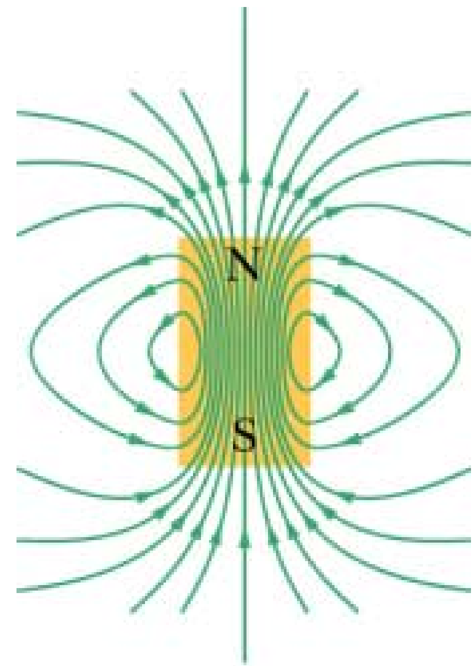
October 13th

Magnetic Fields Due to Currents

Chapter 30

Review

- Set up a *B* field in two ways:
- Intrinsic magnetic field
  - Magnetic field of electrons in a material add together to give a net magnetic field around the material – i.e. permanent magnet
- Electrically charged particles which are moving – i.e. current in a wire



(a)

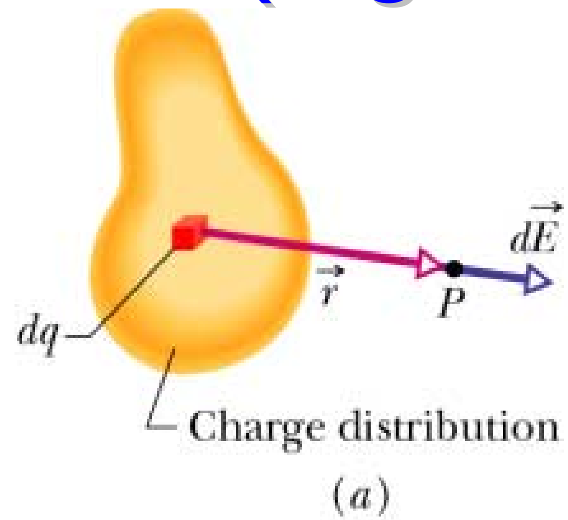
B Fields from Currents (Fig. 30-1)

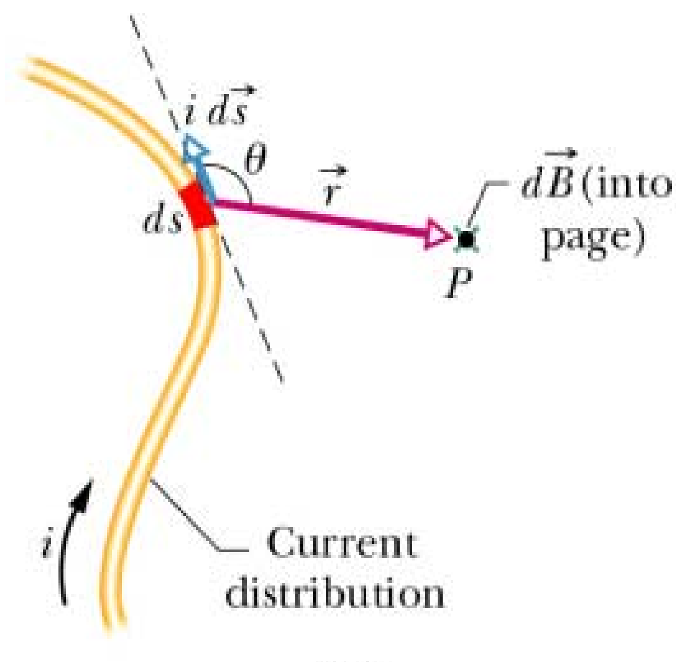
- *E* field produced by a distribution of charges

