October 14th

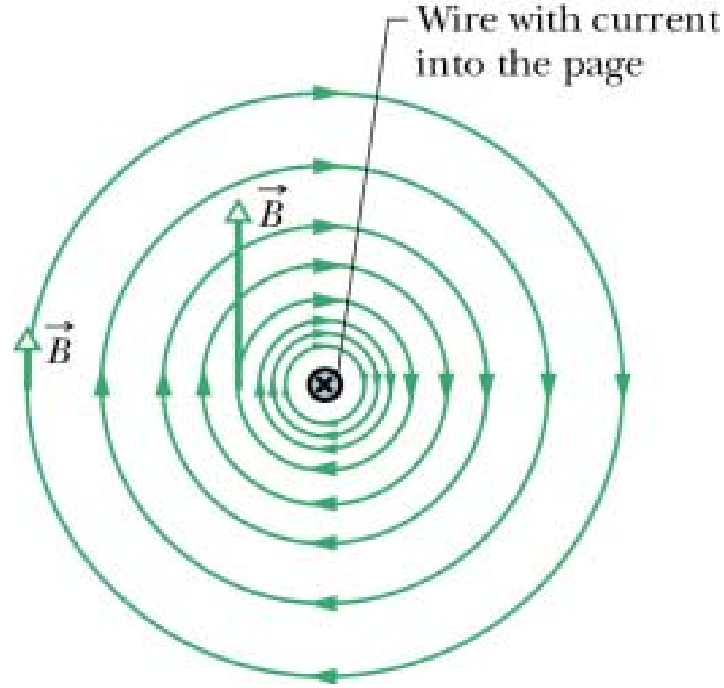
Magnetic Fields Due to Currents

Chapter 30

B Fields from Currents (Figs. 30-2)

- Wire with a current produces a *B* field

$$B = \frac{\mu_0 i}{2\pi R}$$



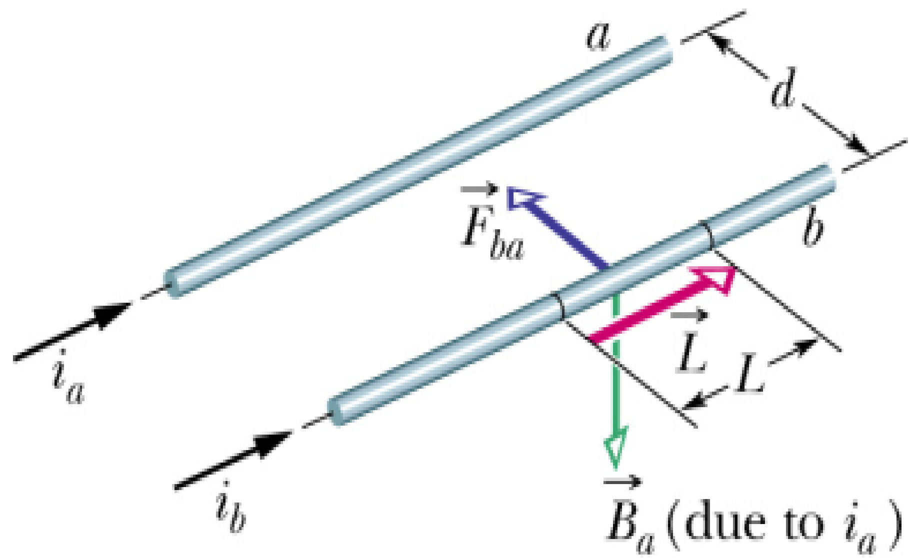
- What happens if we bring two wires, each carrying a current, near each other?
- Will it matter if the currents are in the same direction or opposite each other?

B Fields from Currents (Figs. 30-9)

- What is the force, F_{ba} on wire *b* due to the current in wire *a* ?

