Homework Problems Chapter 9

1. A force is not gravity, nor a nuclear force. What kind of force is it?

Answer: It is **Electromagnetic** in nature.

2. A net force acts on a mass in the direction opposite to its motion. As the mass moves the work done on it by that force is *positive* or *negative*.

Answer: The work done on the mass by the force is **negative**.

3. The net force acting on a mass is in the same direction as its motion. As it moves, is the work done on it *positive* or *negative*?

Answer: The work done on the mass by the force is **positive**.

4. Compared to the direction of a net force that does a positive amount of work on a mass, does the mass move in the *same* or *opposite* direction?

Answer: When a net force does negative work on a mass it is moving in the direction **opposite** to the force. When a net force does positive work on a mass it is moving in the **same** direction as the force.

5. The two forces acting on a mass when it is slowly raised by hand from the floor to a table by a human being are the human's force and gravity. Are the signs of the work done by these two forces the *same* or *opposite*?

Answer: The two forces acting on a mass when it is slowly raised by hand from the floor to a table by a human being are the **human's force** (electromagnetic) and **gravity**. The signs of the work done by these two forces are **opposite** .

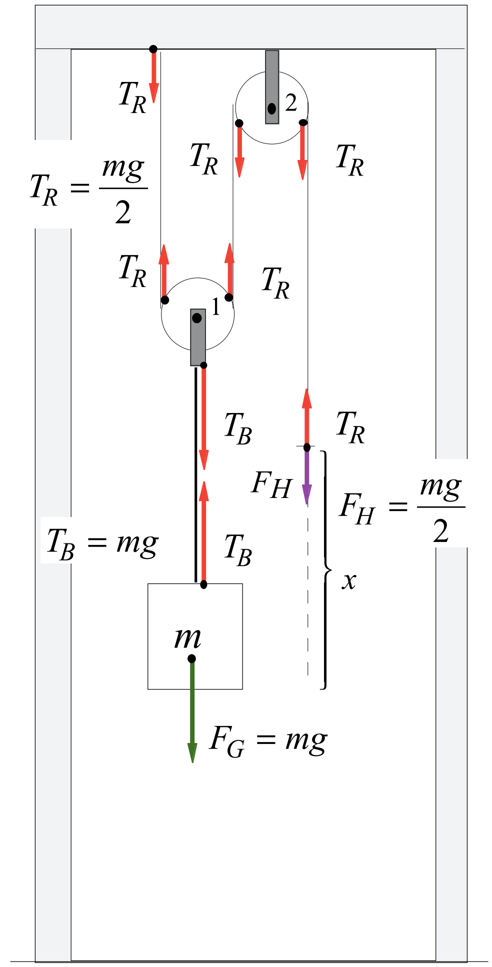
6. A mass travels 100 m in the same direction as the applied force of 50 N. How much work was done by this force? Express this answer in joules. What is the compression of a spring with spring constant 50 N/m, that stores the same energy?

Answers:

$$\boxed{\begin{aligned} w &= \langle \mathbf{F} \rangle \cdot \mathbf{s} = (+50\ \text{N})(+100\ \text{m}) = \underline{5000\ \text{Nm}} \\ &= \underline{5000\ \text{J}} \end{aligned}}$$

$$\boxed{\begin{aligned} &PE = \tfrac{1}{2} k x^2 \\ &x = \sqrt{\frac{2PE}{k}} = \sqrt{\frac{10000\ \text{Nm}}{50\ \text{N/m}}} = \sqrt{200\ \text{m}^2} = \underline{14\ \text{m}} \end{aligned}}$$

The mass in the figure above (all other objects are massless) is raised when the end of the rope is *slowly* pulled down a distance x (see Ch. 5, HW problems 18 – 24, for information relevant to this situation).



First, draw and label the force vectors active on the mass (use *m*), on the end of the rope, and objects at the end of each piece of rope and bar.

7. What force must be applied to the end of the rope to keep the mass stationary. **Answer:** $mg/2$

8. What force must be applied to the end of the rope to move the end of the rope slowly downward a distance x ? **Answer:** $mg/2$.

9. How much work is done by the person pulling down on the rope a distance x ? What is the sign of this work?

Answer: The work done on the rope is, $+mgx/2$ (positive – force and displacement are in the same direction).

10. What magnitude of force does the bar apply to the mass during the motion?

Answer: During the motion (as in 8) at a constant speed upward, the force of the bar acting on the mass is, mg .

11. How far upward does the mass move when the rope is pulled down the distance x ?

Answer: The mass moves upward a distance, $x/2$, when the rope is pulled down the distance, x .

12. What is the work done is done on the mass by the bar? What is its sign?

Answer: The work done on the mass by the bar is $+mgx/2$.