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

- *B* field produced by distribution of currents

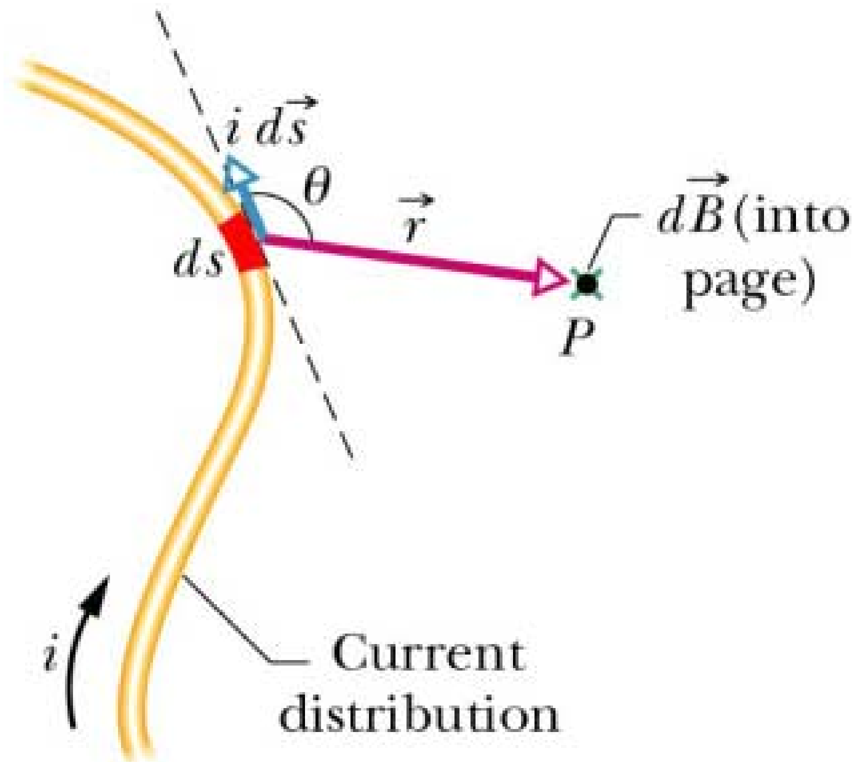
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$





B Fields from Currents (Fig. 30-1b)

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\sin\theta}{r^2}$$



- Current-length element,

$$i\,d\vec{s}$$

  is product of a scalar and a vector.
- Find net B field by integrating.
- A new constant - Permeability constant, μ_0

