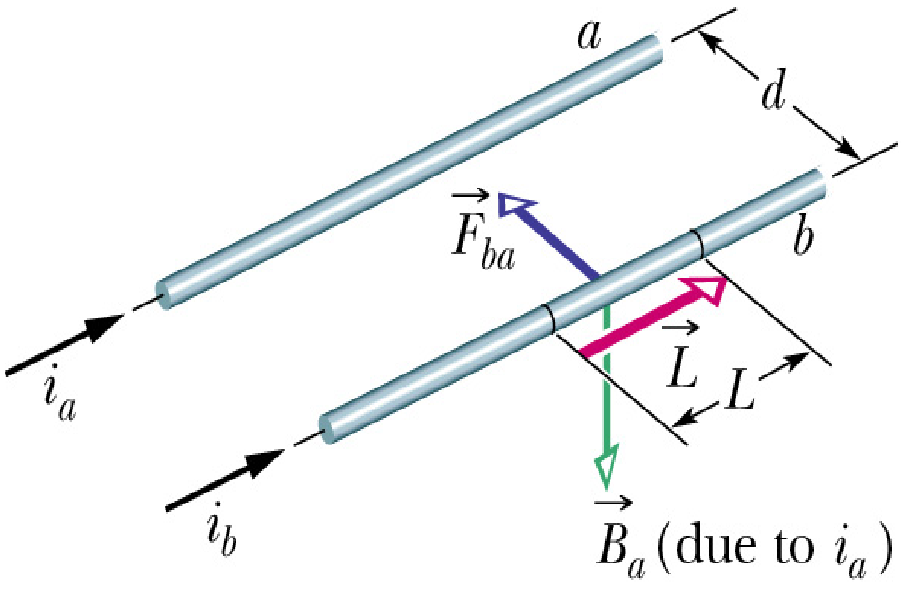
October 15th

Magnetic Fields Due to Currents

Chapter 30

Review (Fig. 30-9)



- Current carrying wires will exert a force on one another
- Calculate *B* field from wire *a* at site of wire *b*

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

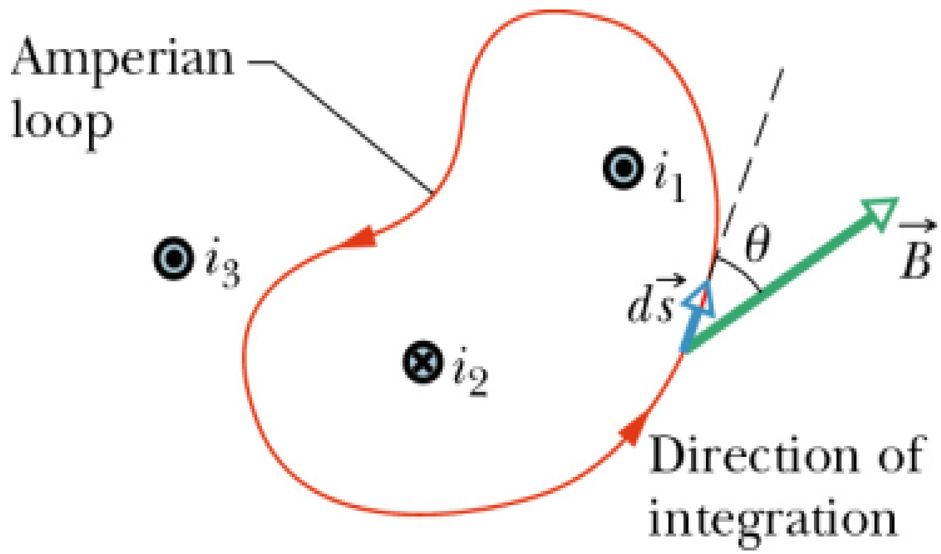
- Force on *b* from *a* is $$F_{ba} = i_b L B_a = \frac{\mu_0 L i_a i_b}{2\pi d}$$
- Parallel currents attract, anti-parallel currents repel

B Fields from Currents (Fig. 30-11)

- For symmetric distributions of charge use Ampere's law to calculate *B* field

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Integral around closed loop called Amperian loop

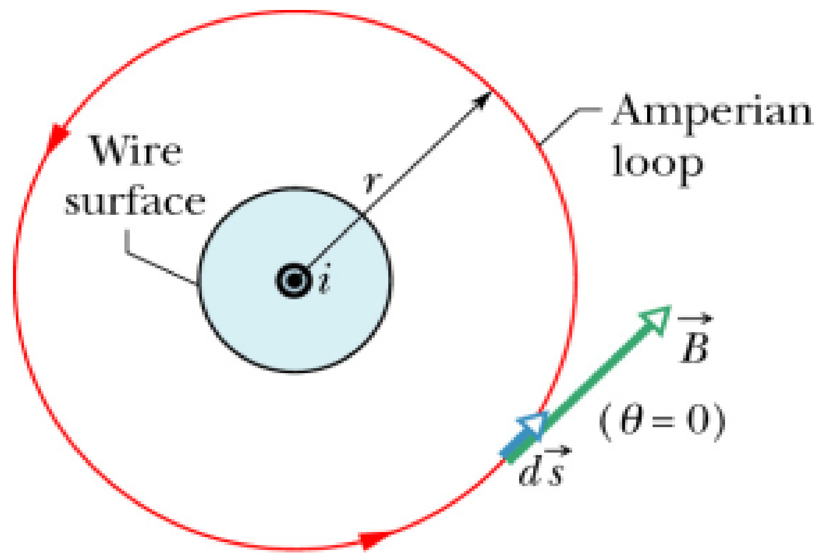


B Fields from Currents (Fig. 30-13)

