PHY-852 QUANTUM MECHANICS II

Homework 8, 25 points

March 27 - April 3, 2002

Spin.

Reading: *Merzbacher*, Chapter 16.

1. /3/ *Merzbacher*, Exercise 16.9.
2. /5/ *Merzbacher*, Exercise 16.12.
3. /5/ *Merzbacher*, Exercise 16.19.
4. /3/ Calculate $(\mathbf{a}\cdot\vec{\sigma})^8$, where $\mathbf{a}$ is an arbitrary fixed vector and $\vec{\sigma}$ is a vector whose Cartesian components are spin Pauli matrices σ_i.
5. /6/ *a.* Let χ and φ be two arbitrary spinors for a particle of spin 1/2. Using three Pauli matrices $\vec{\sigma}$, construct the quantity

$$\mathbf{V} = \varphi^\dagger \vec{\sigma} \chi. \tag{1}$$

   Show that under rotation the components V_i are transformed as the components of a vector. Check your results for a rotation around the z-axis.

   b. /3/ A proton state is characterized by the momentum $\mathbf{p}$ along the z-axis and by the certain value $h = +1$ of the helicity (projection $h = (\vec{\sigma}\cdot\mathbf{p})/|\mathbf{p}|$ of the proton spin ($\mathbf{s} = (1/2)\vec{\sigma}$) on the direction of the momentum $\mathbf{p}$). Let $\hat{\mathbf{j}}$ be the total angular momentum operator of the proton. What are the characteristics of the state obtained from the original one by the action of the operator $\exp(-i\hat{j}_y\alpha)$?