PHY-852 QUANTUM MECHANICS II
Homework 10, 40 points
April 10 - 17, 2002
Stationary perturbations.
Reading: *Merzbacher*, Chapters 8, 18.

1. /8/ *Merzbacher*, Problems 2 and 3, p. 177. [*It might be convenient to use for the solution the creation and annihilation operators.*]

2. *a.* /5/ A static electric field is applied along the axis of vibration of a charged linear harmonic oscillator. Find the first and second order corrections to the energy levels of the oscillator; compare the result with the exact solution.

   b. /5/ A mass m of the linear harmonic oscillator is increased by δm. Find the first and second order corrections to the energy levels; compare the result with the exact solution and find the convergence condition for perturbation theory.

3. A small electric dipole $\mathbf{d}$ with the moment of inertia I can freely rotate in its plane xy; the corresponding Hamiltonian is

$$\hat{H} = \frac{\hbar^2 \hat{l}^2}{2I}, \tag{1}$$

   where $\hat{l} = \hat{l}_z$ is the angular momentum component for the rotation in plane. A static electric field $\vec{\mathcal{E}}$ is applied along the x-axis.

   a. /5/ Considering the electric field as a perturbation find the polarizability of the ground state.

   b. /8/ Find the first and second order corrections to the energy levels.

4. $\star$ /9/ Derive the uncertainty relations for the Heisenberg operators of coordinate $\hat{x}(t_1)$ and momentum $\hat{p}(t_2)$ in case of (i) free motion; (ii) a particle in a uniform field of a force; (iii) a harmonic oscillator.