1. a. Consider a particle of spin 1/2 in a spherically symmetric potential field. Taking into account the existence of spin-orbit coupling and selecting the z-axis as that of quantization, list all quantum numbers necessary for the description of a stationary state.

b. Show that the stationary states are eigenfunctions of the operator \(\mathbf{l} \cdot \mathbf{s}\) and find the corresponding eigenvalues.

c. Show that any stationary state with quantum numbers \(j, m\) of the total angular momentum and its z-projection is a superposition of two states with given projections of the orbital momentum, \(l_z\), and spin, \(s_z\), and write down the amplitudes of the superposition in a standard form of the Clebsch-Gordan coefficients.

d. Requiring that the superposition be an eigenfunction of the operator \((\mathbf{l} \cdot \mathbf{s})\), obtain a set of two coupled equations for these coefficients. Show that the determinant of this set vanishes for the same eigenvalues of \((\mathbf{l} \cdot \mathbf{s})\) as found in point (b).

e. Solve the set of equations, normalize the solutions and give a table of two sets of the Clebsch-Gordan coefficients for two possible (at fixed value of \(l\)) values of the total angular momentum \(j\).

f. Write down the spin-angular part of the wave function as a two-component spinor \((\alpha(n), \beta(n))\) where \(\alpha\) and \(\beta\) are, correspondingly, the amplitudes of probability to find the particle at the angle \(n(\theta, \phi)\) with the spin up or down with respect to the z-axis. Determine what is the direction of the spin polarization at point \(r\) in space.


4. * Consider an electron bound near the surface \(x = 0\) of a metal. An electric field \(E\) is applied along the \(x\)-axis pulling the electron out. Using the semiclassical approximation calculate the electron transmission coefficient through the surface. Find the condition of validity of the semiclassical approximation (assume the binding energy \(\epsilon = 1\) eV).