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m / A$$

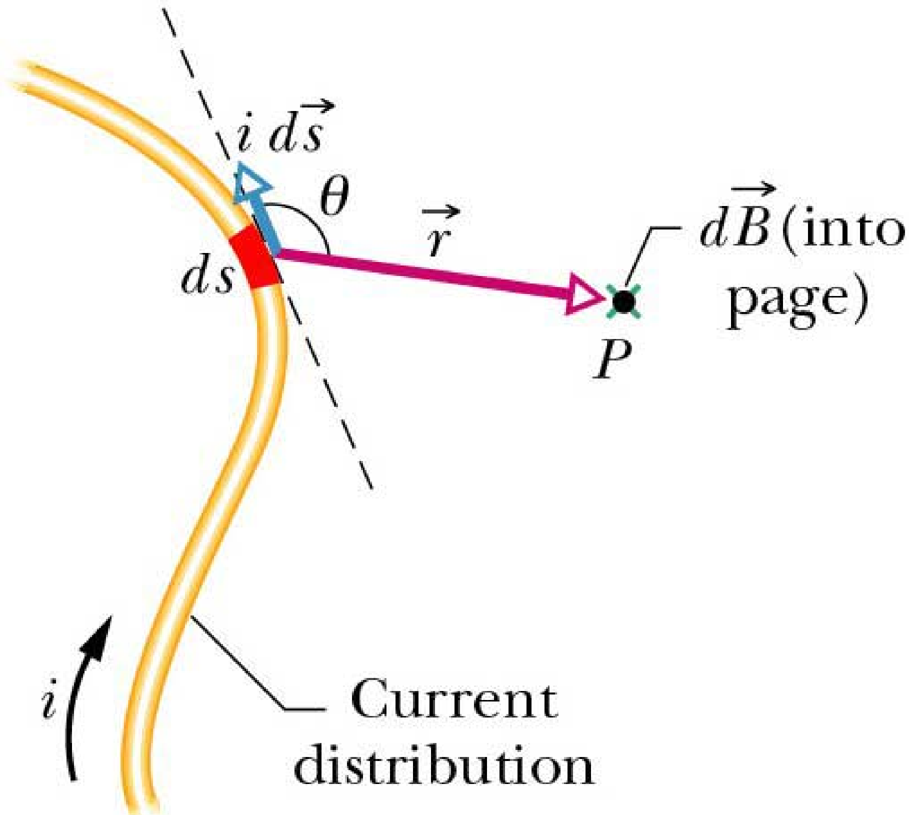
Review

- Biot-Savart Law

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

- In vector form

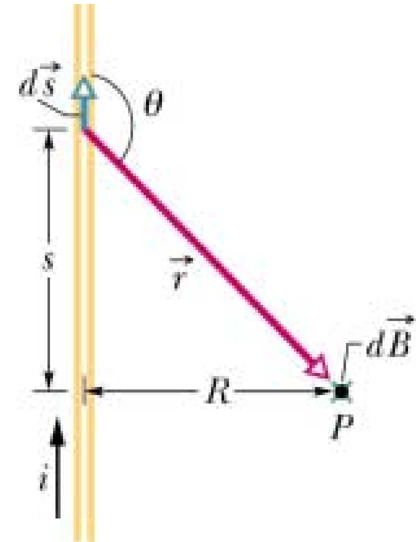
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}$$



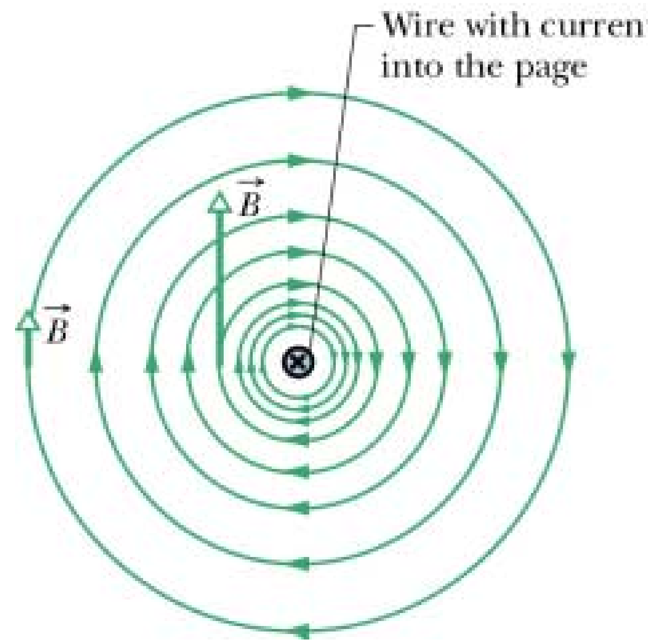
Review

- For a distance *R* away from a long straight wire, which carries current *i*, the *B* field is:

$$B = \frac{\mu_0 i}{2\pi R}$$



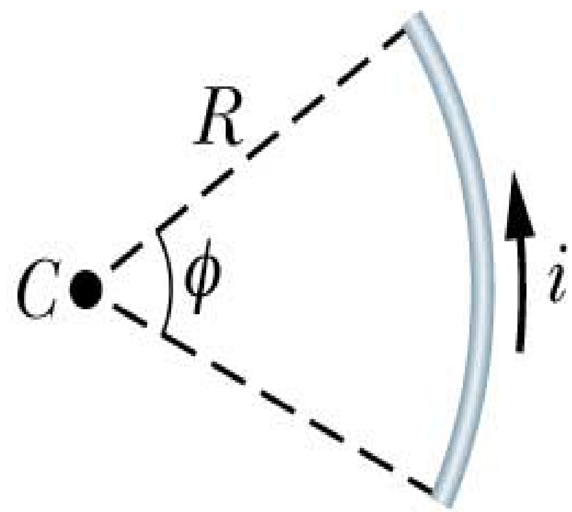
- *B* field forms concentric rings whose direction is given by the right-hand rule.
- Magnitude of *B* decreases with distance as 1/*R* (so spacing of the lines deceases)