- Wires are separated by a distance, *d*, and have currents, i_a and i_b

- First calculate *B* field from wire *a* at the site of wire *b*

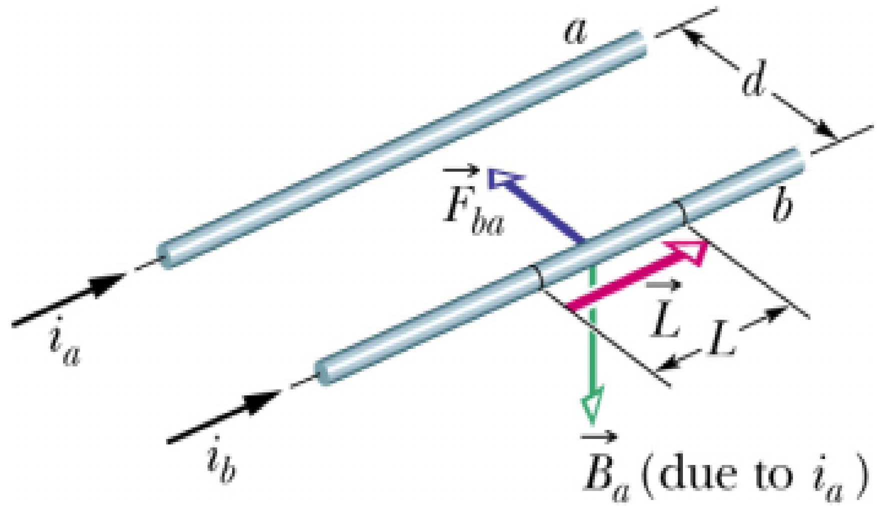


$$B_a = \frac{\mu_0 \, i_a}{2\pi d}$$

B Fields from Currents (Figs. 30-9)

- Using right-hand rule find that the B_a field is directed down

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$



- Now calculate force F_{ba} on wire *b* using

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

- Where the current i_b is current in wire *b*
- *B* field is from wire *a*
- *L* is length of wire *b*

B Fields from Currents (Figs. 30-9)

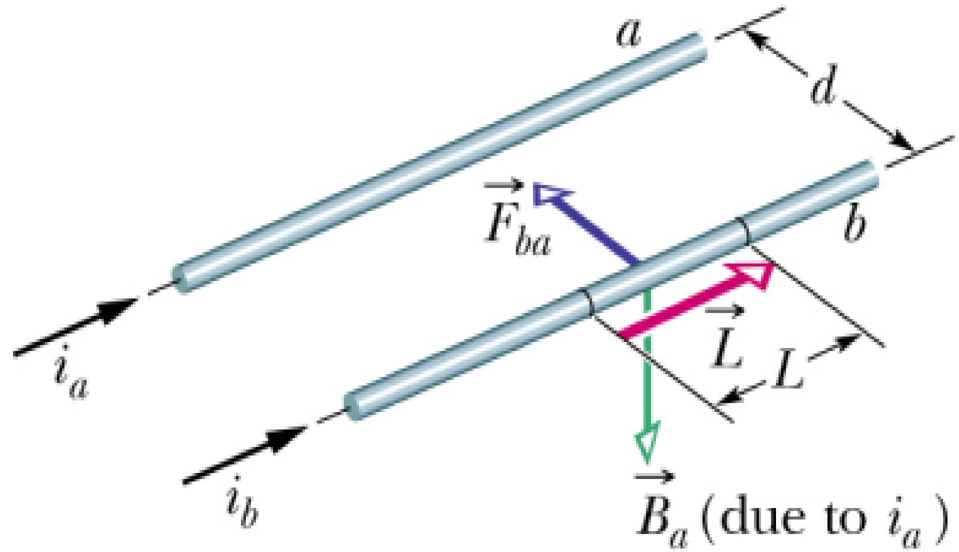
- Current i_b and B_a are $\perp$ to each other

