October 14th

Magnetic Fields Due to Currents
Chapter 30
Wire with a current produces a $B$ field

$$B = \frac{\mu_0 i}{2\pi R}$$

What happens if we bring two wires, each carrying a current, near each other?

Will it matter if the currents are in the same direction or opposite each other?
What is the force, $F_{ba}$ on wire $b$ due to the current in wire $a$?

- Wires are separated by a distance, $d$, and have currents, $i_a$ and $i_b$

- First calculate $B$ field from wire $a$ at the site of wire $b$

\[
B_a = \frac{\mu_0 i_a}{2\pi d}
\]
B Fields from Currents (Figs. 30-9)

- Using right-hand rule find that the $B_a$ field is directed down

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

- Now calculate force $F_{ba}$ on wire $b$ using

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

- Where the current $i_b$ is current in wire $b$
- $B$ field is from wire $a$
- $L$ is length of wire $b$
B Fields from Currents (Figs. 30-9)

- Current $i_b$ and $B_a$ are $\perp$ to each other

$$F_{ba} = i_b L B_a \sin 90$$

- Substituting $B_a$

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

- Applying right-hand rule, direction of $F_{ba}$ is towards wire $a$
What is the force, \( F_{ab} \) on wire a due to the current in wire b?

Calculate \( B \) field from wire b at site of wire a

\[
B_b = \frac{\mu_0 i_b}{2\pi d}
\]

Force on a from b is

\[
F_{ab} = i_a LB_b = \frac{\mu_0 L i_a i_b}{2\pi d}
\]

Apply right-hand rule and find \( F_{ab} \) has same magnitude as \( F_{ba} \) but opposite direction.
What happens if we flip current in wire $a$ so that its moving opposite to the current in wire $b$?

- Use right-hand rules
- $B_a$ points up
- $F_{ba}$ points away from wire $a$

Parallel currents attract, anti-parallel currents repel
Three long, straight, parallel wires are equally spaced with identical currents, either into or out of page. Rank the wires according to the magnitude of the force on each wire due to the currents in the other two wires, greatest first.
What's the force on wire a due to wires b and c?

First find net $B$ field at a from wires b and c

- Calculate magnitude for $B_b$ and $B_c$
- Use right-hand rule to find direction $B_b$ and $B_c$
- Add $B_b$ and $B_c$ as vectors to get $B_{bc}$

Find force on wire a with

$$\vec{F}_{abc} = i_a \vec{L} \times \vec{B}_{bc}$$
What's the force on wire \( a \) due to wires \( b \) and \( c \)?

- \( B_b \) at \( a \) is down
- \( B_c \) at \( a \) is up
- \( B_b > B_c \)

Net \( B \) field is

Force is

\[
B_b = \frac{\mu_0 i_b}{2\pi d} \quad B_c = \frac{\mu_0 i_c}{2\pi (2d)}
\]

\[
B_{bc} = \frac{\mu_0 i}{2\pi d} - \frac{\mu_0 i}{4\pi d} = \frac{\mu_0 i}{4\pi d}
\]

\[
F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}
\]
What’s the force on wire $b$ due to wires $a$ and $c$?

- $B_a$ at $b$ is up
- $B_c$ at $b$ is up

$$B_a = B_c$$

- Net $B$ field is

- Force is

$$F_{bac} = i_b LB_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$
**Checkpoint #2**

- **What’s the force on wire c due to wires a and b?**
- $B_a$ at c is up
- $B_b$ at c is up
  - $B_b > B_a$
- Net B field is
- Force is

\[ B_a = \frac{\mu_0 i_a}{2\pi (2d)} \]
\[ B_b = \frac{\mu_0 i_b}{2\pi d} \]
\[ B_{ab} = \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{4\pi d} = \frac{3\mu_0 i}{4\pi d} \]
\[ F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d} \]
Checkpoint #2

- Forces on each wire due to other two are:

\[ F_{abc} = i_a LB_{bc} = \frac{\mu_0 i^2 L}{4\pi d} \]
\[ F_{bac} = i_b LB_{ac} = \frac{\mu_0 i^2 L}{\pi d} \]
\[ F_{cab} = i_c LB_{ab} = \frac{3\mu_0 i^2 L}{4\pi d} \]

b, c, a
Review

- For certain symmetric distributions of charge, we used Gauss’ law to calculate the $E$ field.

\[ \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

- This is an integral over the Gaussian surface $\mathcal{A}$. 


B Fields from Currents (Fig. 30-11)

- For symmetric distributions of charge use Ampere’s law to calculate $B$ field

\[ \oint C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \]

- Integral around closed loop called Amperian loop
Use the **right-hand rule** to determine the signs for the currents encircled by the Amperian loop.

- Curl right hand around Amperian loop with fingers pointing in direction of integration.
- Current going through loop in the same direction as thumb is positive.
- Current going in the opposite direction is negative.
Checkpoint # 3

• Rank the loops according to the magnitude of \( \oint \vec{B} \cdot d\vec{s} \)

Assume loops are counterclockwise:

a: \( i_{\text{enc}} = i + i - i = i \)

b: \( i_{\text{enc}} = i - i = 0 \)

c: \( i_{\text{enc}} = -i \)

d: \( i_{\text{enc}} = i + i = 2i \)

d, a & c, b

(a & c tie: same magnitude)

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \]
Use Ampere’s law to calculate $B$ field from long straight wire

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \]

Draw Amperian loop as a circle surrounding the wire (like the magnetic field lines)

At every point of the loop

- Magnitude of $B$ is constant
- $B$ and $ds$ are $||$ (both tangent to the circle)
$\mathbf{B}$ Fields from Currents (Fig. 30-13)

- $\mathbf{B}$ and $ds$ are $\parallel$ so

$$\cos \theta = \cos 0 = 1$$

$$\mathbf{B} \cdot d\mathbf{s} = B ds$$

- $B$ constant on loop so

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds$$

$$\oint ds = 2\pi r$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B (2\pi r)$$
Ampere’s law becomes

\[ B(2\pi r) = \mu_0 i_{enc} \]

Current enclosed is just \( i \) so

\[ B = \frac{\mu_0 i}{2\pi r} \]

Same result as with Biot-Savart law (we used \( r=R \) before).