B Fields from Currents (Fig. 30-6)

- *B* field at the center of an arc is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



- Express ϕ in radians not in degrees
- For a complete loop ($\phi = 2\pi$) then *B* is

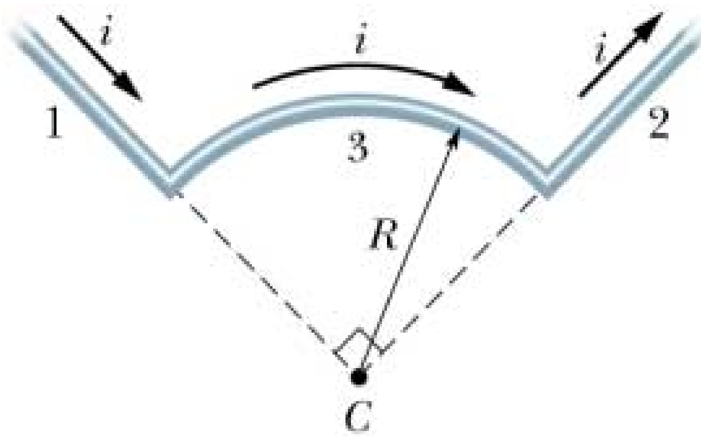
$$B = \frac{\mu_0 i}{2R}$$

B Fields from Currents (Fig. 30-7)

- Calculate the *B* field at point C
- Use Biot-Savart law

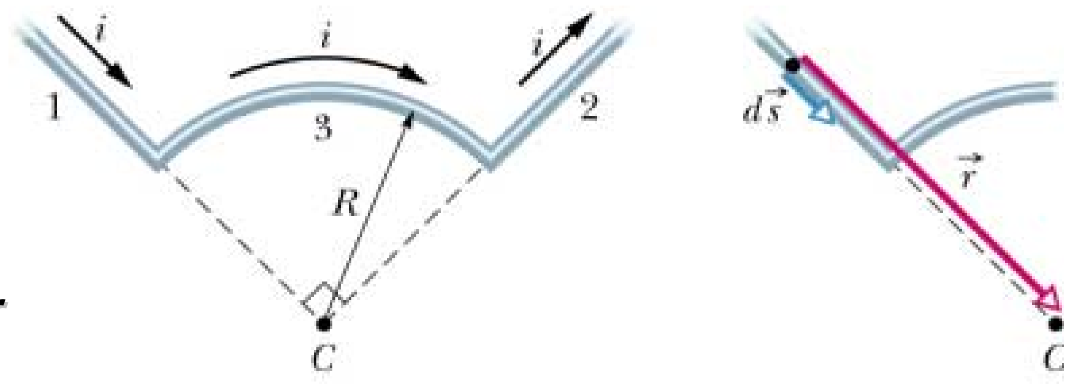
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

- Simplify problem by separating into 3 parts – sides 1, 2 & 3



- Side 1 – straight section on the left
- Side 2 – straight section on the right
- Side 3 – circular arc

B Fields from Currents (Fig. 30-7)



