PHY-852 QUANTUM MECHANICS II
Homework 7, 30 points
March 20 - 27, 2002

Scattering. Angular momentum.
Reading: Merzbacher, Chapters 11, 13, 16.

1. /7/ A particle is scattered on a molecule that has a form of a plane square
with a side length $a$ and four identical atoms in the corners; the inter-
action potential between the particle and each atom is $U(r)$. Using the
Born approximation find the relation between the scattering amplitudes,
differential and total cross sections for the scattering off the molecule and
scattering off an individual atom (consider the cases of low and high en-
ergy).

2. /4/ Taking into account the s- and p-waves, find the angular distribution
of the scattered particles and the total cross section.

3. /5/ Merzbacher, Exercise 11.9.

4. /10/ a. For a particle with orbital momentum $l = 1$ construct the wave
function $\psi(\theta, \phi)$ for the state with the zero projection of the orbital mo-
mentum vector onto the axis defined by the polar angle $\alpha$ and asimuthal
angle $\beta$.

b. The same for the state with projection $l_x = 1$ onto the $x$-axis.

c. For a particle in the state with the orbital momentum quantum numbers
$l = 1$ and $m$ find the probabilities of various values $m'$ of the orbital
momentum projection on the axis $z'$ which has an angle $\alpha$ with respect to
the $z$-axis;

d. For the same state $|l = 1, m\rangle$ find the expectation values $\langle \hat{l}_x \rangle$ and $\langle \hat{l}_y \rangle$,
where $n$ is a positive integer.

5. /4/ Show that the components of the angular momentum $\hat{J}$ of an arbitrary
system satisfy the relation

$$e^{-i\beta \hat{J}_y} \hat{J}_x e^{i\beta \hat{J}_y} = \hat{J}_x \cos \beta + \hat{J}_z \sin \beta.$$  (1)