PHY-852 QUANTUM MECHANICS II
Homework 7, 30 points
March 20 - 27, 2002
Scattering. Angular momentum.
Reading: *Merzbacher*, Chapters 11, 13, 16.

1. /7/ A particle is scattered on a molecule that has a form of a plane square with a side length a and four identical atoms in the corners; the interaction potential between the particle and each atom is $U(\mathbf{r})$. Using the Born approximation find the relation between the scattering amplitudes, differential and total cross sections for the scattering off the molecule and scattering off an individual atom (consider the cases of low and high energy).

2. /4/ Taking into account the s- and p-waves, find the angular distribution of the scattered particles and the total cross section.

3. /5/ *Merzbacher*, Exercise 11.9.

4. /10/ *a.* For a particle with orbital momentum $l = 1$ construct the wave function $\psi(\theta, \phi)$ for the state with the zero projection of the orbital momentum vector onto the axis defined by the polar angle α and asimuthal angle β.

   b. The same for the state with projection $l_x = 1$ onto the x-axis.

   c. For a particle in the state with the orbital momentum quantum numbers $l = 1$ and m find the probabilities of various values m' of the orbital momentum projection on the axis z' which has an angle α with respect to the z-axis;

   d. For the same state $|l = 1, m\rangle$ find the expectation values $\langle \hat{l}_x^n \rangle$ and $\langle \hat{l}_y^n \rangle$, where n is a positive integer.

5. /4/ Show that the components of the angular momentum $\hat{\mathbf{J}}$ of an arbitrary system satisfy the relation

$$e^{-i\beta \hat{J}_y} \hat{J}_z e^{i\beta \hat{J}_y} = \hat{J}_z \cos\beta + \hat{J}_x \sin\beta. \tag{1}$$