- Use Ampere's law to calculate *B* field from long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Draw Amperian loop as a circle surrounding the wire (like the magnetic field lines)

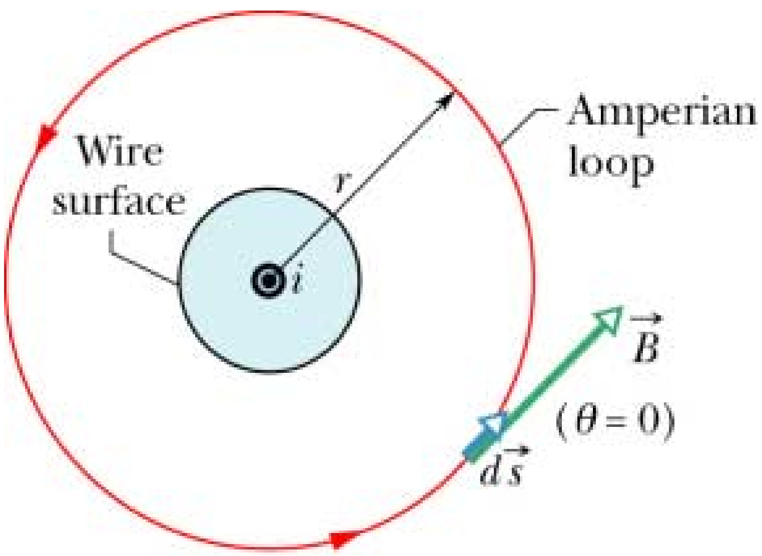


- At every point of the loop
  - Magnitude of *B* is constant
  - *B* and *ds* are || (both tangent to the circle)

B Fields from Currents (Fig. 30-13)

- Ampere's law becomes

$$B(2\pi r) = \mu_0 i_{enc}$$



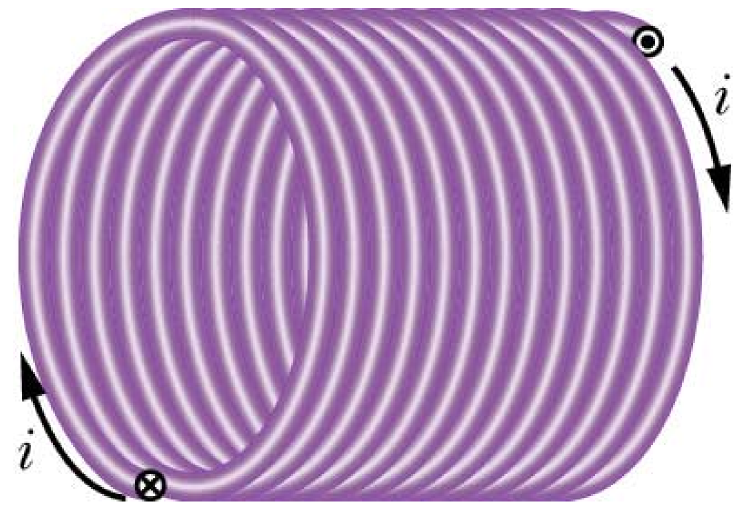
- Current enclosed is just *i* so

$$B = \frac{\mu_0 i}{2\pi r}$$

- Same result as with Biot-Savart law (we used *r=R* before).

B Fields from Currents (Figs. 30-16,20)

