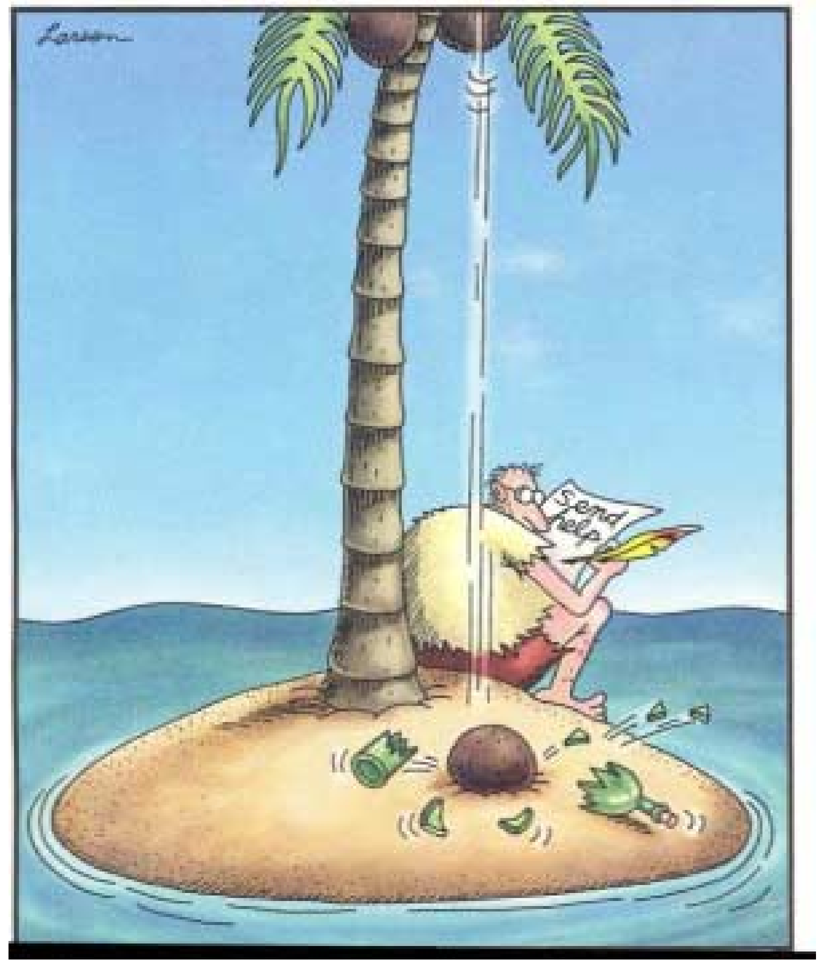
October 21th

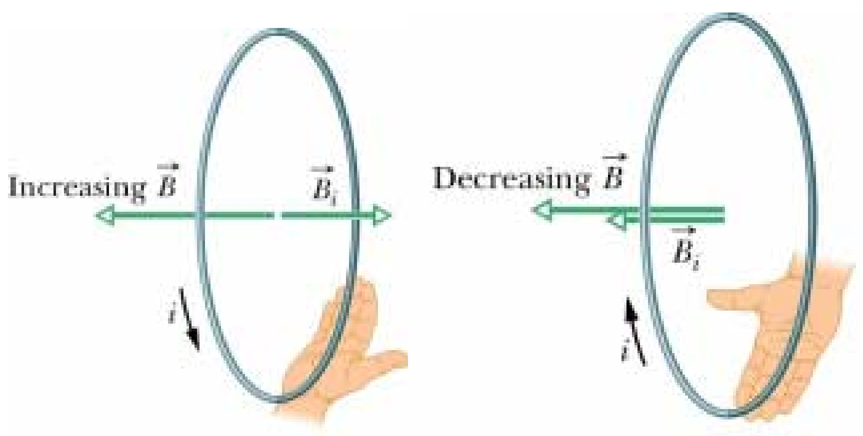
Induction and Inductance

Chapter 31



Review

- Magnetic flux $$\Phi_B = \int \vec{B} \bullet d\vec{A}$$
- Faraday's law (one loop) for emf (E) (induced voltage) $$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
- Faraday's law (N loops) $$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$
- Lenz's law – induced emf gives rise to a current whose *B* field opposes the change in flux that produced it



Review

- Induced emf of a conductor moving with velocity, *v*, in a ⊥ *B* field is given by

