October 16/17th

Induction and Inductance

Chapter 31

"Now, you've got him, Vinnie!"

Review

- Forces due to *B* fields
  - On a moving charge

Lorentz force

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

  - On a current

$$\vec{F}_B = i\vec{L}\times\vec{B}$$

  - Current carrying coil feels a torque

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

$$\mu = NiA$$

- Currents generate *B* field
  - Biot-Savart law

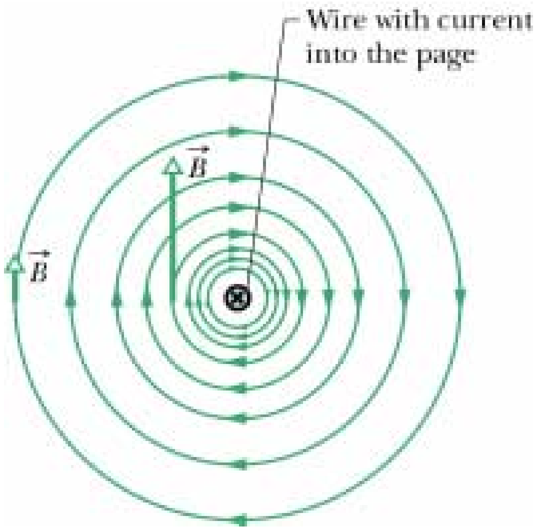
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}$$

  - Ampere's law

$$\oint \vec{B}\bullet d\vec{s} = \mu_0 i_{enc}$$

- Calculated *B* field for
  - Long, straight wire

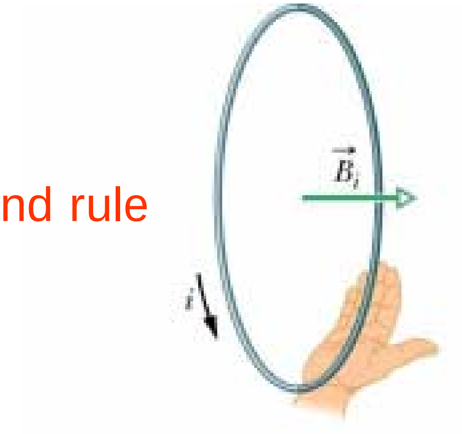
$$B = \frac{\mu_0 i}{2\pi r}$$

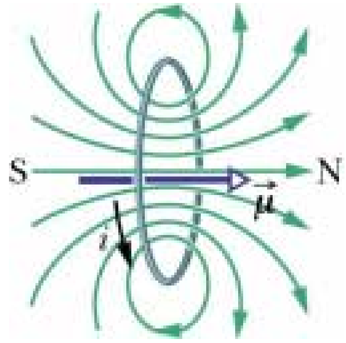


  - Center of loop

$$B_i = \frac{\mu_0 i}{2R}$$

Note right-hand rule
For the loop

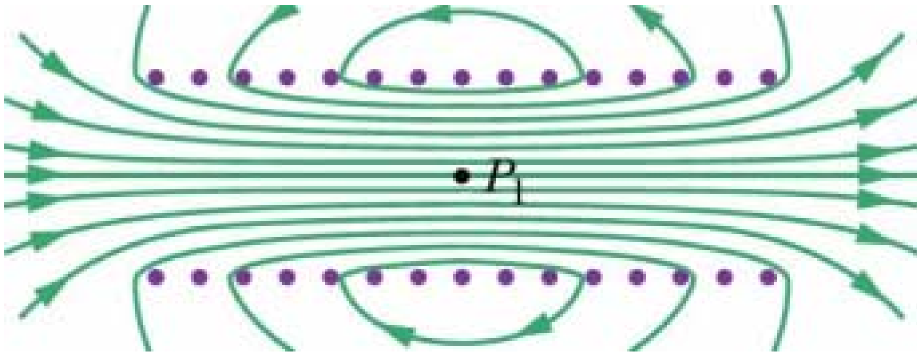




  - Inside solenoid (P_1)

$$B = \mu_0 i n$$

$$n = N / L$$

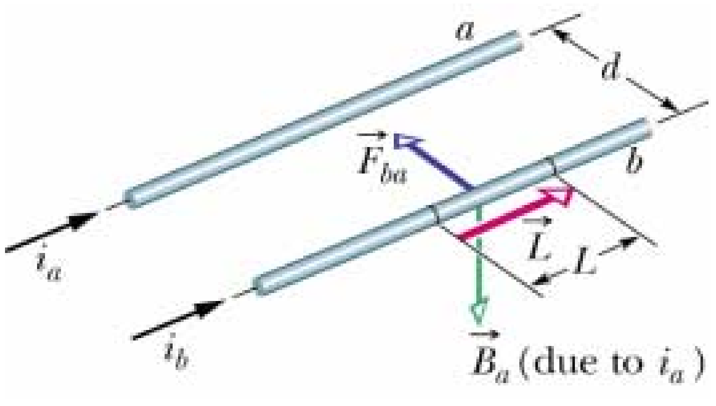


Review

- Force on a wire carrying current, i_1, due to B of another parallel wire with current i_2