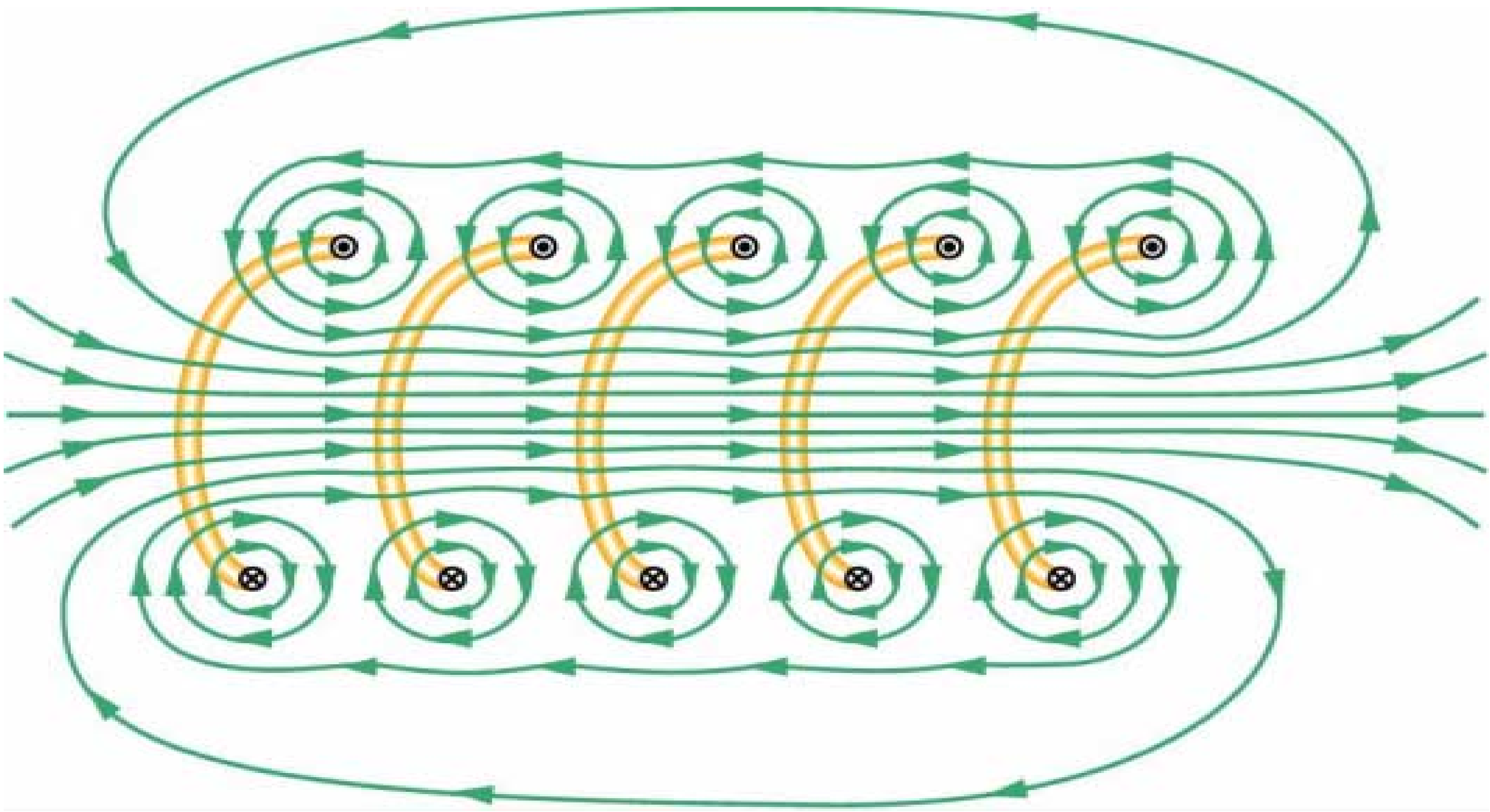
- What happens if there are several loops of wire put together?
- A long, tightly wound helical coil of wire is called a solenoid
- Bend solenoid so ends meet to make a hollow donut gives a toroid



- Use Ampere's law to calculate *B* field for a solenoid and a toroid

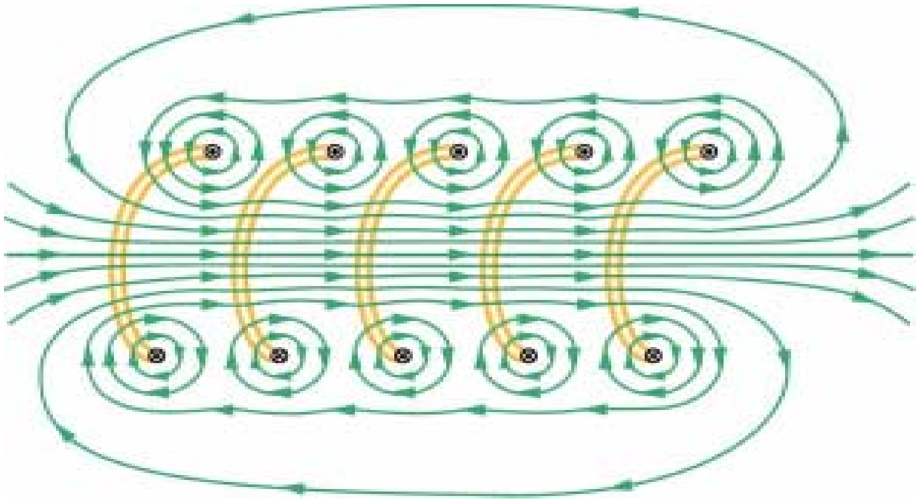
B Fields from Currents (Fig. 30-17)

Solenoid's *B* field is vector sum of fields produced by each turn (loop) in solenoid



B Fields from Currents (Fig. 30-17)

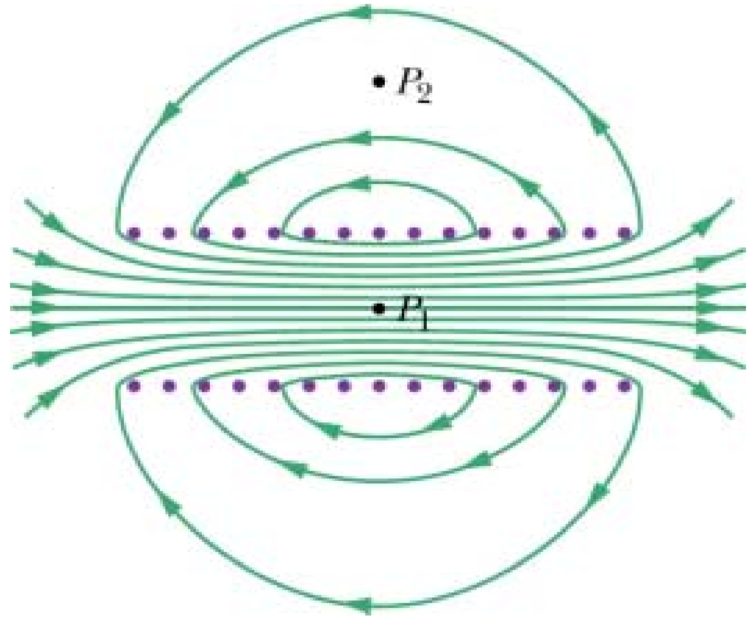
- Near each loop it looks like an infinite straight wire
- Between the loops fields tend to cancel
- Inside the solenoid, far from the wire, *B* field is parallel to axis



