

**PHY-852 QUANTUM MECHANICS II**

**Homework 1**, 30 points

January 16-23, 2002

**Semiclassical approximation.**

Reading: *Merzbacher*, Chapter 7, sections 1 - 4.

1. /6/ *Merzbacher*, Exercise 7.5, p. 125.

2. A particle is moving in the potential field

$$U(x) = U_0 \left| \frac{x}{a} \right|^q, \quad U_0 > 0, \quad q > 0. \quad (1)$$

a. /8/ Using the semiclassical approximation find the energy spectrum, the spacing between the nearest levels  $s = E_{n+1} - E_n$  in the region of validity of the approximation and the level density  $\rho(E)$  as a function of the parameter  $q$ . Does the spacing  $s$  increase or decrease with the increase of  $n$ ?

b. /8/ For a semiclassical wave function  $\psi_n(x)$  calculate expectation values of potential energy,  $\langle n|U|n \rangle$ , and kinetic energy,  $\langle n|K|n \rangle$ , and check the virial theorem /*Merzbacher*, eq. (3.89)/.

3. /8/ Find in the semiclassical approximation the transmission coefficient through the “semicircular” barrier ( $U_0 > 0$ )

$$U(x) = \begin{cases} U_0 \left( 1 - \frac{x^2}{a^2} \right), & |x| < a, \\ 0, & |x| > a. \end{cases} \quad (2)$$

Discuss the criteria of validity of the result.