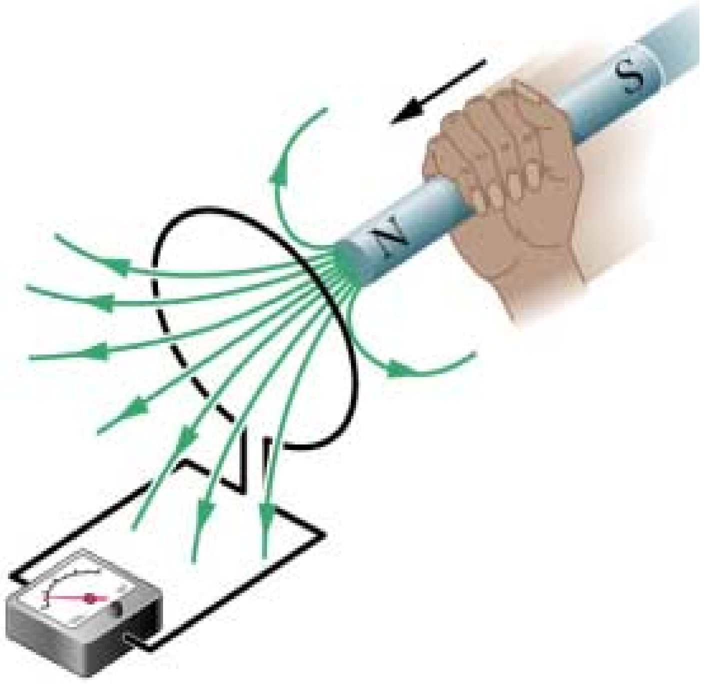
$$F = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

- Force is attractive (repulsive) if current in both wires are same (opposite) directions



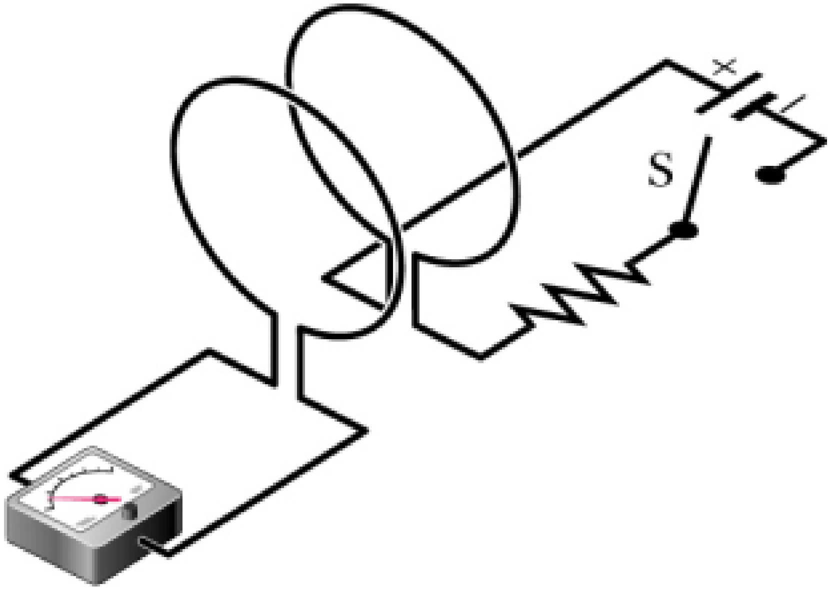
Induced currents (Fig. 31-1)

- A current can produce a *B* field
- Can a *B* field generate a current?
- Move a bar magnet in and out of loop of wire
  - Moving magnet towards loop causes current in loop
  - Current disappears when magnet stops
  - Move magnet away from loop current again appears but in opposite direction
  - Faster motion produces a greater current