- An ideal solenoid
  - is infinity long with closely packed turns of wire
  - has uniform *B* field which is parallel to solenoid axis

B Fields from Currents (Fig. 30-17)

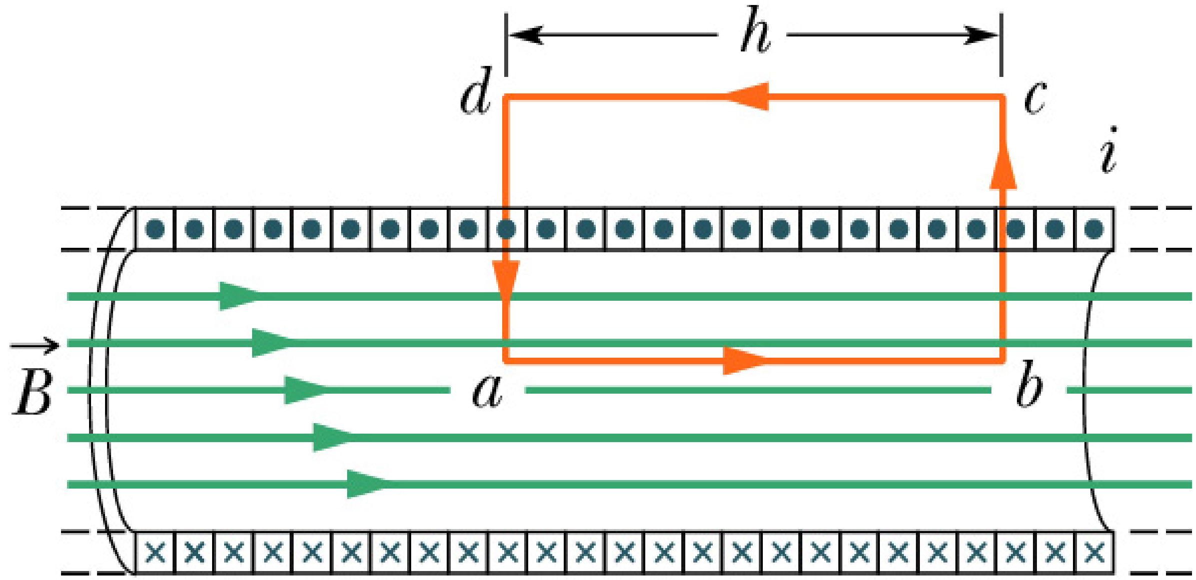
- For points outside the solenoid *B* fields from the upper parts of the turns tend to cancel the lower
- Ideal solenoid $B_{outside}=0$
- For a real solenoid can assume $B_{outside}=0$ if
  - length >> diameter
  - Only consider points not near ends of solenoid



- Use right-hand rule to find direction of *B* field
  - Grasp solenoid so fingers follow direction of *i* in loops, thumb points in *B*

B Fields from Currents (Fig. 30-19)

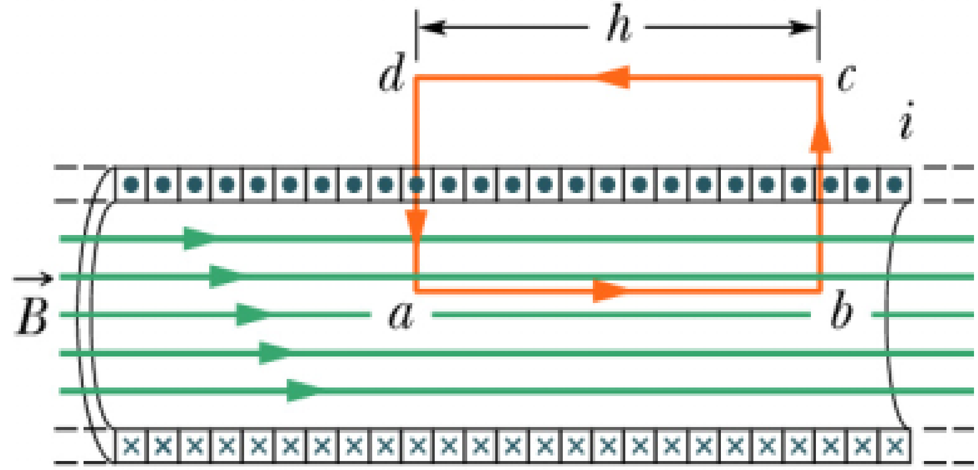
- Use Ampere's law to calculate *B* field of ideal solenoid



- Draw Amperian loop a-b-c-d-a intersecting solenoid

B Fields from Currents (Fig. 30-19)