$$F_{ba} = i_b L B_a \sin 90$$

- Substituting B_a

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

$$\boxed{F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}}$$



- Applying right-hand rule, direction of F_{ba} is towards wire *a*

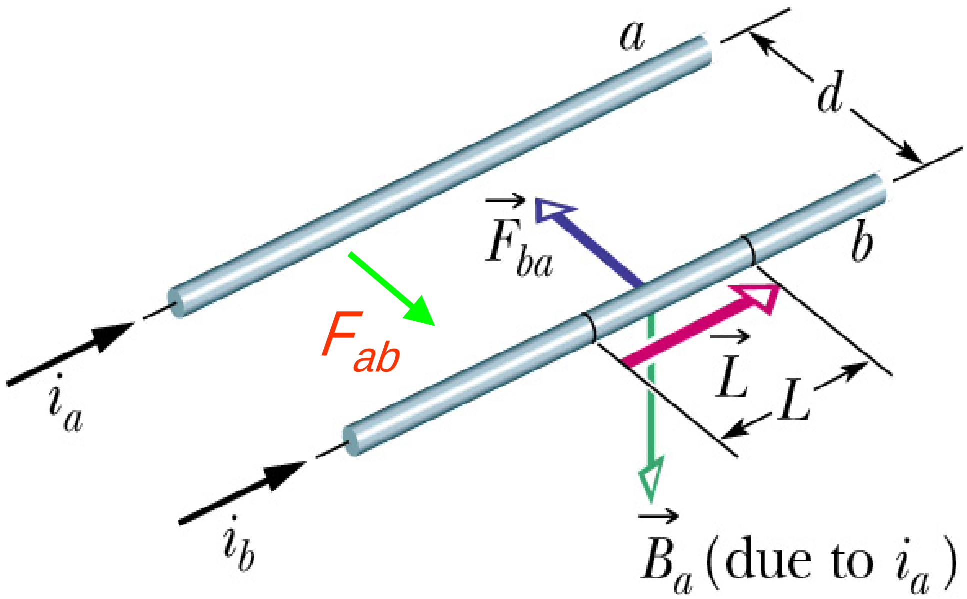
B Fields from Currents (Figs. 30-9)

- What is the force, F_{ab} on wire a due to the current in wire *b* ?
- Calculate *B* field from wire *b* at site of wire *a*

$$B_b = \frac{\mu_0 i_b}{2\pi d}$$

- Force on *a* from *b* is

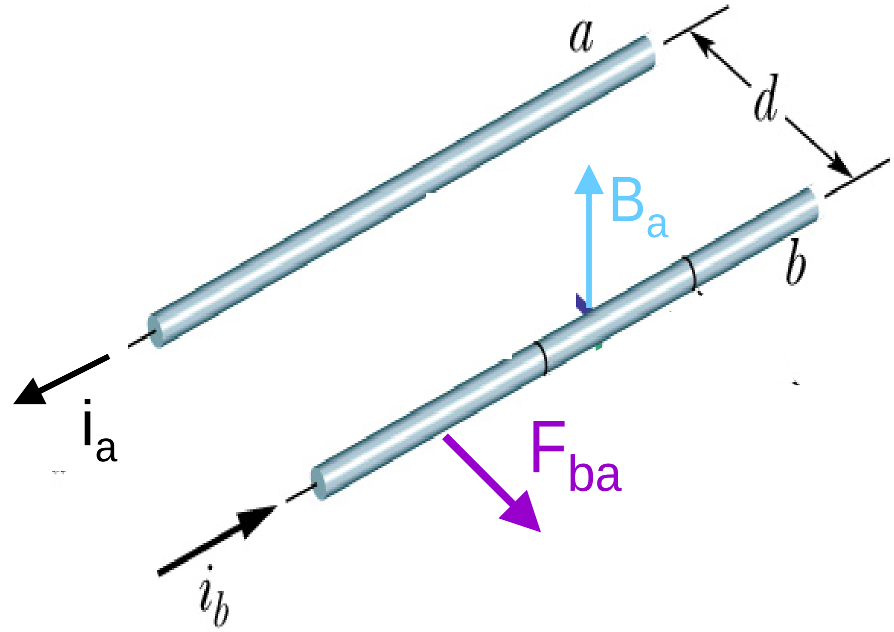
$$F_{ab} = i_a L B_b = \frac{\mu_0 L i_a i_b}{2\pi d}$$



- Apply right-hand rule and find F_{ab} has same magnitude as F_{ba} but opposite direction

B Fields from Currents (Figs. 30-9)

- What happens if we flip current in wire *a* so that its moving opposite to the current in wire *b* ?