- Side 1 – Angle, θ, between *ds* and *r* is zero so

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 0}{r^2} = 0$$

$$B_1 = 0$$

- Side 2 – Angle, θ, between *ds* and *r* is 180 so

$$B_2 = 0$$

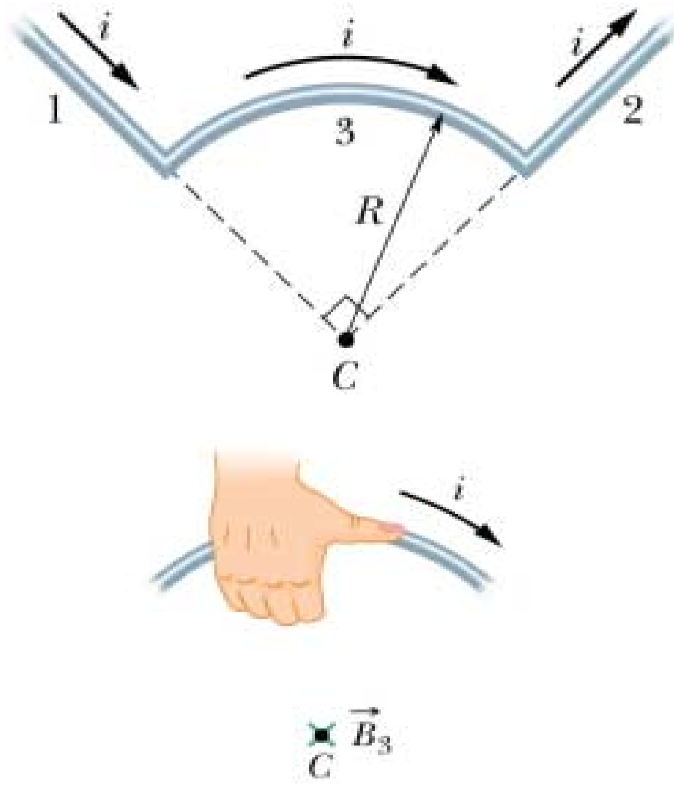
B Fields from Currents (Fig. 30-7)

- Side 3 – circular arc
- Just derived *B* field at center of arc as

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

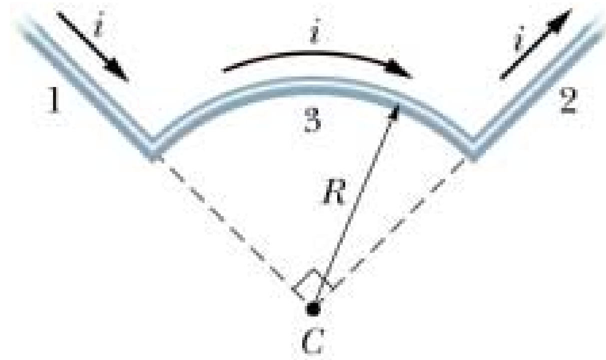
- Given that $\phi = \pi/2$ so

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}$$



- Use right-hand rule to find that B_3 is directed into page

B Fields from Currents (Fig. 30-7)



- Find net *B* field by combining the 3 fields
- Remember they combine as vectors!

$$B_1 = 0$$

$$B_2 = 0$$

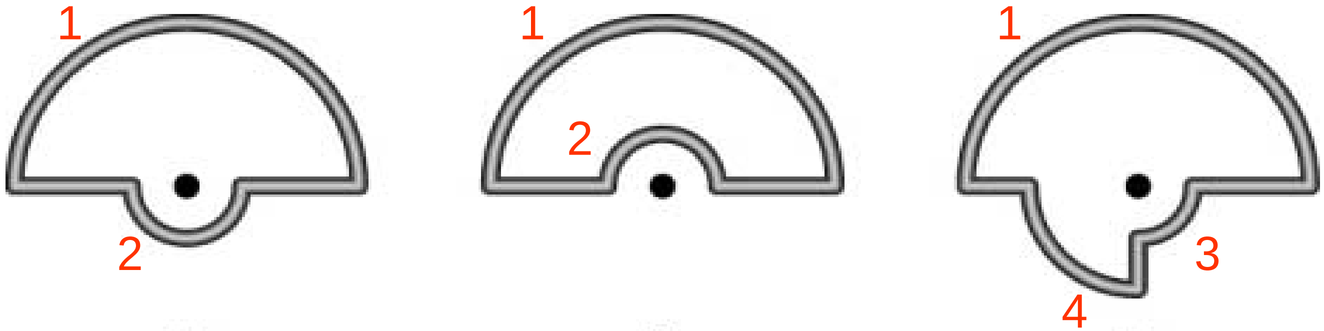
$$B_3 = \frac{\mu_0 i}{8R}$$

- Total *B* field is into the page and has magnitude

$$B = \frac{\mu_0 i}{8R}$$

Checkpoint #1

- Three circuits with same *i* and various circular arcs of half (π) or quarter circles ($\pi/2$) and radii *r*, *2r* and *3r*. Rank magnitude of *B* field produced at the center (the dot), greatest first.



