NAME...

PHY-852: QUANTUM MECHANICS II
Midterm (makeup), 30 points
March 28, 2002

PROBLEM 1. *a.* /3/ A wave function of a particle is

$$\psi(\mathbf{r}) = A(x^2 - y^2)ze^{-\beta r}. \tag{1}$$

Determine the probabilities of various values of l, m, and parity in this state.

b. /5/ The same for the function

$$\psi(\mathbf{r}) = A(x^2 + y^2)ze^{-\beta r}. \tag{2}$$

Here it may be useful to use the Legendre polynomial

$$P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta. \tag{3}$$

PROBLEM 2. /6/ For a spinless particle in a three-dimensional isotropic potential $U(r)$ (of arbitrary shape),

a. is it possible to have a level with the degeneracies $d = 2, d = 7, d = 9$?

b. what are the possible degeneracies of the first excited level?

For each case give an example.

PROBLEM 3. /16/ *a.* Determine the scattering amplitude and cross section for a low energy particle (formulate explicitly what is the low energy approximation and when it can be valid) from a repulsive potential

$$U(r) = \begin{cases} U_0 > 0, & r < R, \\ 0, & r > R. \end{cases} \tag{4}$$

b. Check your answer in the limit $U_0 \to \infty$.

c. Compare the result (a) with the Born approximation. What is the condition of validity for the Born approximation?

SOLUTION, *Problem 1.*

a. This case is quite similar to that in the main Midterm. In terms of raising, r_+, and lowering, r_-, component of the vector $\mathbf{r}$, the function (1), can be expressed as

$$\psi(\mathbf{r}) = \frac{A}{2}(r_+^2 + r_-^2) z e^{-\beta r} \propto (Y_{22} + Y_{2-2})Y_{10}. \tag{5}$$

Therefore we uniquely determine negative parity and $l = 3$ since another possibility with negative parity, $l = 1$, is incompatible with the projections $m = \pm 2$ which appear equiprobably.

b. This case is a little more complicated. It is convenient to rewrite the function separating the spherically invariant part r^2,

$$\psi(\mathbf{r}) = A(r^2 - z^2) z e^{-\beta r} \propto \eta - \eta^3, \quad \eta = \cos\theta. \tag{6}$$

Here the projection $m = 0$ is defined uniquely and parity is still negative. However this function is a superposition of Y_{10} and Y_{30}, or, in terms of the Legendre polynomials $P_l(\eta)$,

$$\eta - \eta^3 = aP_1 + bP_3 = a(P_1 - P_3), \tag{7}$$

where the last equality follows from the fact that in the forward direction ($\eta = 1$) all $P_l = 1$, and the left hand side vanishes, so that $a + b = 0$. Using the relation between Legendre polynomials and spherical functions, we obtain

$$\eta - \eta^3 = a\sqrt{4\pi}\left(\frac{Y_{10}}{3} - \frac{Y_{30}}{7}\right). \tag{8}$$

Since the spherical function are orthonormalized, the needed probabilities w_l of $l = 3$ and $l = 1$ are, respectively,

$$w_1 = \frac{7}{10}, \quad w_3 = \frac{3}{10}. \tag{9}$$