

EXPERIMENT: PROJECTILE MOTION

OBJECTIVES

- to ascertain that the parabolic motion of an object in two dimensions can be analyzed by considering the motion in each dimension separately
- to check for the existence of possible systematic errors in the data by comparing the obtained values for acceleration in the horizontal and vertical directions with the expected values

APPARATUS

A pre-recorded video of object in projectile motion will be used.

THEORY

This lab involves a two-dimensional projectile motion problem. Therefore, it goes one step beyond the one-dimensional studies of forces and acceleration done in the previous two labs. The most general projectile motion would involve three dimensions and could be described by referring to the x-, y-, and z-components of the velocity and acceleration vectors in the problem. This component-wise approach greatly simplifies the understanding of the motion. We are able to make this simplification when analyzing parabolic motion because, for example, the x-component of the acceleration affects only the x-component of the velocity, etc. Therefore the equations describing three-dimensional motion simplify to three sets of equations, each of which we already understand from our one-dimensional studies.

Further simplification comes through a shrewd choice of the orientation of the coordinate system to be used. If there is only one acceleration vector in the problem, we may choose one axis to be in the direction of that acceleration. This makes two components of the acceleration equal to zero and simplifies the equations of motion considerably. Proper choice of the second axis can often make $v_z = 0$ as well as $a_z = 0$, and then the problem involves only two dimensions (x and y) since there is no motion at all in the z-direction. This is the situation in the present experiment.

At any time t , the instantaneous velocity of a projectile is a vector, \mathbf{v} , tangent to the flight path. This velocity vector can be split up into a horizontal component, v_x , and a vertical component, v_y . If we neglect air friction then the only force acting on the projectile AFTER it leaves the launcher is the gravitational force, which is directed vertically downward. In this case, there are no forces acting in the x-dimension.

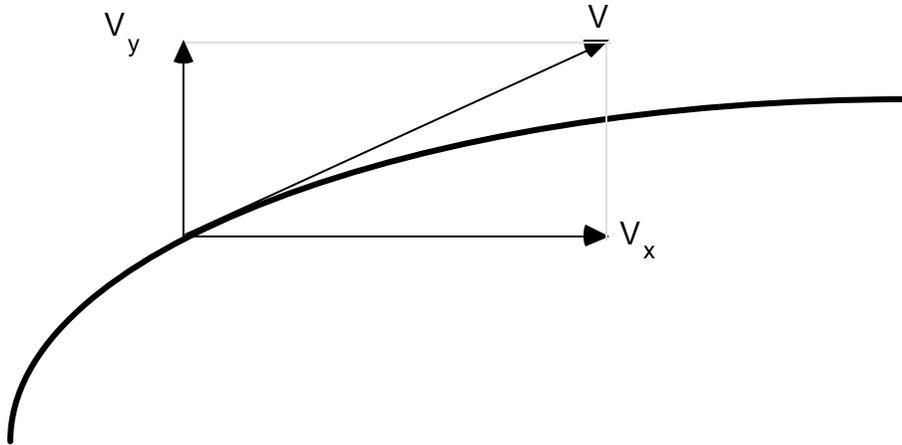


Figure 1

Recall from Newton's Second Law that:

$$\mathbf{F} = m\mathbf{a} \quad (1)$$

This is a vector equation. The vector on the left (\mathbf{F}) can equal the vector on the right ($m\mathbf{a}$) if and only if each component of the left vector equals each component of the right vector. Therefore, the motion of a projectile can be separated mathematically into independent components. In the x -dimension

$$F_x = ma_x \quad (2)$$

But if $F_x = 0$, then ma_x must equal zero also, and since we know that m , the mass, is not equal to zero, we must have:

$$a_x = 0 \quad (3)$$

Similarly, in the y -dimension:

$$F_y = ma_y \quad (4)$$

F_y is the force of gravity, so a_y is expected to be g , the acceleration due to gravity:

$$\begin{aligned} F_y = ma_y &= -mg \\ a_y &= -g \end{aligned} \quad (5)$$

where the minus sign indicates that the force (and thus also the acceleration) is directed vertically downward.

In Summary - According to equations (3) and (5), the two-dimensional projectile motion can be described in terms of two one-dimensional motions:

- 1) Motion with constant velocity (no acceleration) in the horizontal direction, and
- 2) Motion with constant acceleration in the vertical direction. That is,

$$v_x = v_{x0} \quad (6a)$$

$$v_y = v_{y0} - gt, \quad (6b)$$

where v_{x0} and v_{y0} are the initial components of velocity in the horizontal and vertical directions, respectively.

PROCEDURE

In this lab you will use a set of recorded video images to calculate the position and speed of a ball moving in projectile motion. The video images have been prerecorded because it takes more time than is available in class to create a good one.

The trajectory of the ball has been recorded with a video camera. The camera takes 30 images (frames) per second, so the time between images is roughly 0.033 seconds. These images are captured on our computers and are available for analysis. The movie of the ball is in the Lab 5 folder for this week. The software called *Videopoint* will allow you to select a particular frame and measure the position of the ball in that frame. Start the program from the *Videopoint* icon on the desktop. After starting the program, select the “Open Movie” command and open the movie stored in the Lab 5/Projectile Motion folder for this week. When you are asked by the program, indicate that you will use only one object for digitization.