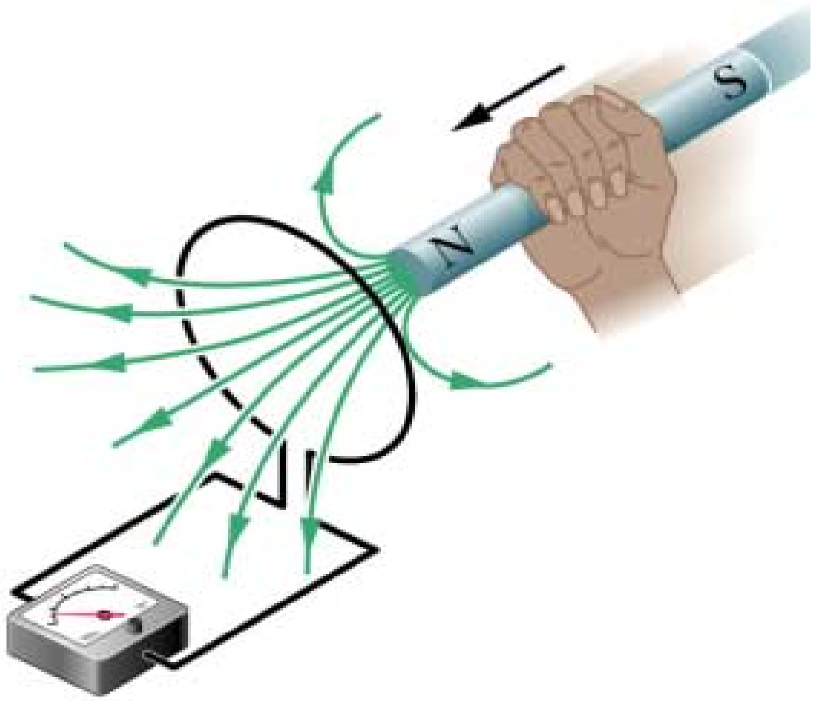
Induced currents (Fig. 31-2)

- Have two conducting loops near each other
  - Close switch so current flows in one loop, briefly register a current in other loop
  - Open switch, again briefly register current in other loop but in opposite direction



Induced currents (Fig. 31-1)

- Current produced in the loop is called induced current
- The work done per unit charge to produce the current is called an induced emf
- Process of producing the current and emf is called induction

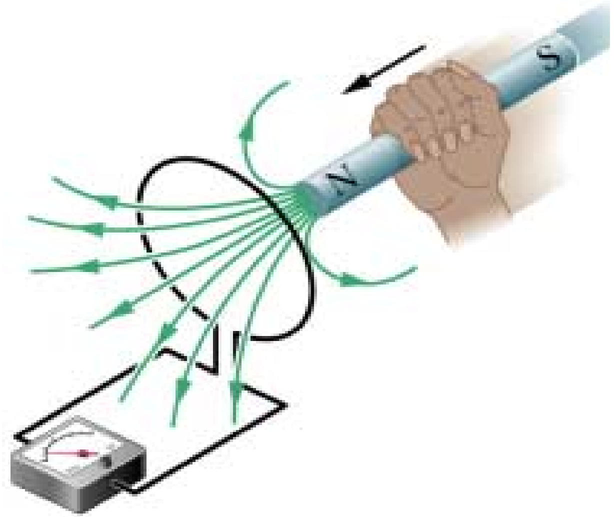


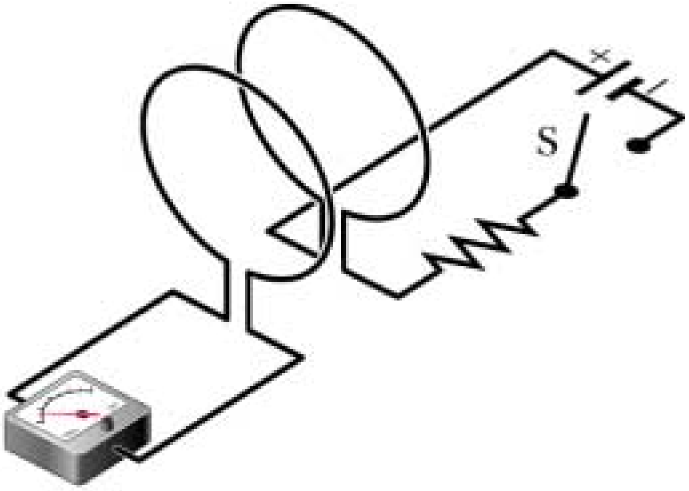
Faraday’s law

- Faraday observed that an induced current (and an induced emf) can be generated in a loop of wire by:
  - Moving a permanent magnet in or out of the loop
  - Holding it close to a coil (solenoid) and changing the current in the coil
  - Keep the current in the coil constant but move the coil relative to the loop
  - Rotate the loop in a steady B field
  - Change the shape of the loop in a B field

Faraday's law

- Faraday concluded that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop
- Need to calculate the amount of magnetic field through the loop so define magnetic flux analogous to electric flux