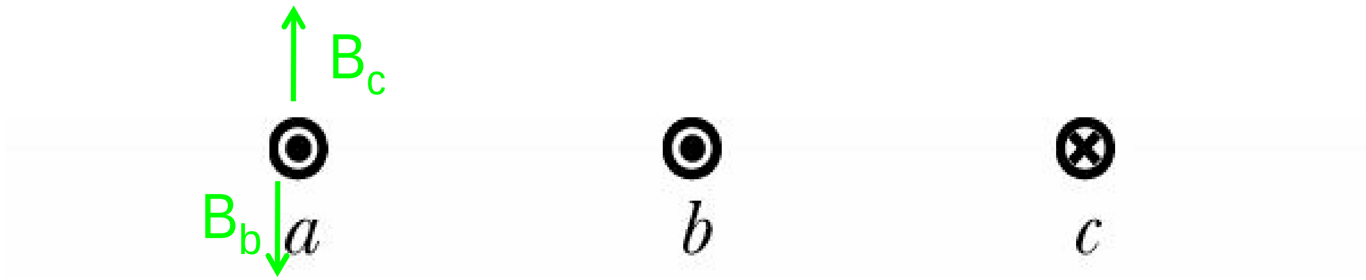
- Use right-hand rules
- B_a points up
- F_{ba} points away from wire *a*

- Parallel currents attract, anti-parallel currents repel

Checkpoint #2

- Three long, straight, parallel wires are equally spaced with identical currents, either into or out of page. Rank the wires according to the magnitude of the force on each wire due to the currents in the other two wires, greatest first.

Checkpoint #2

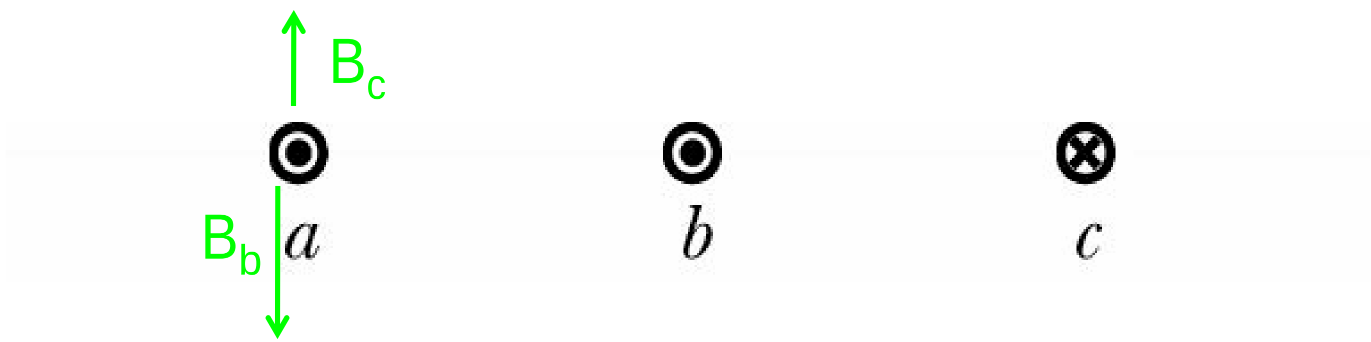


- What's the force on wire *a* due to wires *b* and *c*?
- First find net *B* field at *a* from wires *b* and *c*
  - Calculate magnitude for B_b and B_c
  - Use right-hand rule to find direction B_b and B_c
  - Add B_b and B_c as vectors to get B_{bc}

$$B = \frac{\mu_0 i}{2\pi d}$$

- Find force on wire a with $\vec{F}_{abc} = i_a \vec{L} \times \vec{B}_{bc}$

Checkpoint #2



- What's the force on wire *a* due to wires *b* and *c* ?
- B_b at *a* is down
- B_c at *a* is up