After opening the movie, watch it by clicking on the “play” button which is located on the control bar at the bottom of the image viewer. *Videopoint* shows you the frame number that is currently being displayed in the image viewer. After watching the whole movie, go to the first frame that corresponds to the moment when the ball leaves the launcher and becomes visible. Do this by clicking the “forward” or “rewind” buttons on the control bar until you reach the appropriate frame.

On your screen you will also see that there is a table already created for you which contains the image number, the time associated with each image and locations for storing x and y coordinates. If you simply click on the “bull’s eye” icon, and then use the bull’s eye to click on the position of the ball in each frame, *Videopoint* will store the x and y coordinates in the reserved locations in the table. As you click on the ball in the first frame to measure its position, the program will automatically enter the x and y coordinates into the table and it will advance the movie to the next frame. Therefore you can simply keep on clicking on the position of the ball without any additional steps until you have gone through the whole movie. Make sure not to click on the ball in any of the final frames after the ball has hit the table, and is therefore no longer subject to free-fall.

The x and y coordinates will be measured for you in the “Origin 1” coordinate system in units of “pixels.” These are not the units we want, so we must convert them. In order to do this you must first click on the “ruler” icon in the toolbar. You will be asked to enter the dimensions of the object of known length. In our movie, the distance between the two “poles” with ball-shaped tops is one meter. Therefore enter “1.00” m in the appropriate place and then click “continue.” You must then click on one the top of one pole and then the top of the other pole. You should now see a line connecting the two poles. You have now told the program that the number of pixels along that line corresponds to one meter. Make sure to click on the top “ball” of each pole as close to its center as possible, so that your conversion factor is as exact as is possible. In the origin window you will now see a conversion factor from pixels to meters. From now on all your x and y locations will be measured and displayed in meters and any that were already stored are automatically converted to meters. **DO NOT** change the origin of the “Origin 1” coordinate system, that is, leave it at the lower left corner of the frame.

Make your first set of graphs in *Videopoint* by clicking the “graph” icon. Make the following three graphs:

- 1) y versus x

- 2) y versus time
- 3) x versus time

Make sure these graphs have appropriate titles and axes titles with units. At the bottom of each graph write a brief graph analysis.

Copy your data table to the clipboard and transfer it to this week's Excel spreadsheet. Only copy the part of the table that contains actual data.

From the transferred coordinates, you will calculate v_x and v_y for each image using the equations

$$v_x = \frac{\Delta x}{\Delta t}, \quad (7a) \qquad v_y = \frac{\Delta y}{\Delta t}, \quad (7b)$$

where Δx refers to the distance between the image PRECEDING and the image FOLLOWING the actual frame you are considering in the x view, similarly for Δy in the y view. Δt is the time interval between these two images ($t_{i+1} - t_{i-1}$). The formulae are given on the spreadsheet. On the spreadsheet you will calculate the quantity "DeltaT" first. This is the time elapsed between two adjacent images and is simply a check of the time between every two images, which naturally should be constant. **NOTE: DeltaT is not the same as Δt .**

According to equations 6a and 6b, the graphs of v_x vs. t and v_y vs. t should be straight lines, the slopes of which are zero and g respectively. The equations also tell us that if the instant at which the projectile left the launcher can be established from your movie, then the vertical intercepts of your v vs. t graphs should be v_{x0} and v_{y0} .

In order to see how linear the two graphs are, make graphs of v_x vs. t and v_y vs. t. Do this by transferring the spreadsheet data for time and speed into *Kaleidagraph* and making ONE graph containing both v_x vs. t and v_y vs. t. Once the data is plotted have the program calculate a best-fit line for BOTH the v_x and v_y data. Determine the slopes of each from the best-fit lines and compare the accelerations with the expected values.

CHECKLIST

Your lab report should include the following six items:

- 1) a graph of y vs. x of your original data (from *Videopoint*) with analysis
- 2) graphs of y vs. t and x vs. t from *Videopoint* with analyses
- 3) one graph showing v_x vs. t and v_y vs. t and with linear fits and equations of lines
- 4) comments upon how well you obtained the expected acceleration within the uncertainties. Are any systematic errors present in your data? Give a couple of examples of possible systematic errors.*
- 5) estimates of how accurately you measure an x, y point on the video. In other words, give a discussion of the random error present in your data.*

*If you need a refresher on random and systematic error, review the section “Measurements, Uncertainties, and Errors” at the beginning of the lab manual.

- 6) your conclusions, based on the results of your experiment, as to how successfully the motion of a projectile can be described in terms of two independent components. Remember:

The equations describing displacement in the x- and y-dimensions are

$$y = v_{yo} t - \frac{at^2}{2} \quad \text{and} \quad x = v_{xo}t.$$

These two equations can be combined by eliminating the variable t, that is, by substituting t from the second equation into the first:

$$t = \frac{x}{v_{xo}}$$
$$\therefore y = v_{yo} \frac{x}{v_{xo}} - \frac{g}{2} \left(\frac{x}{v_{xo}} \right)^2$$

When simplified this can be seen to be the equation of a parabola that opens downward. Is this what you observe in your graph of y vs. x?