Faraday's law

- Magnetic flux through area *A*

- *dA* is vector of magnitude *dA* that is ⊥ to the differential area, *dA*

$$\boxed{\Phi_B = \int \vec{B} \bullet d\vec{A}}$$

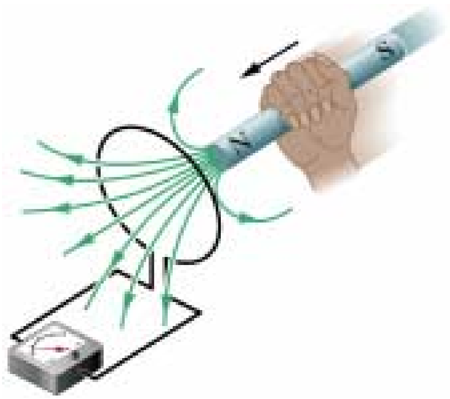
- If *B* is uniform and ⊥ to *A* then

$$\Phi_B = BA$$

- SI unit is the weber, Wb

$$\boxed{Wb = T \cdot m^2}$$

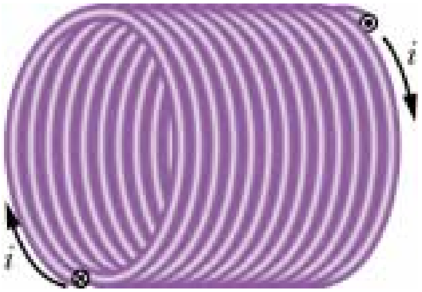
Faraday’s law



- Faraday’s law of induction – induced emf in loop is equal to the rate at which the magnetic flux changes with time

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Minus signs means induced emf tends to oppose the flux change
- If magnetic flux is through a closely packed coil of N turns



$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Faraday's law

- If B is constant within coil

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

- We can change the magnetic flux through a loop (or coil) by
  - Changing magnitude of *B* field within coil
  - Changing area of coil, or portion of area within *B* field
  - Changing angle between *B* field and area of coil (e.g. rotating the coil)

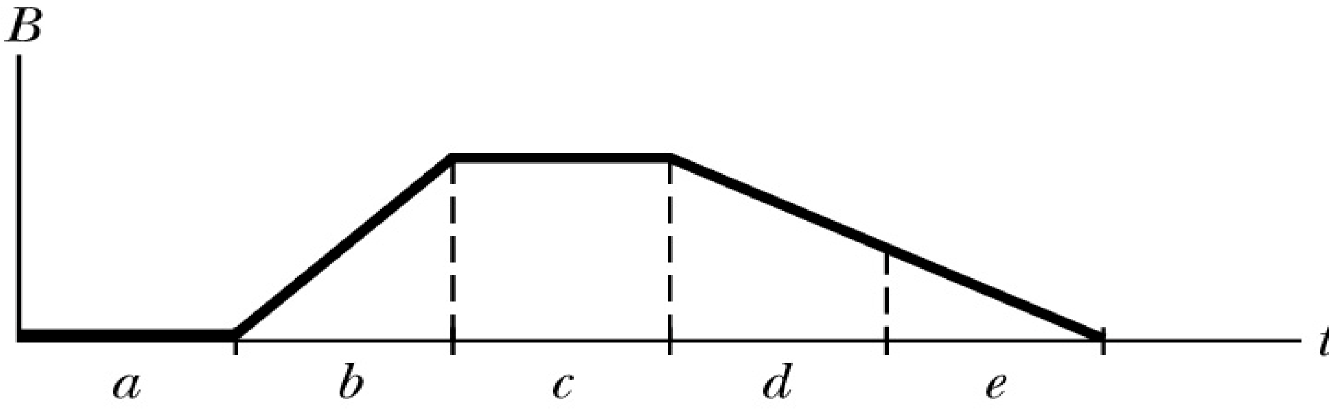
$$\mathcal{E} = -NA\cos\theta\frac{dB}{dt}$$

$$\mathcal{E} = -NB\cos\theta\frac{dA}{dt}$$

$$\mathcal{E} = -NBA\frac{d(\cos\theta)}{dt}$$

Checkpoint #1

- Graph shows magnitude *B(t)* of uniform *B* field passing through loop, ⊥ to plane of the loop. Rank the five regions according to magnitude of emf induced in loop, greatest first.