$$B_b = \frac{\mu_0 i_b}{2\pi d}$$

$$B_c = \frac{\mu_0 i_c}{2\pi(2d)}$$

$$B_b > B_c$$

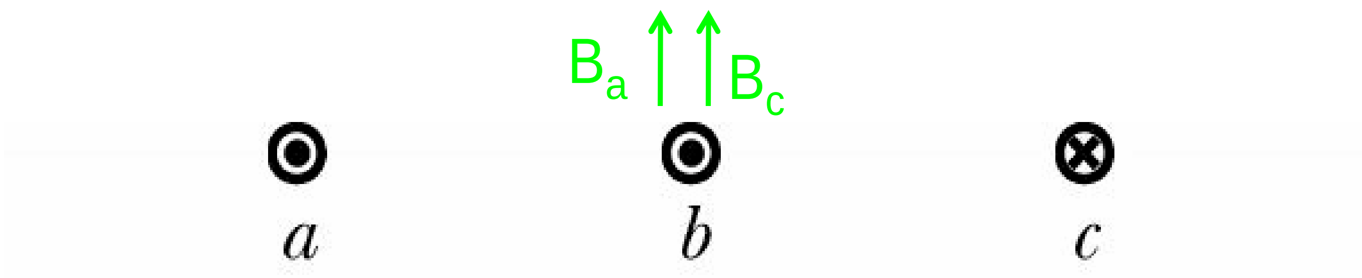
- Net *B* field is

$$B_{bc} = \frac{\mu_o i}{2\pi d} - \frac{\mu_0 i}{4\pi d} = \frac{\mu_0 i}{4\pi d}$$

- Force is

$$F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}$$

Checkpoint #2



- What's the force on wire *b* due to wires *a* and *c*?
- B_a at *b* is up
- B_c at *b* is up

$$B_a = \frac{\mu_0 i_a}{2\pi d} \qquad B_c = \frac{\mu_0 i_c}{2\pi d}$$

$$B_a = B_c$$

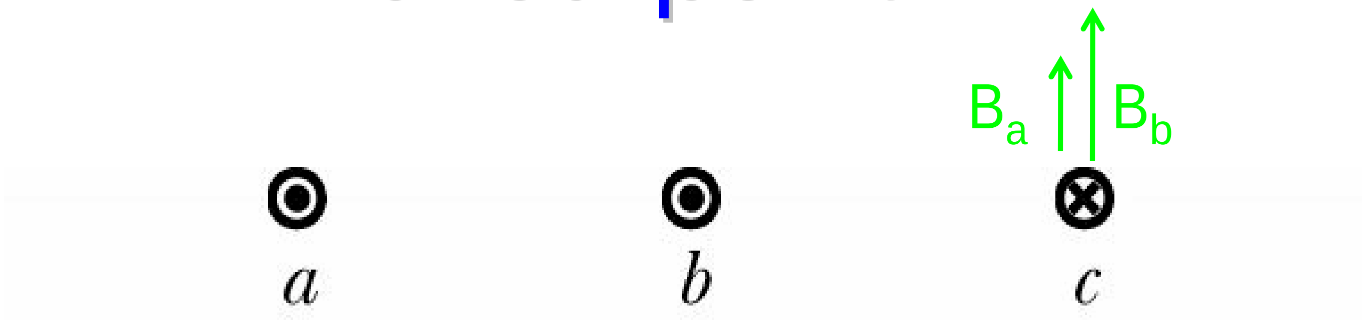
- Net *B* field is

$$B_{ac} = \frac{\mu_o i}{2\pi d} + \frac{\mu_0 i}{2\pi d} = \frac{\mu_0 i}{\pi d}$$

- Force is

$$F_{bac} = i_b L B_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$

Checkpoint #2



- What's the force on wire *c* due to wires *a* and *b* ?
- B_a at *c* is up
- B_b at *c* is up

$$B_a = \frac{\mu_0 i_a}{2\pi(2d)}$$

$$B_b = \frac{\mu_0 i_b}{2\pi d}$$

$$B_b > B_a$$

- Net *B* field is

$$B_{ab} = \frac{\mu_o i}{2\pi d} + \frac{\mu_0 i}{4\pi d} = \frac{3\mu_0 i}{4\pi d}$$

