PHY-852 QUANTUM MECHANICS II
Homework 9, 25 points
March April 3 - 10, 2002
Angular momentum coupling.
Reading: *Merzbacher*, Chapter 17.

1. *a*. Consider a particle of spin 1/2 in a spherically symmetric potential field. Taking into account the existence of spin-orbit coupling and selecting the z-axis as that of quantization, list all quantum numbers necessary for the description of a stationary state.

   b. Show that the stationary states are eigenfunctions of the operator $(\mathbf{l}\cdot\mathbf{s})$ and find the corresponding eigenvalues.

   c. Show that any stationary state with quantum numbers j, m of the total angular momentum and its z-projection is a superposition of two states with given projections of the orbital momentum, l_z, and spin, s_z, and write down the amplitudes of the superposition in a standard form of the Clebsch-Gordan coefficients.

   d. Requiring that the superposition be an eigenfunction of the operator $(\mathbf{l}\cdot\mathbf{s})$, obtain a set of two coupled equations for these coefficients. Show that the determinant of this set vanishes for the same eigenvalues of $(\mathbf{l}\cdot\mathbf{s})$ as found in point (b).

   e. Solve the set of equations, normalize the solutions and give a table of two sets of the Clebsch-Gordan coefficients for two possible (at fixed value of l) values of the total angular momentum j.

   f. Write down the spin-angular part of the wave function as a two-component spinor $(\alpha(\mathbf{n}), \beta(\mathbf{n}))$ where α and β are, correspondingly, the amplitudes of probability to find the particle at the angle $\mathbf{n}(\theta, \phi)$ with the spin up or down with respect to the z-axis. Determine what is the direction of the spin polarization at point $\mathbf{r}$ in space.

2. *Merzbacher*, Problem 4, p. 449.

3. *Merzbacher*, Problem 6, p. 449.

4. * Consider an electron bound near the surface $x = 0$ of a metal. An electric field $\mathcal{E}$ is applied along the x-axis pulling the electron out. Using the semiclassical approximation calculate the electron transmission coefficient through the surface. Find the condition of validity of the semiclassical approximation (assume the binding energy $\epsilon = 1$ eV).