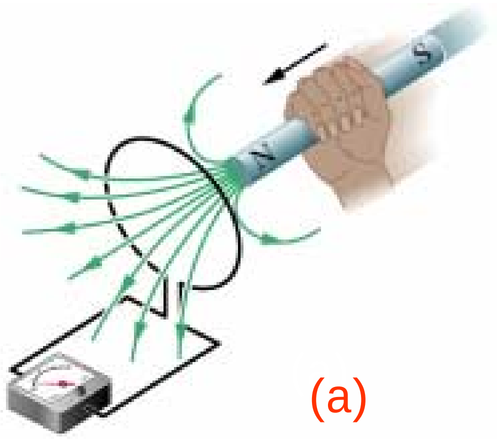


$$\mathcal{E} = -NA\cos\theta \frac{dB}{dt} = -NA\frac{dB}{dt}$$

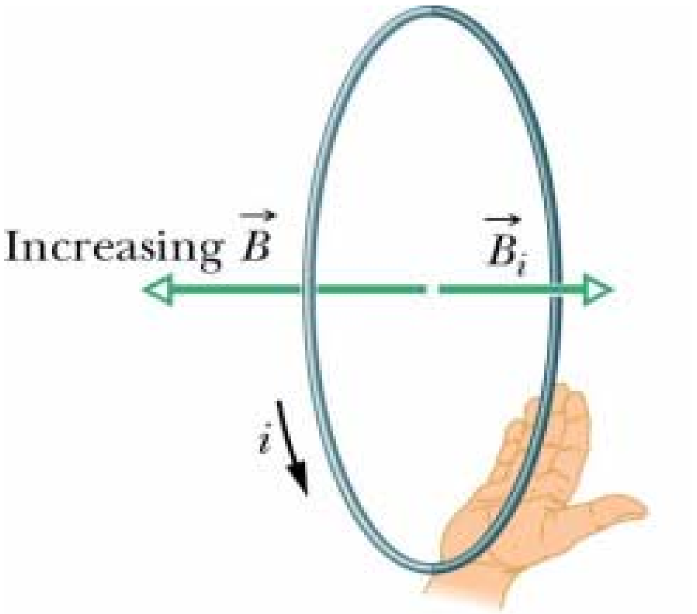
b, then d & e tie,
then a & c (zero)

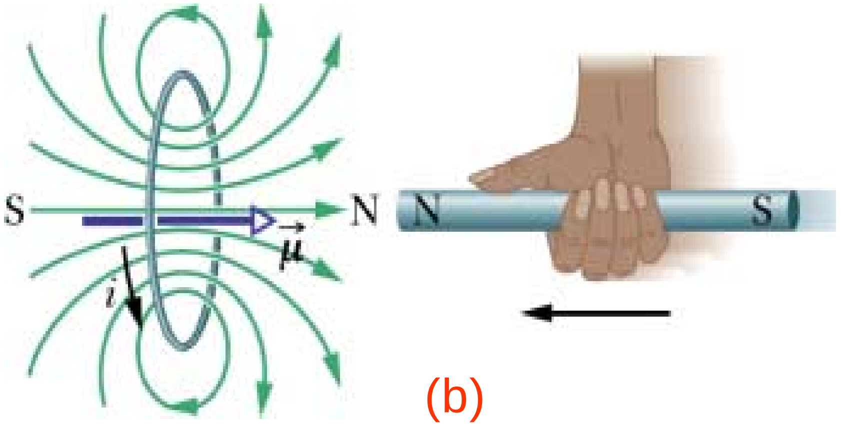
Lenz’s law (Figs. 31-4, 31-5)

- Lenz’s law – An induced emf gives rise to a current whose *B* field opposes the change in flux that produced it
- As the magnet moves towards, the loop the flux in loop increases (a), so the induced current sets up B_{i} field in the opposite direction (b)



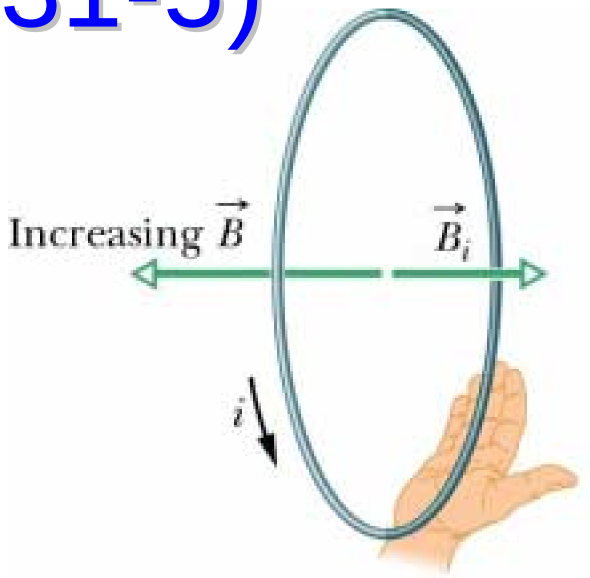
(a)



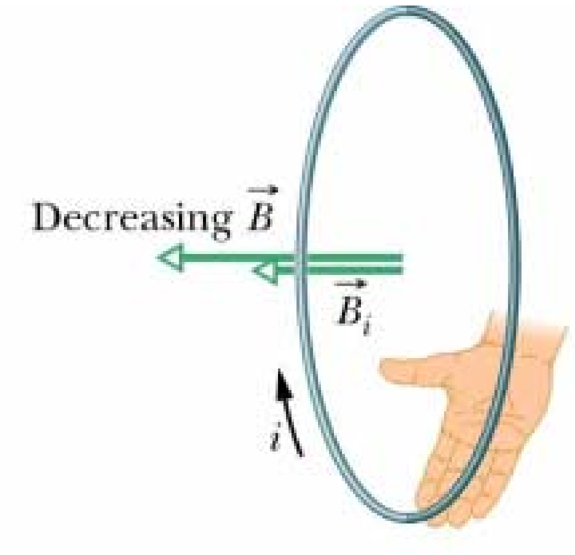


(b)

