PHY-852 QUANTUM MECHANICS II
Homework 1, 30 points
January 16-23, 2002
Semiclassical approximation.
Reading: *Merzbacher*, Chapter 7, sections 1 - 4.

1. /6/ *Merzbacher*, Exercise 7.5, p. 125.

2. A particle is moving in the potential field

$$U(x) = U_0 \left| \frac{x}{a} \right|^q, \quad U_0 > 0, \quad q > 0. \tag{1}$$

   a. /8/ Using the semiclassical approximation find the energy spectrum, the spacing between the nearest levels $s = E_{n+1} - E_n$ in the region of validity of the approximation and the level density $\rho(E)$ as a function of the parameter q. Does the spacing s increase or decrease with the increase of n?

   b. /8/ For a semiclassical wave function $\psi_n(x)$ calculate expectation values of potential energy, $\langle n|U|n\rangle$, and kinetic energy, $\langle n|K|n\rangle$, and check the virial theorem /*Merzbacher*, eq. (3.89)/.

3. /8/ Find in the semiclassical approximation the transmission coefficient through the "semicircular" barrier ($U_0 > 0$)

$$U(x) = \begin{cases} U_0 \left(1 - \frac{x^2}{a^2}\right), & |x| < a, \\ 0, & |x| > a. \end{cases} \tag{2}$$

   Discuss the criteria of validity of the result.