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



- Integral can be written as sum of four integrals, one for each side

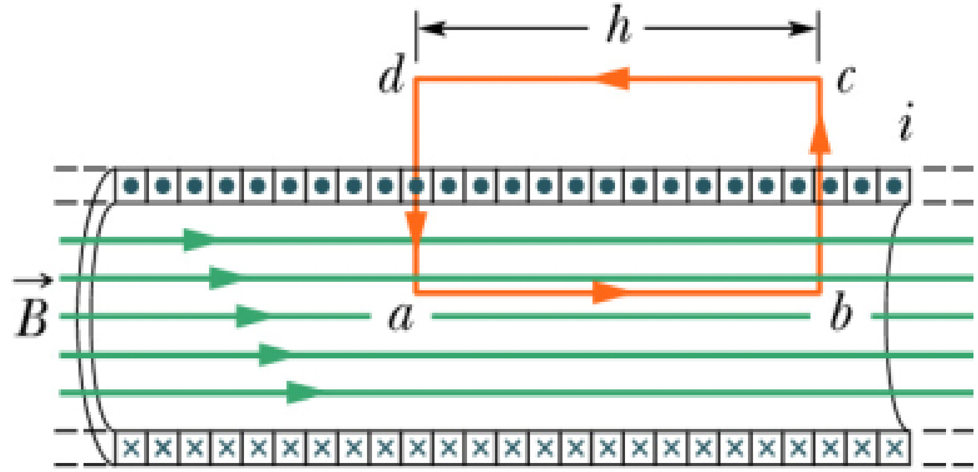
$$\oint \vec{B} \bullet d\vec{s} = \int_a^b \vec{B} \bullet d\vec{s} + \int_b^c \vec{B} \bullet d\vec{s}$$

$$+ \int_c^d \vec{B} \bullet d\vec{s} + \int_d^a \vec{B} \bullet d\vec{s}$$

B Fields from Currents (Fig. 30-19)

- First integral *B* field is || to *ds*

$$\int_a^b \vec{B} \bullet d\vec{s} = B[s]_a^b = Bh$$



- For sides *bc* and *da* *B* is ⊥ to *ds* so

$$\int_b^c \vec{B} \bullet d\vec{s} = \int_d^a \vec{B} \bullet d\vec{s} = 0$$

- For the length outside the solenoid B = 0

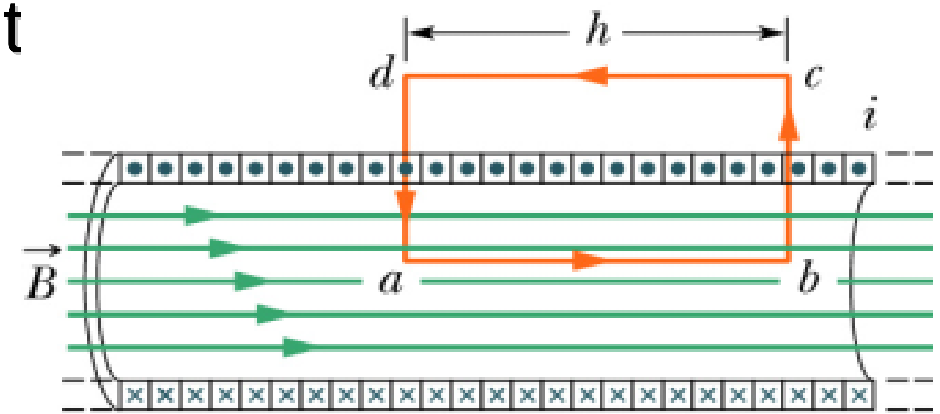
$$\int_c^d \vec{B} \bullet d\vec{s} = 0$$

$$\oint \vec{B} \bullet d\vec{s} = Bh$$

B Fields from Currents (Fig. 30-19)

- Now need to find amount of current enclosed

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



- Single coil has current *i*
- But Amperian loop encloses several coils so total current is

$$i_{enc} = inh$$

- *where* *n* is the number of turns per unit length

$$n = \frac{N}{L}$$

- *N* = total # of turns
- *L* = length

B Fields from Currents (Fig. 30-19)

- Substituting into Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

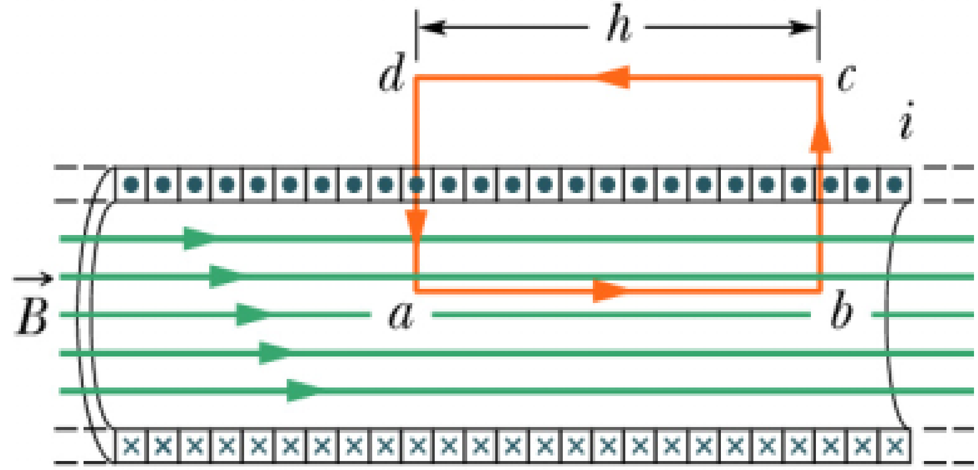
$$Bh = inh$$

- For ideal solenoid:

$$B = \mu_0 in$$

- n is total # of turns (N) / length (L)

$$n = \frac{N}{L}$$



- *B* field of solenoid
  - does not depend on diameter or length of solenoid
  - is uniform over its cross section