Lenz’s law (Fig. 31-5)



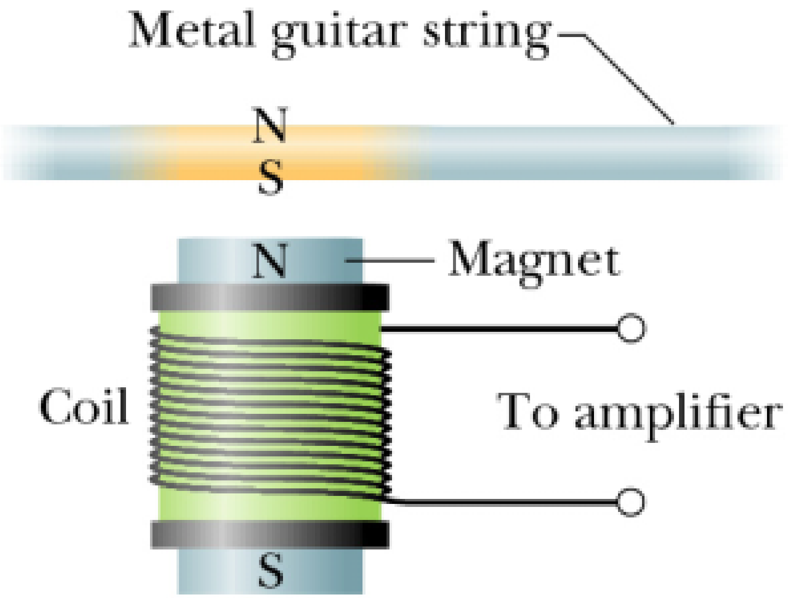
- (a) Magnet moves towards loop; the flux in loop increases so induced current sets up B_i field in the opposite direction to cancel the increase:
- B_i is in the opposite direction to increasing B



- (b) Magnet moves away from loop; the flux decreases so induced current has a B_i field in the same direction to cancel the decrease:
- B_i is in the same direction as decreasing B

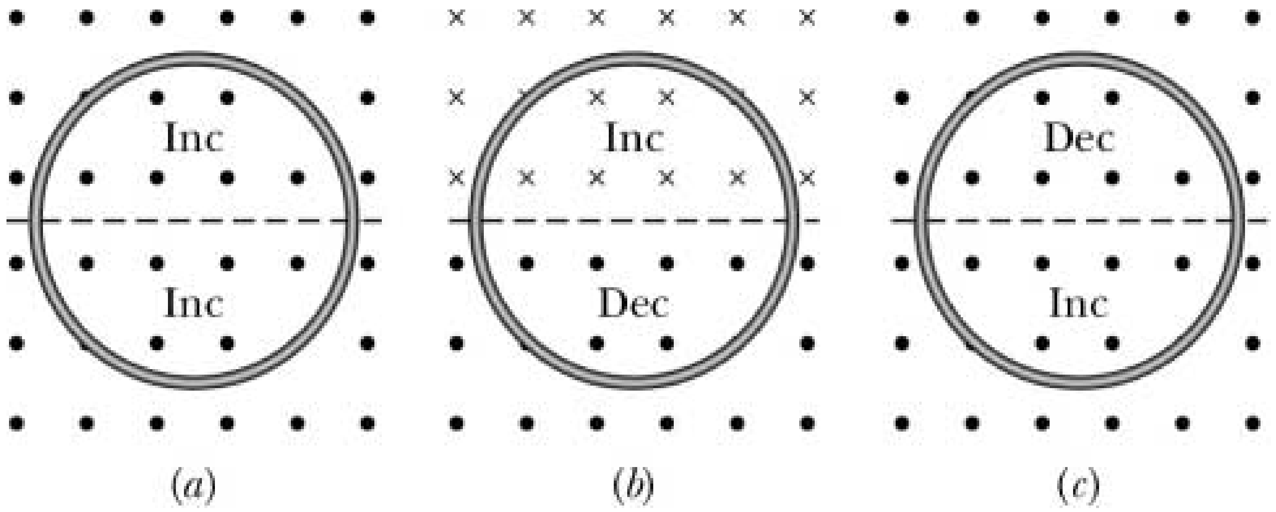
Lenz's law (Figs. 31-7)