(a) (b) (c)

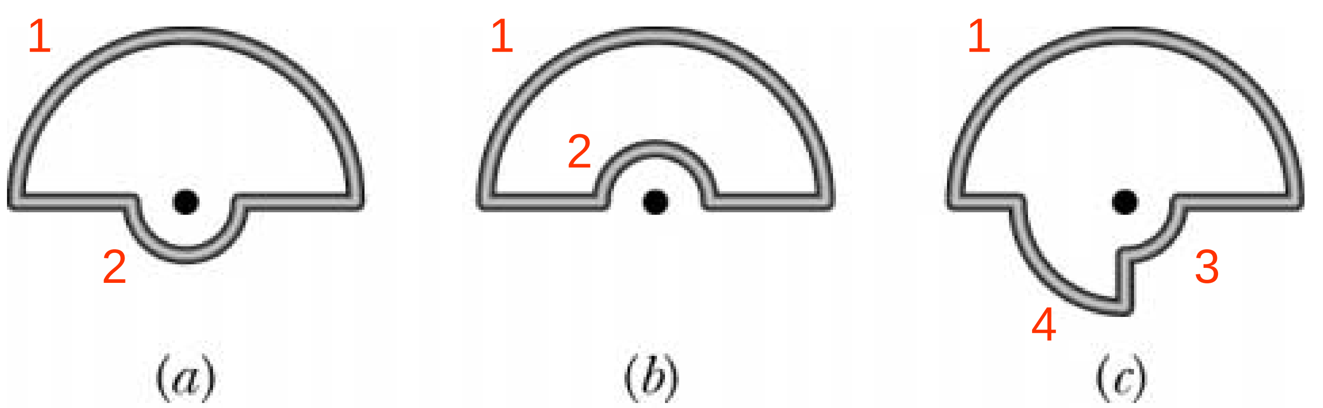
- For all straight sections $\theta = 0$ or $\theta = 180$ so

$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin\theta}{r^2} = 0$$

Checkpoint #1

- Recall *B* field at center of circular arc
- Find magnitude of *B* field for each arc and then at the end add them as vectors

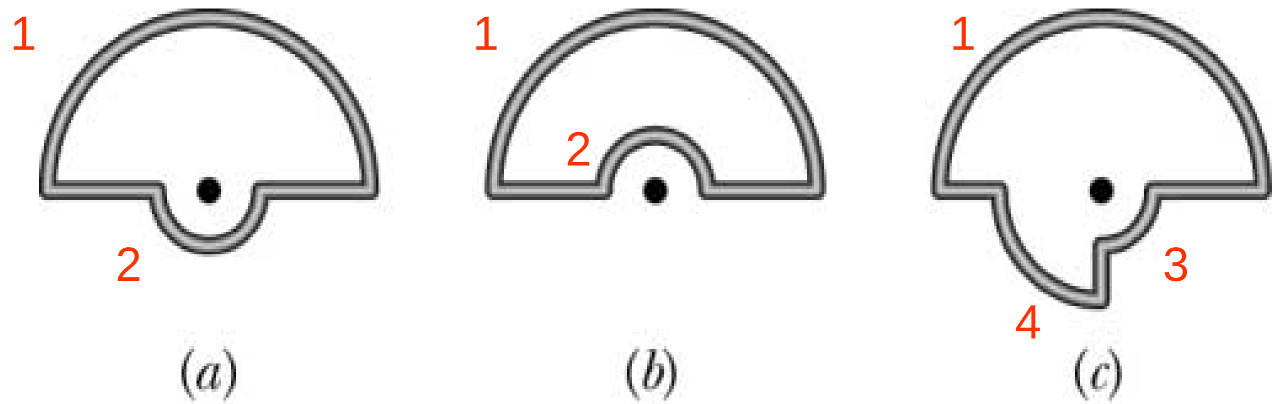
$$B = \frac{\mu_0 i \phi}{4\pi R}$$



- All circuits have large upper arc (*R=3r*) with *B* field

$$B_1 = \frac{\mu_0 i \phi}{4\pi R} = \frac{\mu_0 i \pi}{4\pi(3r)} = \frac{\mu_0 i}{12r}$$