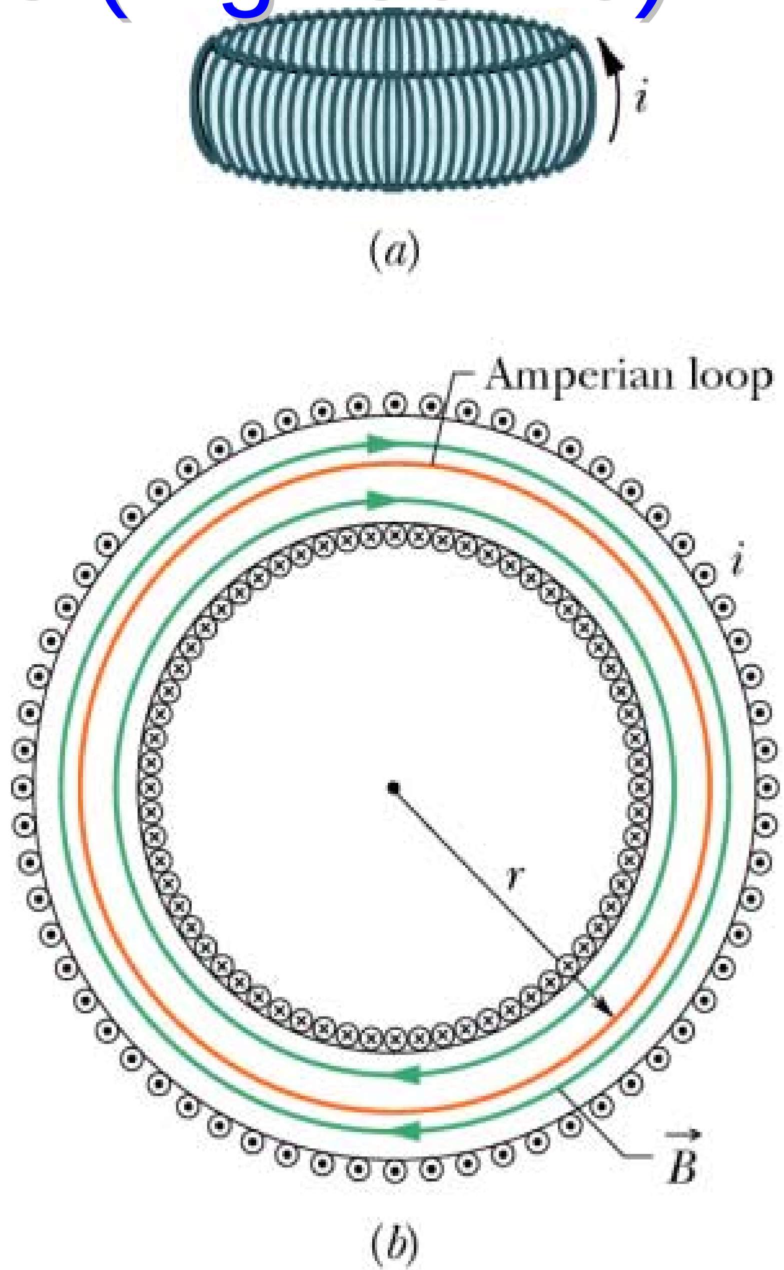
B Fields from Currents (Fig. 30-20)

- Calculate *B* field for a toroid using Ampere's law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

- Choose Amperian loop to be a concentric circle inside toroid
- *B* and *ds* are parallel along entire loop so

$$\oint \vec{B} \bullet d\vec{s} = B\int ds = B(2\pi r)$$



B Fields from Currents (Fig. 30-20)

- Current enclosed by loop is

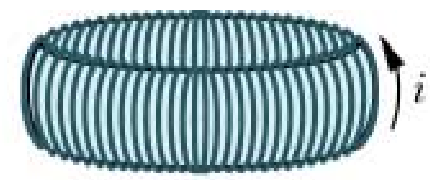
$$i_{enc} = iN$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

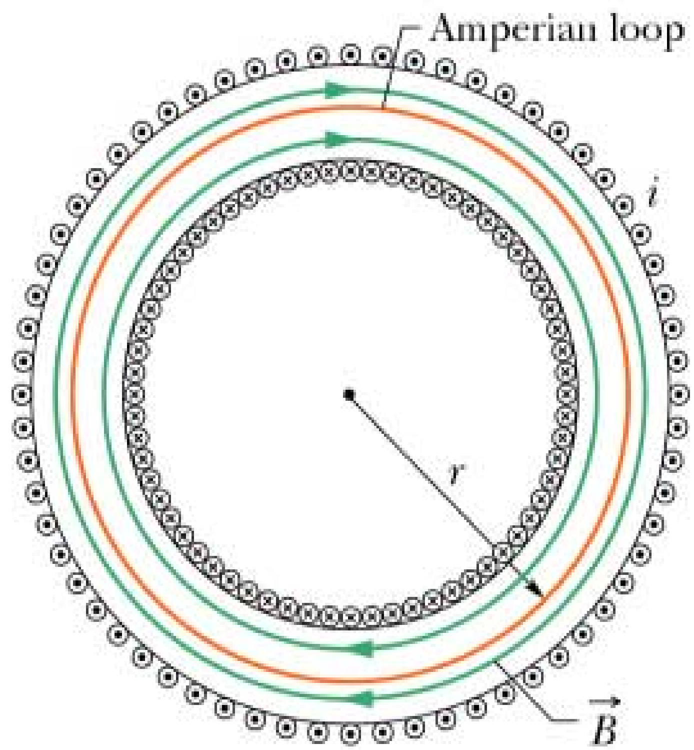
$$B(2\pi r) = \mu_0 iN$$

- *B* field for toroid is

$$B = \frac{\mu_0 iN}{2\pi r}$$



(a)