Example: electric guitar

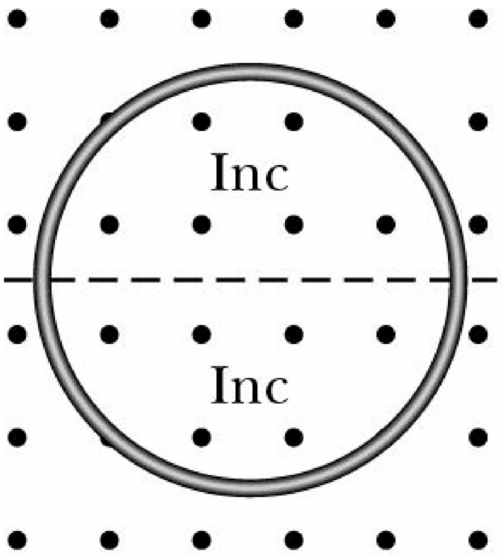


Checkpoint #2

- Three identical circular conductors in uniform *B* fields that are either increasing or decreasing in magnitude at identical rates. Rank according to magnitude of current induced in loop, greatest first.



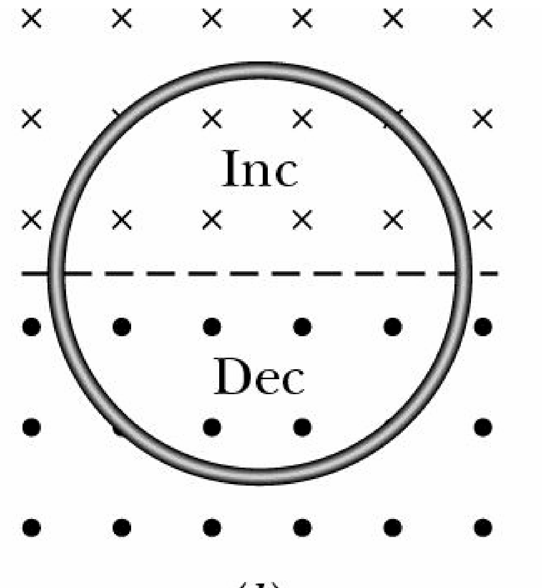
  - Use Lenz's law to find direction of B_i
  - Use right-hand rule to find direction of current



(a)

INC=increase

- Situation (a):
  - B increases out of page, so B_i is into page
  - From right-hand rule, induced current is clockwise



(b)

INC=increase
DEC=decrease

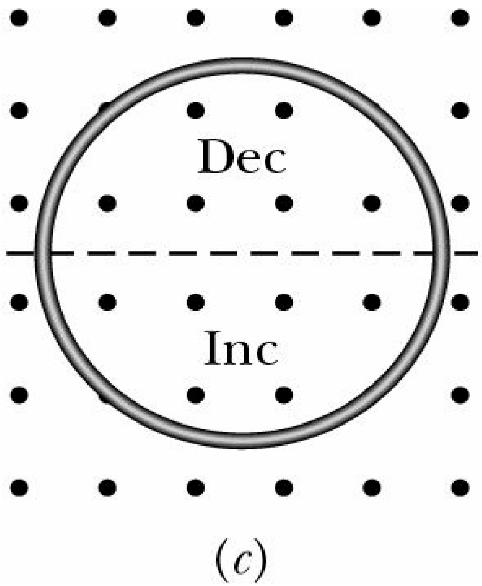
- Situation (b) top:

  B increases into the page, so B_i is out of the page

- Situation (b) bottom:

  B decreases out of the page, so B_i is out of the page

- In both cases from the right-hand rule, induced current is counter-clockwise



INC=increase
DEC=decrease

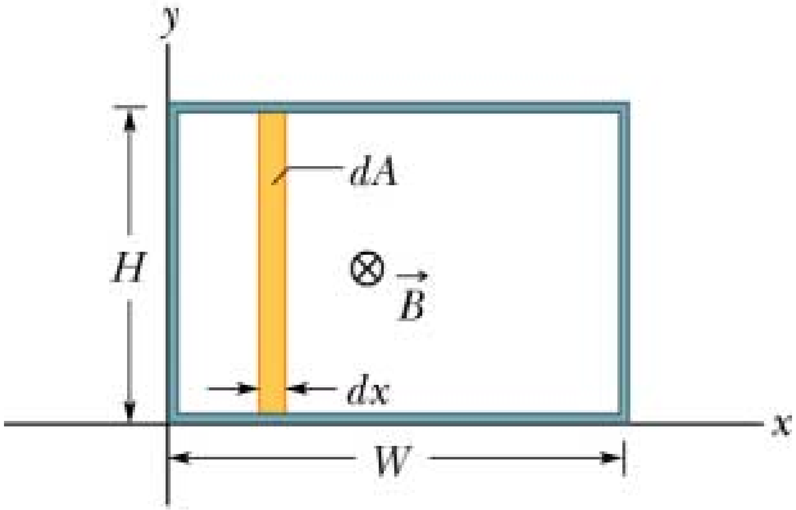
- Situation (c) top:
  - B decreases out of page, so B_i is out of the page
- Situation (c) bottom:
  - B increases out of the page, so B_i is into the page
- Total B_i is zero and total current is zero
- Rank magnitude of current induced in loops

a & b tie, then c

Problem 31-3 (Fig.31-9)

