PHY-852 QUANTUM MECHANICS II
Homework 4, 30 points
February 6 - 13, 2002
Motion in a central field.
Reading: *Merzbacher*, Chapter 11, sections 3 - 5; Chapter 12.

1. /8/ *a.* Find the first order differential equation for the spherical function $Y_{ll}(\theta,\phi)$ with the maximum projection $m = l$, solve this equation and normalize the solution. Explain the corresponding physical picture.

   b. Using the lowering operator construct the function $Y_{l,l-1}(\theta,\phi)$.

2. /4/ Consider a particle of mass m in a spherically symmetric potential well with the potential equal to $U(r) = -U_0, U_0 > 0$, for $r < R$ and $U(r) = 0$ for $r > R$. Find the interval of values U_0 where the well supports two bound states with orbital momentum $l = 0$.

3. /7/ *a.* An object of mass M confines a loosely bound light particle of mass m with binding energy ϵ. The particle is suddenly released from the system by an external perturbation, and the distribution of probabilities $W(\mathbf{p})$ of different values of the particle momentum $\mathbf{p}$ is measured. Assuming that the particle in the bound state spends most of the time outside the well, obtain a simple analytical expression for $W(\mathbf{p})$.

   b. In many experiments, instead of the full three-dimensional momentum distribution $W(\mathbf{p})$, only the "longitudinal" distribution $W_x(p_x)$ of the momentum component p_x along a specific axis x is measured. Using the result *a*, predict the function $W_x(p_x)$.

   c. Assume that the neutron binding energy in a "halo" nucleus which consists of 3 protons and 8 neutrons is only 0.3 MeV. The experiment measuring $W_x(p_x)$ shows a bell-shaped curve with the full width at half maximum equal to $\Delta p_x = 44$ MeV/c where the convenient units for the momentum are used, c is the speed of light. Does this result agree with the predictions of the point *b*?

4. /5/ An electron is in the ground state of the tritium atom. The tritium nucleus ^{3}H (one proton + two neutrons) suddenly undergoes beta-decay into the helium nucleus ^{3}He (two protons and one neutron); the new electron and antineutrino created in the beta-decay carry away the electric charge and energy. Find the probability for the original atomic electron to remain in the ground state of the helium atom.

5. /6/ Consider the dicrete spectrum of states of a particle in an attractive central field. Let $E_{min}(l)$ be the lowest energy level with the orbital momentum l. Prove that

$$E_{min}(l) < E_{min}(l') \quad \text{if} \quad l < l'. \tag{1}$$