PHY-852 QUANTUM MECHANICS II
Homework 2, 40 points
January 23 - 30, 2002
Separation of variables. Orbital momentum.
Reading: *Merzbacher*, Chapter 11, sections 3 - 5; Chapter 12.

1. *a.* /6/ *Merzbacher*, Problem 1, p. 255.

   b. /5/ For an arbitrary state ψ_{lm} of a particle with orbital momentum l and projection $l_z = m$ find expectation values $\langle \hat{l}_x \rangle$, $\langle \hat{l}_y \rangle$, $\langle \hat{l}_x^2 \rangle$, $\langle \hat{l}_y^2 \rangle$, $\langle \hat{l}_x \hat{l}_y \rangle$, $\langle \hat{l}_y \hat{l}_x \rangle$.

2. /2/ *a.* Consider a particle of mass m moving in the (x, y)-plane in the harmonic oscillator trap described by the anisotropic potential

$$U(x, y) = \frac{1}{2} m w_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2. \tag{1}$$

   Separate the Cartesian coordinates x, y and find the energy spectrum and the stationary wave functions.

   b. /3/ In the limit of an *isotropic* oscillator, $\omega_x = \omega_y$, find the degeneracy of a given energy value E (a number of independent states with the same energy).

   c. /10/ Continue to consider the isotropic case. Introduce the polar coordinates ρ and ϕ in the plane, write the Schrödinger equation in these coordinates, and separate the variables. Solve the angular and radial parts, see a hint in *Merzbacher*, Problem 13, p. 276; it is convenient to introduce a variable $z = (m\omega/\hbar)\rho^2$. Show that the energy spectrum and the degree of degeneracy found in polar coordinates coincide with those in Cartesian coordinates, point *b*.

   d. /4/ For the lowest 6 states of point *c* establish the explicit correspondence between the solutions in Cartesian and polar coordinates.

3. Often molecules and nuclei can be modeled by a plane *rotor*, a system with only one degree of freedom (a polar angle ϕ), described by a rotational Hamiltonian $\hat{H} = \hbar^2 \hat{l}_z^2/(2I)$ with the moment of inertia I.

   a. /3/ Find the normalized eigenfunctions of the plane rotor, energy eigenvalues and their degeneracy. Construct the combinations of the degenerate eigenfunctions with certain parity under reflection in the plane with respect to the x-axis.

   b. /7/ Consider the state of the rotor with the wave function $\psi(\phi) = A \cos^n \phi$ (n integer ≥ 0). Find the probability distributions of the projection $m = l_z$ and of energy, and expectation values of these quantities.