- Loop has width W=3.0m and height H=2.0m
- Loop in non-uniform and varying *B* field ⊥ to loop and directed into the page

$$B = 4t^2x^2$$



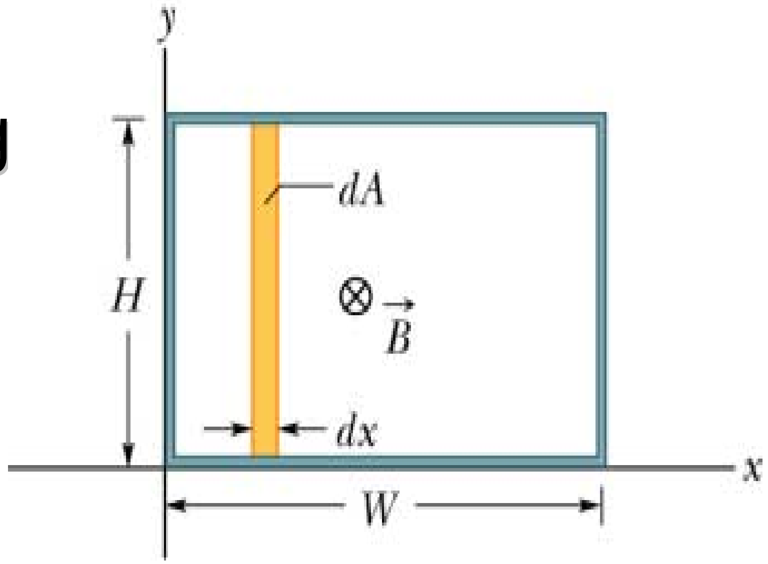
- What is magnitude and direction of induced emf around loop at t=0.10s?
- Since magnitude *B* is changing in time, flux through the loop is changing so use Faraday’s law to calculate induced emf

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Problem 31-3 (Fig.31-9)

- *B* is not uniform so need to calculate magnetic flux using

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$



- *B* ⊥ to plane of loop and only changes in x direction

$$\vec{B} \bullet d\vec{A} = BdA = BHdx$$

$$B = 4t^2x^2$$

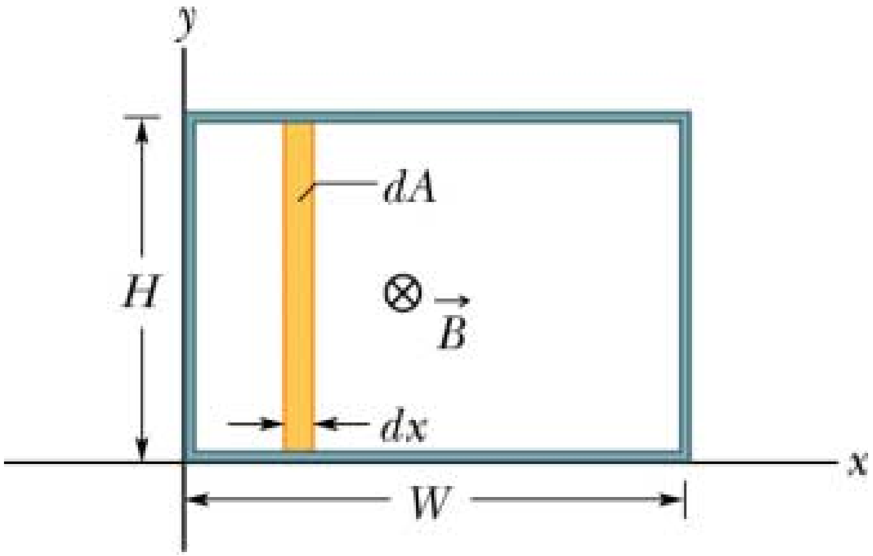
- At time t:

$$\Phi_B = \int BHdx = 4t^2 H \int_0^3 x^2 dx = 4t^2 H \left[\frac{x^3}{3}\right]_0^3 = 72t^2$$

Problem 31-3 (Fig.31-9)

- Now use Faraday's law to find the magnitude of the induced emf

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t$$



- At t=0.10s, emf = 14 V

- Find direction of emf by Lenz's law

$$B = 4t^2x^2$$

  - B is increasing in time directed into the page, so B_i is in opposite direction - out of the page
  - Right-hand rule – current (and emf) are counterclockwise