Checkpoint #1



$$B = \frac{\mu_0 i \phi}{4\pi R}$$

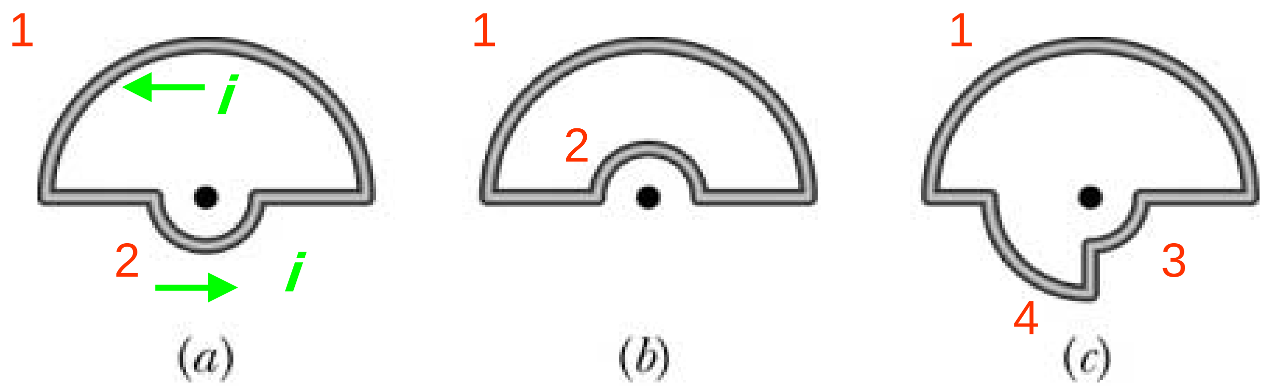
- Circuits a & b each have small (*R=r*) half arc

$$B_2 = \frac{\mu_0 i \pi}{4\pi r} = \frac{\mu_0 i}{4r}$$

- Circuit c has a small and medium (*R=2r*) quarter arc

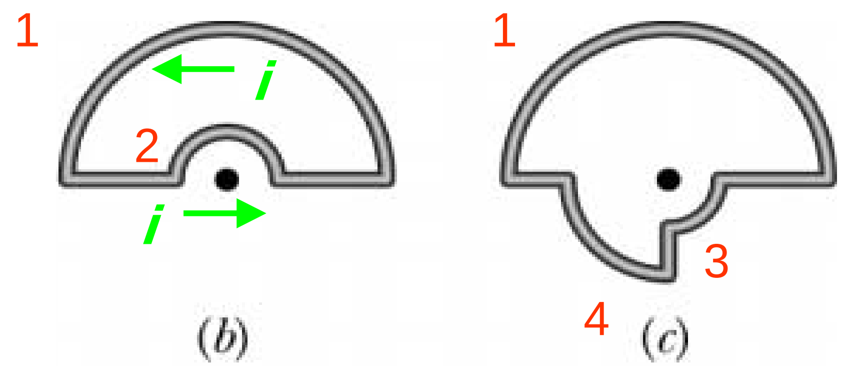
$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi r} = \frac{\mu_0 i}{8r}$$

$$B_4 = \frac{\mu_0 i(\pi/2)}{4\pi(2r)} = \frac{\mu_0 i}{16r}$$



- Assume *i* is flowing counterclockwise
- Use right-hand rule to find direction of *B*
- For all upper arcs (1) *B* field is out of page
- For circuit a
  - Small arc: B field is also out of page so

$$B_a = B_1 + B_2 = \frac{\mu_0 i}{12r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{3r}$$