- Force is

$$F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d}$$

Checkpoint #2

- Forces on each wire due to other two are:

$$F_{abc} = i_a L B_{bc} = \frac{\mu_0 i^2 L}{4\pi d}$$

$$F_{bac} = i_b L B_{ac} = \frac{\mu_0 i^2 L}{\pi d}$$

$$F_{cab} = i_c L B_{ab} = \frac{3\mu_0 i^2 L}{4\pi d}$$

b, c, a

Review

- For certain symmetric distributions of charge we used Gauss' law to calculate the *E* field

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

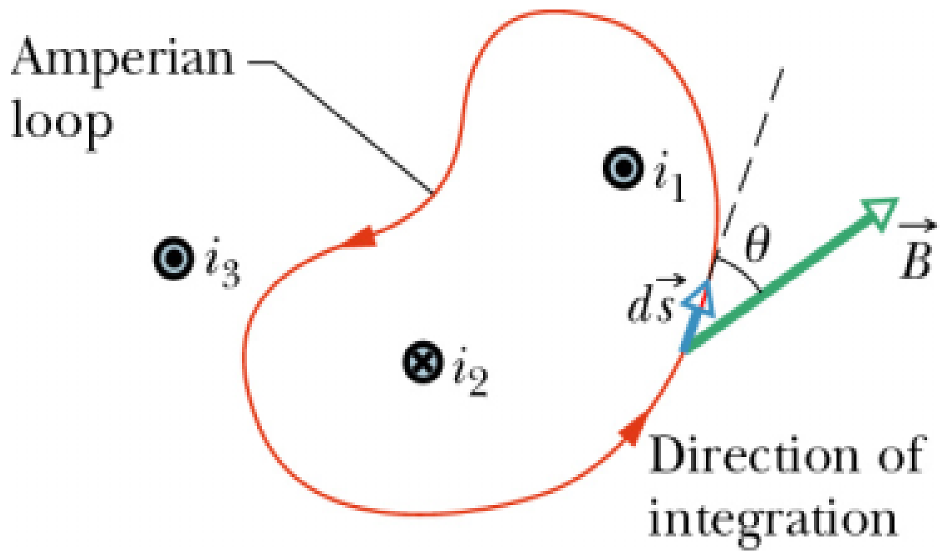
- This is an integral over the Gaussian surface *A*

B Fields from Currents (Fig. 30-11)

- For symmetric distributions of charge use Ampere's law to calculate *B* field

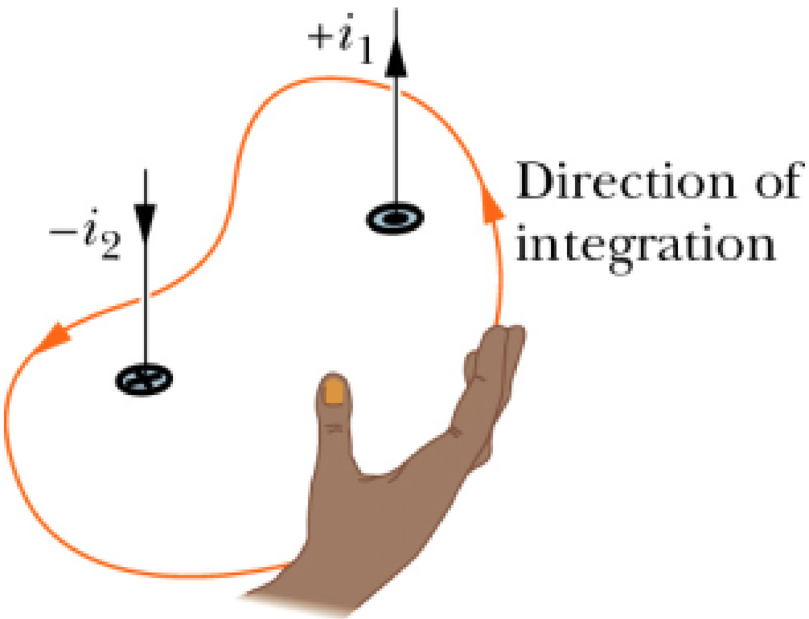
$$\boxed{\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}}$$

