PRACTICE PROBLEMS - 2

1. Find the eigenvalues and the eigenfunctions of the operator $\mathcal{O} = a + (\mathbf{b}\cdot\mathbf{s})$, where a is a number, $\mathbf{b}$ a non-operator vector, and $\mathbf{s}$ spin operator for a particle of spin 1/2.

2. Reduce the operator $(a + \mathbf{b}\cdot\vec{\sigma})^8$ to the expression linear in Pauli matrices.

3. Find the explicit matrix expressions for the raising and lowering operators $s_{\pm}$ and their squares for a particle of spin 1/2.

4. Find the transformation of the spin function of a particle of spin 1/2 under rotation by an angle γ around the axis characterized by the unit vector $\mathbf{n}_0$.

5. Show that $\chi_1^{\dagger}\chi_2$ does not change under rotations.

6. *a.* Find the possible quantum numbers l, j, m for the particle of spin 1/2 in the state described by the wave function

$$\psi = R(r)(\mathbf{s}\cdot\mathbf{n})\chi, \tag{1}$$

   where χ is an arbitrary spinor and $\mathbf{n} = \mathbf{r}/r$.

   b. Normalize this function assuming that the radial function $R(r)$ is already normalized to 1.

   c. Find the angular distribution of probability for a particle described by the wave function (1).

7. A beam of particles of spin 1/2 is polarized in the direction $\mathbf{e}$. The beam is going through an analyzer that transmits only the particles polarized in the direction $\mathbf{e}'$. Find the transmission coefficient (a fraction of the original intensity transmitted through the analyzer).

8. A neutral particle of spin 1/2 with magnetic moment μ is placed in the uniform magnetic field which has a fixed direction $\mathbf{e}$ but changes in time, $\mathbf{B} = B(t)\mathbf{e}$. Find the time dependence of the spinor wave function and of the expectation values of spin components s_i.

9. A perturbation $u(x) = g\delta(x - a/2),\ g > 0$, is applied to a particle moving in the infinitely high potential box $0 \le x \le a$. Find the energy shifts in the first order of perturbation theory and compare to the exact solution.

10. Find the energy shifts of the levels of the one-dimensional harmonic oscillator under the cubic perturbation $u(x) = gx^3$. Discuss the validity of perturbation theory.

11. Find the splitting of the first excited state of the two-dimensional harmonic oscillator (in the xy-plane) in the presence of the perturbation $u(x) = Axy$.

12. Find the magnetic moment of an atom in the states ${}^1F$; ${}^2D_{3/2}$.

13. Find the possible values of the atomic magnetic moment in the state ${}^4S$.

14. An experiment shows that in an atomic state with quantum numbers ${}^7H$ the magnetic moment is equal to zero. Find the value of the total angular momentum of the atom.

15. Construct the scheme of possible radiative transitions between the states ${}^1D \rightarrow {}^1P$ of an atom in a static magnetic field.

16. A linear harmonic oscillator was in the kth stationary state in the remote past, $t \rightarrow -\infty$. Then the uniform variable weak electric field along the oscillation axis was applied,

$$\mathcal{E}(t) = \mathcal{E}_0 e^{-(t/\tau)^2}. \tag{2}$$

Find in the first order of perturbation theory the probability for the oscillator to be in the nth state at $t \rightarrow \infty$. Qualitatively explain what will change in the second order of perturbation theory.

17. A right hand side wall of an infinitely high potential box, $0 \leq x \leq a$ is suddenly shifted to a position $b > a$. Find the probability to find a particle in the box in the nth stationary state if originally the particle was in the ground state. What is the total probability of the excitation of the particle?

18. Estimate the radiation lifetime and the width of the excited $2p$ state in the μ-mesoatom (a hydrogen-like bound system of the negatively charged muon, $m_\mu \approx 207\, m_e$, and the proton); compare this lifetime with the decay lifetime of the free muon $\tau_\mu = 2.2 \times 10^{-6}$ c.

19. *a.* Show that the electric dipole transitions between the fine structure components of the same atomic term are forbidden.

    b. Show that the electric dipole transitions between the Zeeman components of the same atomic term in the static magnetic field are forbidden.