface of another medium with index of refraction n_2, if $n_1>n_2$, total reflection (i.e., no light is transmitted) will occur when the incident angle (θ_1) is greater than a critical angle (θ_C). The value of the critical angle (θ_C) can be calculated by solving the equation given by Snell's law for θ_1, assuming $\theta_2 = 90°$. The phenomenon of total reflection is the basis of how optical fibers work. When light enters one end of the optical fiber, the beam is repeatedly reflected internally along the optical fiber and finally comes out from the other end.

   a) Calculate θ_C for a beam of light incident from Lucite (n_1=1.45) to air (n_2=1.0).

   b) Replace the rectangular Lucite block with a semi-circular block. Rotate the protractor and observe the reflection inside the Lucite block (look from top down at the light trace inside the block. You may need to turn off the room lights to see it clearly.) At a certain angle, you will see the reflected beam suddenly brightens. At this angle, the beam is totally reflected and this angle is the critical angle.

      Estimated critical angle: θ_C = _______________

      Q: Does the critical angle roughly agree with your calculation?

   c) Shine the laser beam at one end of a bare optical fiber.

      Q: Does the light only come out at the end of the cable? Why?

      Q: Give an example of an application of total internal reflection from medical or some other technology.