- Integral around closed loop called Amperian loop



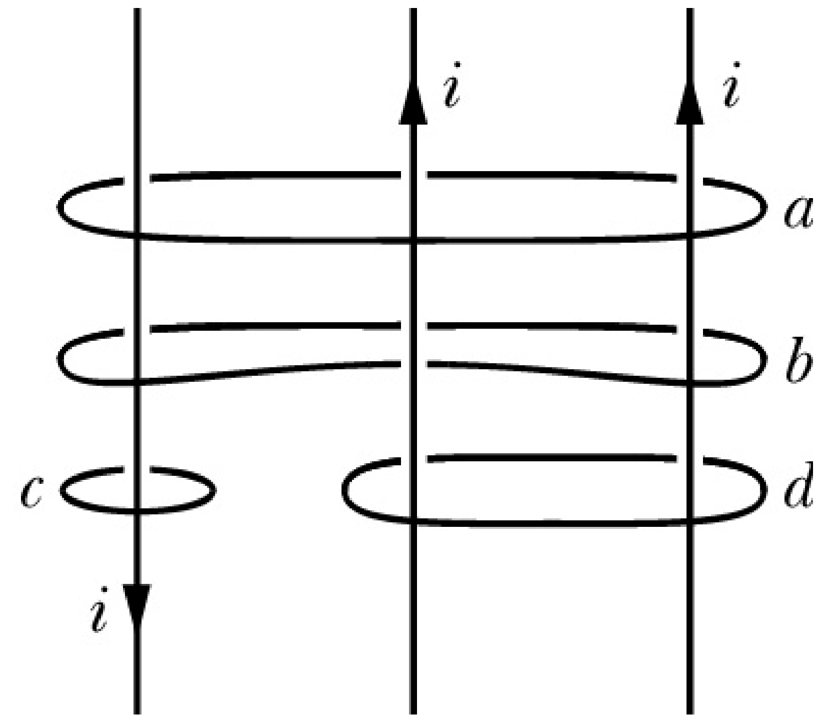
B Fields from Currents (Fig. 30-12)

- Use the right-hand rule to determine the signs for the currents encircled by the Amperian loop
- Curl right hand around Amperian loop with fingers pointing in direction of integration
- Current going through loop in the same direction as thumb is positive.
- Current going in the opposite direction is negative.



Checkpoint # 3

- Rank the loops according to the magnitude of $\oint \vec{B} \bullet d\vec{s}$



Assume loops are counterclockwise:

a: i_{enc} = i + i – i = i

b: i_{enc} = i – i = 0

c: i_{enc} = - i

d: i_{enc} = i + i = 2 i

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

d, a & c, b

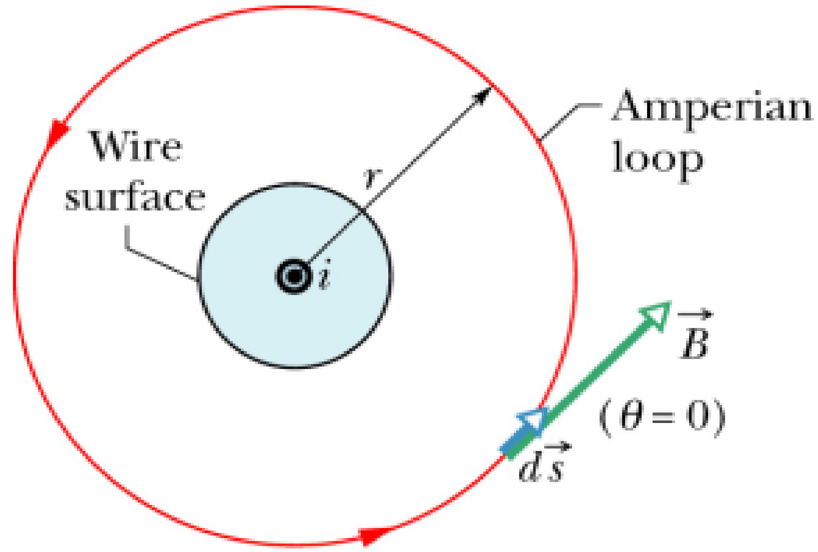
(a & c tie: same magnitude)