13. Compare the work done on the rope to the work done on the mass. Are they equal to each other? Why is that so?

Answer: The work done by the person pulling down on the handle, is **equal to** the work done on the mass by the bar. **The human does work on the rope, the rope's tension forces (2 of them) do work on the pulley and bar, and the bar's tension force does work on the mass. By raising the mass, the work done by the human adds an equivalent amount of energy to the mass.**

14. What is the potential energy stored by an ideal spring with spring constant, k , that is *stretched* a distance, x , from its normal length? Is the stored potential energy the same if the spring is *compressed* by the same distance? What is the sign of the work done on the spring by the external force when it is stretched and when it is compressed?

Answers: A spring stretched a distance, x, from its normal length stores a potential energy of $\frac{1}{2}kx^2$, and compressed a distance, x, also stores a potential energy of $\frac{1}{2}kx^2$. A force stretchind or compressing a spring does **positive** work.

15. Two springs have spring constants, k_1 and k_2, and are stretched from their normal length by distances, x_1 and x_2 , respectively. What is the ratio of the spring constants needed to make the potential energy stored the same in both?

Answer:

$$\frac{1}{2}k_1x_1^2 = \frac{1}{2}k_2x_2^2 \quad \text{(same potential energy)}$$

$$\frac{k_1}{k_2} = \frac{x_2^2}{x_1^2} = \underline{\left(\frac{x_2}{x_1}\right)^2} \quad \text{(solve for ratio of } k\text{'s)}$$

16. Should I connect two identical springs in a parallel or series combination, if I want to store the most energy for a given stretch of the combination?

$$\text{(series) } k_s = \frac{k}{2}; \quad \text{(parallel) } k_p = 2k$$

$$\text{(series) } PE_s = \frac{1}{2}k_sx^2 = \frac{1}{4}kx^2$$

$$\text{(parallel)} PE_p = \frac{1}{2}k_px^2 = kx^2 \quad \text{(4 times as big as } PE_s\text{)}$$

Answer: Connect the two identical springs in **parallel** .

17. Should I connect two identical springs in a parallel or series combination, if I want to store the most energy for a given force applied to the combination?

Find expression for potential energy of a spring distorted by a given force:

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kx(x) = \frac{1}{2}F\left(\frac{F}{k}\right) = \frac{1}{2}\frac{F^2}{k} \quad (PE \text{ for force, } F\text{, on spring, } k)$$

Find PE of the series and parallel combinations with a given force, F.

$$\text{(series) } PE_s = \frac{1}{2}\frac{F^2}{k_s} = \frac{F^2}{k}$$

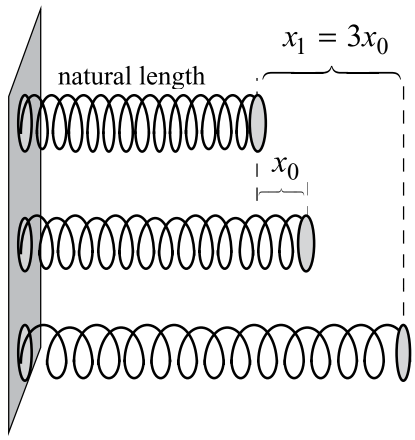
$$\text{(parallel) } PE_p = \frac{1}{2}\frac{F^2}{k_p} = \frac{F^2}{4k} \quad \text{(4 times smaller than } PE_s\text{)}$$

Answer: Connect the two identical springs in **series** .

18. A stretched spring stores an initial potential energy, PE_0. What is the potential energy stored by the spring if the amount of stretch is doubled?

Answer: $4PE_0$

$$PE_0 = \tfrac{1}{2}kx_0^2$$
$$PE = \tfrac{1}{2}kx^2 \quad \text{with } x = 2x_0$$
$$= \tfrac{1}{2}k(2x_0)^2 = 4\left(\tfrac{1}{2}kx_0^2\right) = 4PE_0$$



19. A spring with spring constant, k , starts with a stretch of x_0 and is then stretched to a distance $x = 3x_0$. How much potential energy is *added* (ΔPE) to the spring during the second stretch. (express your answer using only k and x_0)

Answer: $8\left(\tfrac{1}{2}kx_0^2\right)$, or 8 times the starting amount of potential energy.

Method 1: Find the potential energy stored at the two stretches and find the difference.

$$PE_0 = \tfrac{1}{2}kx_0^2$$
$$PE_1 = \tfrac{1}{2}k(3x_0)^2 = \tfrac{1}{2}k(9)(x_0^2) = 9(\tfrac{1}{2}kx_0^2)$$
$$\Delta PE = PE_1 - PE_0$$
$$= 9(\tfrac{1}{2}kx_0^2) - \tfrac{1}{2}kx_0^2 = \underline{8\left(\tfrac{1}{2}kx_0^2\right)}$$

Method 2: Find the work done in stretching from x_0 and $3x_0$.

$$w = \langle \mathbf{F} \rangle \cdot \mathbf{s}$$
$$= \left[\frac{(3kx_0 + kx_0)}{2}\right] \cdot s$$
$$= \left[\frac{4kx_0}{2}\right] \cdot (2x_0) = \underline{8\left(\tfrac{1}{2}kx_0^2\right)}$$