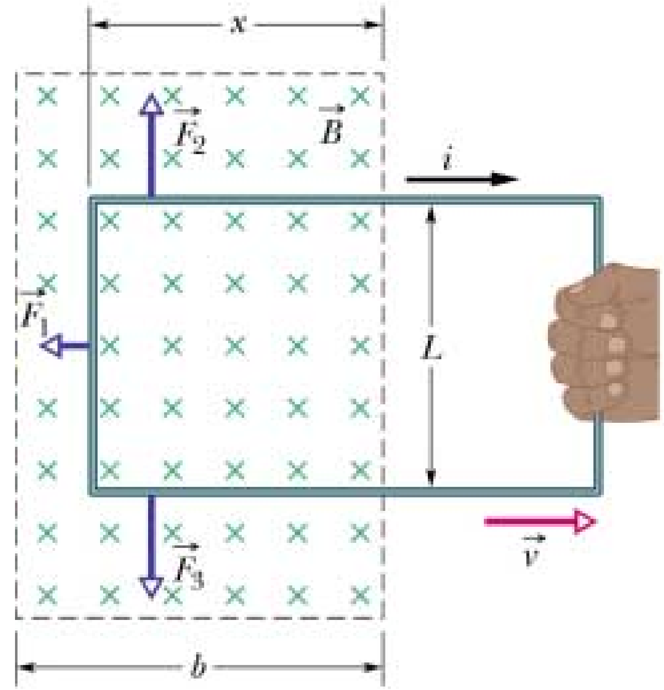
$$\mathcal{E} = BLv$$

- Induced current in loop in a *B* field experiences a force

$$F_B = i\vec{L} \times \vec{B}$$

- Found F_1 opposes your force F_{app}

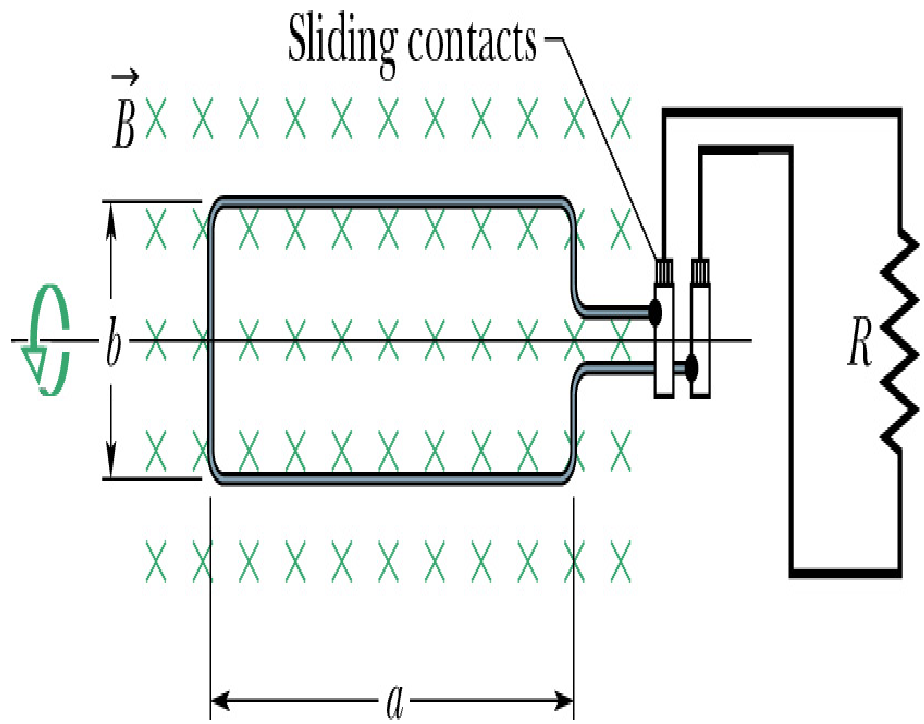
$$\vec{F}_{app} = -\vec{F}_1$$



- Work you do in pulling the loop appears as thermal energy in the loop

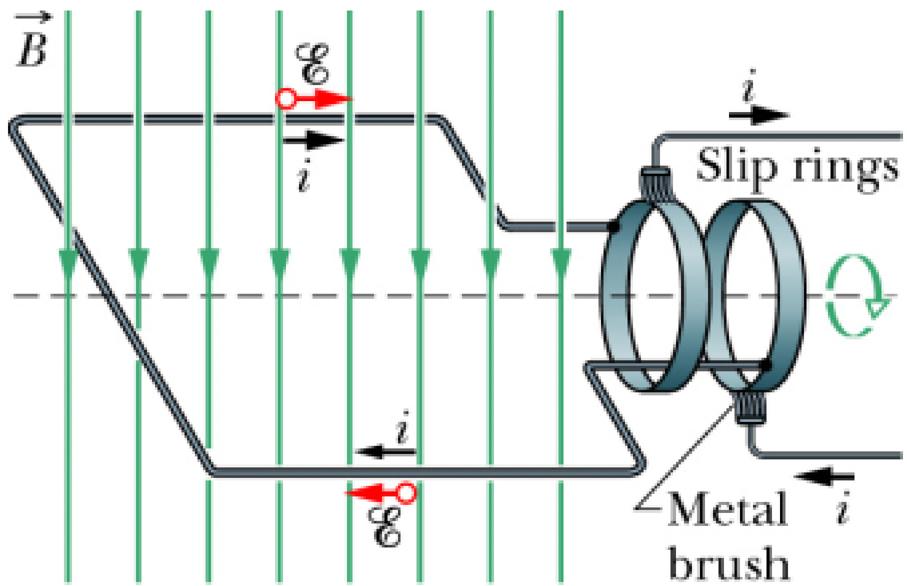
Inductance

