NAME..

PHY-852: QUANTUM MECHANICS I
Quiz 1
January 28, 2002

PROBLEM. An angular part of the wave function of a particle is

$$\psi = A \sin^2 \theta. \tag{1}$$

Find the possible values of l and m in this state and their probabilities.

SOLUTION.

The absence of the ϕ-dependence shows that only $m = 0$ is possible. Because of parity, we can have only even l, and since this is a polynomial of the second order in $\cos\theta$, the allowed values are $l = 0$ and $l = 2$. Therefore we have a superposition of Y_{00} and Y_{20}, or $P_0 = 1$ and $P_2 = (3/2)\cos^2\theta - (1/2)$,

$$\psi = A[1 - \cos^2\theta] = A \cdot \frac{2}{3}(P_0 - P_2) = \frac{2}{3}A\sqrt{4\pi}\left(Y_{00} - \frac{1}{\sqrt{5}}Y_{20}\right). \tag{2}$$

¿From the weights of the relative components of the orthonormalized functions Y_{00} and Y_{20} we find the probabilities w_2 of $l = 2$ and w_0 of $l = 0$:

$$w_2 = \frac{1}{6}, \quad w_0 = \frac{5}{6}. \tag{3}$$