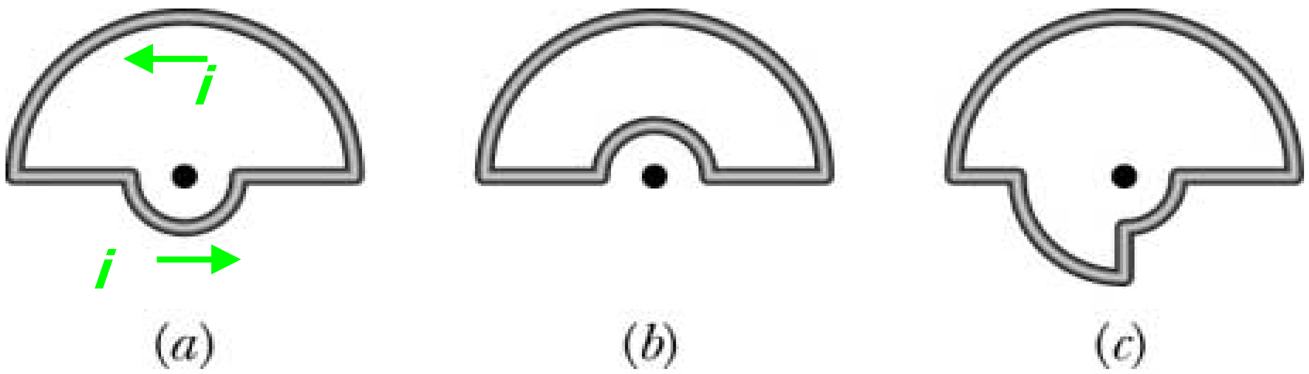
- For circuit b
  - Small arc, B field is into page so

$$B_b = B_1 - B_2 = \frac{\mu_0 i}{12r} - \frac{\mu_0 i}{4r} = -\frac{\mu_0 i}{6r}$$

- Negative sign means net *B* field points into page
- For circuit c

$$B_c = B_1 + B_3 + B_4 = \frac{\mu_0 i}{12r} + \frac{\mu_0 i}{16r} + \frac{\mu_0 i}{8r} = \frac{13\mu_0 i}{48r}$$

Checkpoint #1



- Net *B* field for each circuit is

$$B_a = \frac{\mu_0 i}{3r} \qquad B_b = -\frac{\mu_0 i}{6r} \qquad B_c = \frac{13\mu_0 i}{48r}$$

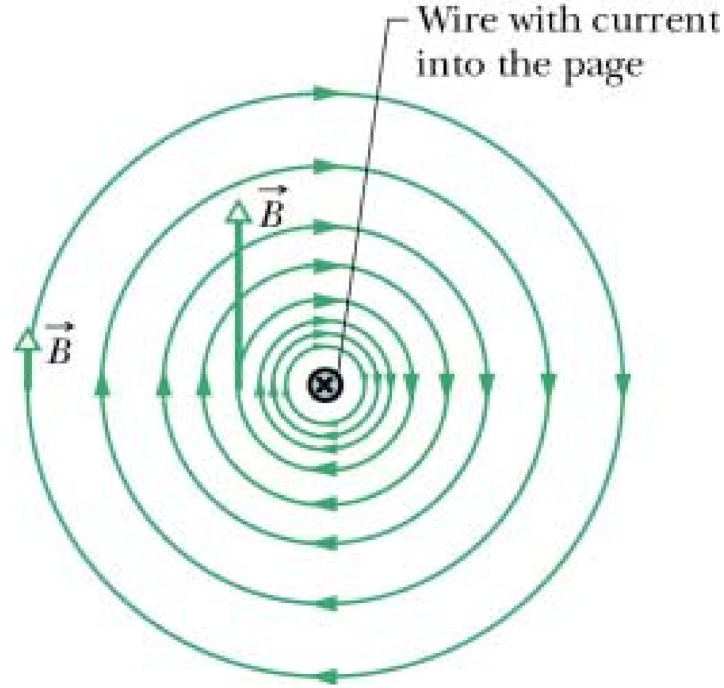
- Rank magnitude of *B* field, greatest first

a, c, b

B Fields from Currents (Figs. 30-2)

- Wire with a current produces a *B* field

$$B = \frac{\mu_0 i}{2\pi R}$$



- What happens if we bring two wires, each carrying a current, near each other?
- Will it matter if the currents are in the same direction or opposite each other?