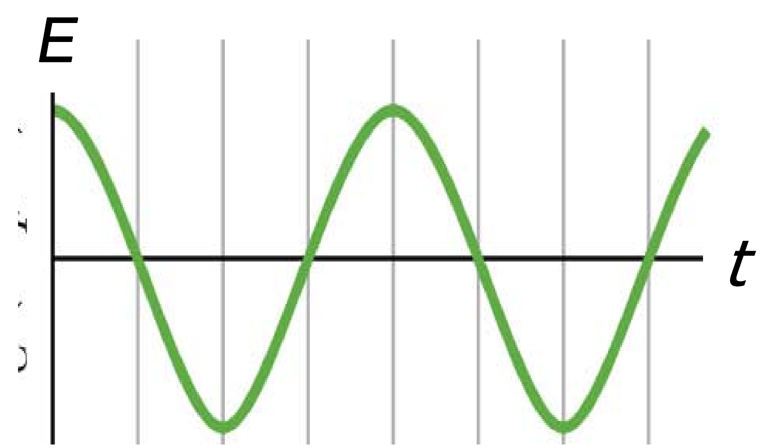
- Generators – convert mechanical energy to electrical energy
- External agent rotates loop of wire in B field
  - Hydroelectric plant
  - Coal burning plant
- Changing Φ_B induces an emf and current in an external circuit



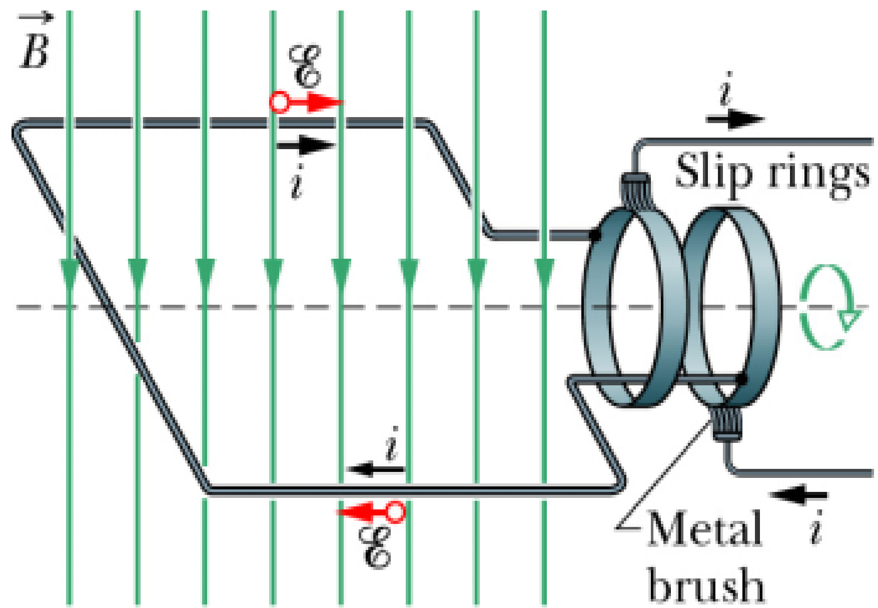
Inductance

- Alternating current (ac) generator
  - Ends of wire loop are attached to slip rings which rotate with loop
  - