NAME..

PHY-852: QUANTUM MECHANICS II
Quiz 4
April 2002

PROBLEM. Electrons in an atom are in the stationary state with total angular momentum $\mathbf{J}$. A nucleus of the atom has its own angular momentum $\mathbf{I}$. The hyperfine interaction between the magnetic moments of the electrons and the nucleus is proportional to the scalar product of the angular momenta,

$$V_{h.f.s.} = A(\mathbf{J} \cdot \mathbf{I}). \tag{1}$$

a. Which of the following quantities are conserved by this interaction: components of the vectors $\mathbf{J}, \mathbf{I}, \mathbf{J} \pm \mathbf{I}$, their squares $\mathbf{J}^2, \mathbf{I}^2, (\mathbf{J} \pm \mathbf{I})^2$?

b. For given absolute values of J and I, what splitting of original atomic levels is produced by this interaction (how many split levels and what is the energy spacing between them)?

SOLUTION. *a.* The total angular momentum of the atom

$$\mathbf{F} = \mathbf{J} + \mathbf{I} \tag{2}$$

is the only vector whose components are constants of motion. But all squares are conserved.

b. The split levels carry the possible values of F from $|J - I|$ to $J + I$. The number of split levels is $2J_< + 1$, where $J_<$ is the smaller of J and I. The shift of the level with the total momentum F is

$$\delta E(F) = \frac{A}{2}[F(F+1) - J(J+1) - I(I+1)]. \tag{3}$$

The spacing between the adjacent split levels,

$$\delta E(F) - \delta E(F-1) = \frac{A}{2}[F(F+1) - (F-1)F] = AF. \tag{4}$$

is linear in F.