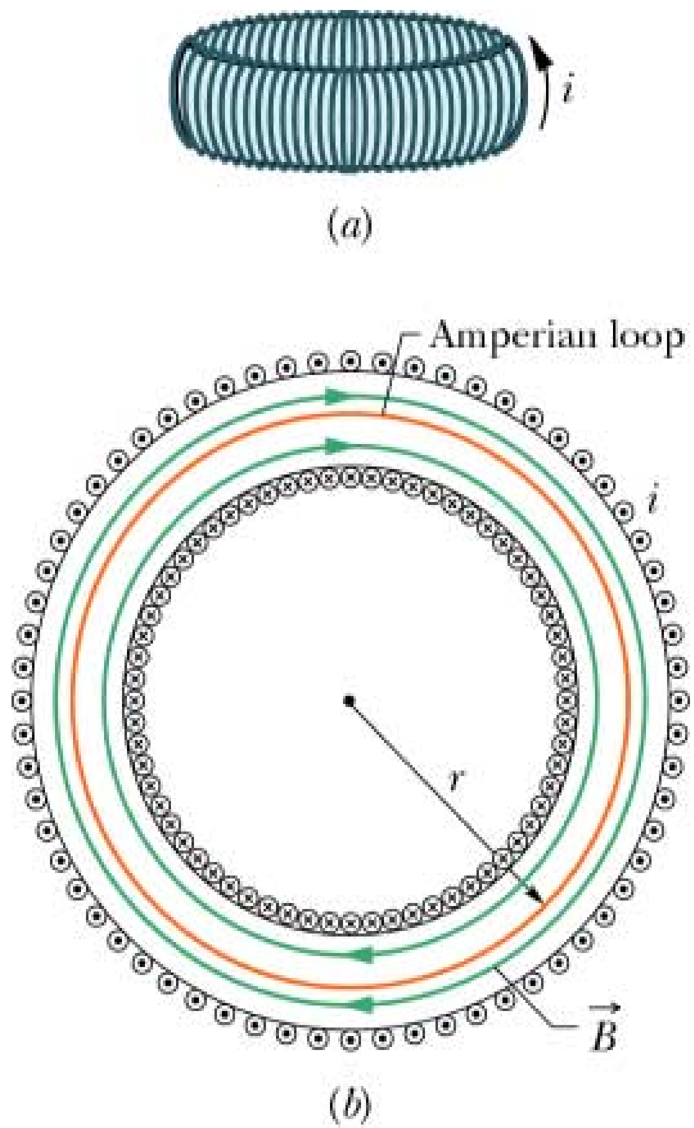
(b)

B Fields from Currents (Fig. 30-20)

- Toroid – *B* field is not constant over its cross section

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

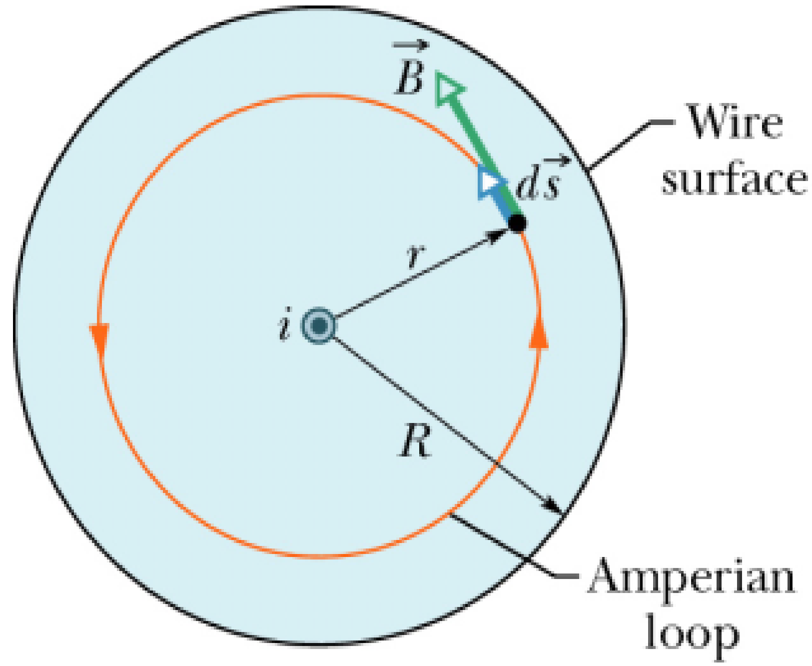
- N = total # of turns
- Use right-hand rule to find direction of *B* field
  - Grasp toroid with fingers in direction of current in windings, thumb points in B
- *B* = 0 outside toroid



