PHY-852 QUANTUM MECHANICS II
Homework 6, 30 points
February 20 - 27, 2002
Elastic scattering and Born approximation.
Reading: *Merzbacher*, Chapters 13 and 20.

1. *a.* /3/ *Merzbacher*, Exercise 13.7.

   b. /5/ Establish the relation between your result and the Bragg condition

   $$n\lambda = 2d \sin\frac{\theta}{2} \tag{1}$$

   for the scattering angles θ that correspond to the interference maxima in the particle scattering by a system of parallel crystallic planes (n is an integer and λ is the wave length of the particle).

2. *a.* /3/ *Merzbacher*, Exercise 13.9.

   b. /3/ *Merzbacher*, Exercise 13.10.

   c. /4/ For the total cross section in problems *a* and *b* consider the limiting cases of low and high energies, $ka \ll 1$ and $ka \gg 1$, respectively (in the *Exercise* 13.9 a is the radius of the well).

3. Consider the standard expression for the elastic scattering amplitude for a particle of mass m in the potential $U(\mathbf{r})$,

   $$f(\mathbf{k}', \mathbf{k}) = -\frac{m}{2\pi\hbar^2}\int d^3r\, e^{-i(\mathbf{k}'\cdot\mathbf{r})}U(\mathbf{r})\psi_{\mathbf{k}}(\mathbf{r}), \quad \mathbf{k}' = k\frac{\mathbf{r}}{r}, \tag{2}$$

   where the wave function $\psi_{\mathbf{k}}(\mathbf{r})$ has at large distances r the asymptotic behavior

   $$\psi_{\mathbf{k}}(\mathbf{r}) \approx e^{i(\mathbf{k}\cdot\mathbf{r})} + f(\mathbf{k}', \mathbf{k})\frac{e^{ikr}}{r}. \tag{3}$$

   a. /4/ For the potential that at large distances falls off as $U(r) \propto r^{-s}$, find which values of s allow the asymptotic solution in the form (2).

   b. /5/ For a potential which becomes negligibly small at distances $r \sim R$, find at what distances the asymptotic representation (3) becomes valid.

   c. /3/ For an arbitrary potential $U(r)$ with central symmetry, show that if the Born approximation is valid, the product $E\sigma(E)$, where $\sigma(E)$ is the total scattering cross section at energy E, is a monotonously growing function of E.