B Fields from Currents (Fig. 30-13)

- Use Ampere's law to calculate *B* field from long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Draw Amperian loop as a circle surrounding the wire (like the magnetic field lines)



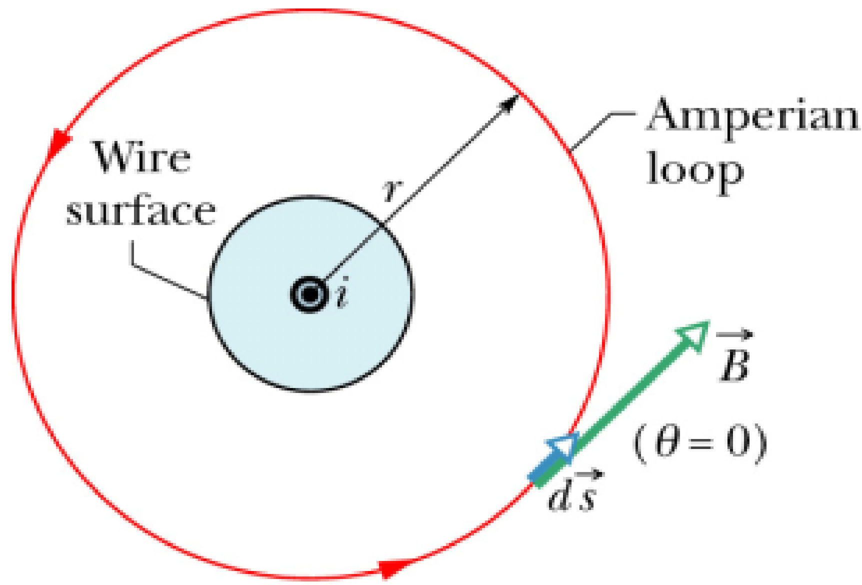
- At every point of the loop
  - Magnitude of *B* is constant
  - *B* and *ds* are || (both tangent to the circle)

B Fields from Currents (Fig. 30-13)

- *B* and *ds* are || so

$$\cos\theta = \cos 0 = 1$$

$$\vec{B} \bullet d\vec{s} = Bds$$



- *B* constant on loop so

$$\oint \vec{B} \bullet d\vec{s} = B \oint ds$$

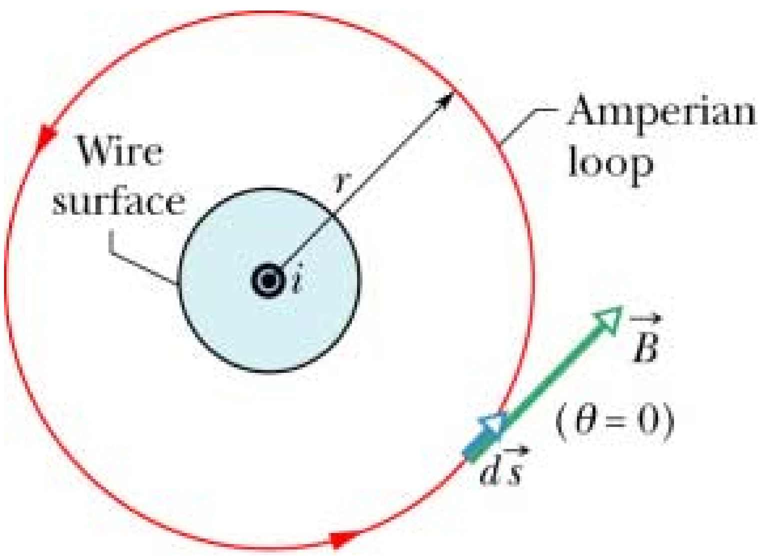
$$\oint ds = 2\pi r$$

$$\oint \vec{B} \bullet d\vec{s} = B(2\pi r)$$

B Fields from Currents (Fig. 30-13)

- Ampere's law becomes

$$B(2\pi r) = \mu_0 i_{enc}$$



- Current enclosed is just *i* so

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

- Same result as with Biot-Savart law (we used *r=R* before).