B Fields from Currents (Fig. 30-14)

- Calculate *B* field inside a long straight wire

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$



- Again *B* and *ds* are || and *B* is a constant so

$$\oint \vec{B} \bullet d\vec{s} = B\oint ds = B(2\pi r)$$

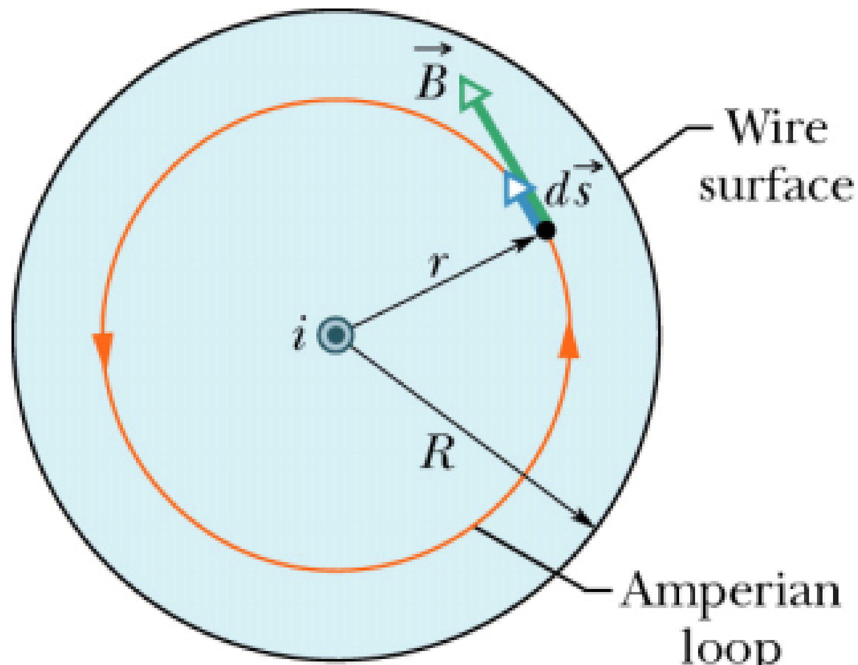
$$B(2\pi r) = \mu_0 i_{enc}$$

B Fields from Currents (Fig. 30-14)

- Need to find i_{enc}
- Current is uniformly distributed so i enclosed by loop is α to area enclosed

$$i_{enc} = i\frac{\pi r^2}{\pi R^2}$$

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$



$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

Chapter 30 - Problem 38P

- A long circular pipe with outside radius R carries a uniformly distributed current *i* into the page. A wire runs parallel to the pipe at a distance of 3R from center to center. Find the magnitude and direction of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

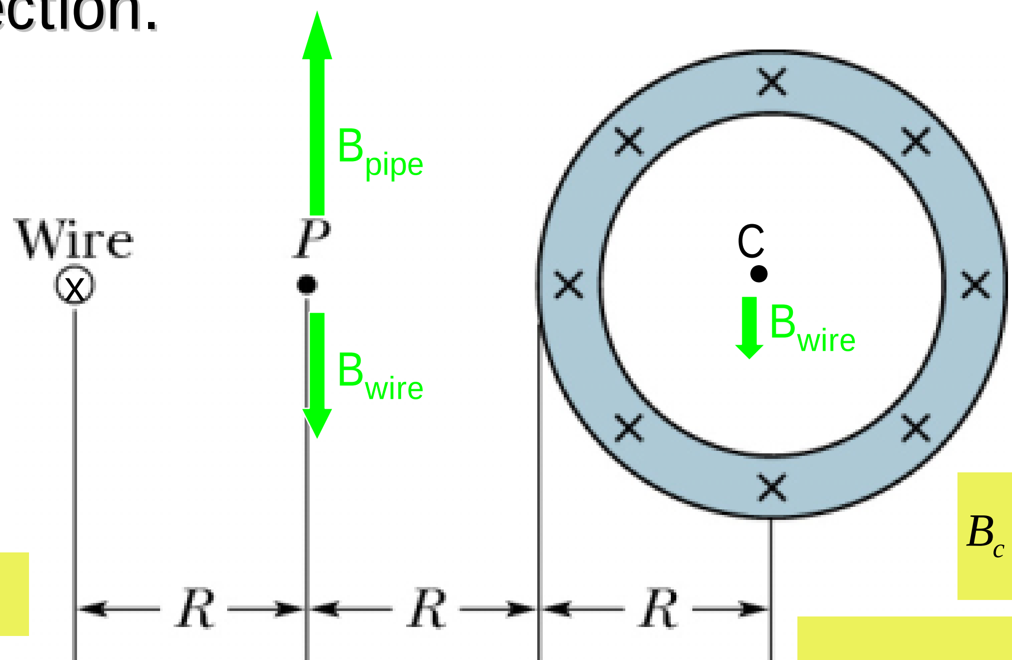
$B_P = -B_C$

BUT

$B_{P,wire} > B_{C,wire}$

SO

i of wire into page



$$B_c = \frac{\mu_0 i_{wire}}{2\pi r} = \frac{\mu_0 i_{wire}}{2\pi(3R)}$$

$$B_P = B_{pipe} - B_{wire} = \frac{\mu_0 i_{pipe}}{2\pi(2R)} - \frac{\mu_0 i_{wire}}{2\pi(R)}$$