**PHY-852 QUANTUM MECHANICS II**
**Homework 9**, 25 points
*March April 3 - 10, 2002*
**Angular momentum coupling**.
Reading: *Merzbacher*, Chapter 17.

1. *a.* Consider a particle of spin 1/2 in a spherically symmetric potential field. Taking into account the existence of spin-orbit coupling and selecting the $z$-axis as that of quantization, list all quantum numbers necessary for the description of a stationary state.

   *b.* Show that the stationary states are eigenfunctions of the operator $(\mathbf{l}\cdot\mathbf{s})$ and find the corresponding eigenvalues.

   *c.* Show that any stationary state with quantum numbers $j, m$ of the total angular momentum and its $z$-projection is a superposition of two states with given projections of the orbital momentum, $l_z$, and spin, $s_z$, and write down the amplitudes of the superposition in a standard form of the Clebsch-Gordan coefficients.

   *d.* Requiring that the superposition be an eigenfunction of the operator $(\mathbf{l}\cdot\mathbf{s})$, obtain a set of two coupled equations for these coefficients. Show that the determinant of this set vanishes for the same eigenvalues of $(\mathbf{l}\cdot\mathbf{s})$ as found in point ($b$).

   *e.* Solve the set of equations, normalize the solutions and give a table of two sets of the Clebsch-Gordan coefficients for two possible (at fixed value of $l$) values of the total angular momentum $j$.

   *f.* Write down the spin-angular part of the wave function as a two-component spinor $(\alpha(\mathbf{n}), \beta(\mathbf{n}))$ where $\alpha$ and $\beta$ are, correspondingly, the amplitudes of probability to find the particle at the angle $\mathbf{n}(\theta, \phi)$ with the spin up or down with respect to the $z$-axis. Determine what is the direction of the spin polarization at point $\mathbf{r}$ in space.

2. *Merzbacher*, Problem 4, p. 449.

3. *Merzbacher*, Problem 6, p. 449.

4. $^{\star}$ Consider an electron bound near the surface $x = 0$ of a metal. An electric field $\mathcal{E}$ is applied along the $x$-axis pulling the electron out. Using the semiclassical approximation calculate the electron transmission coefficient through the surface. Find the condition of validity of the semiclassical approximation (assume the binding energy $\epsilon = 1$ eV).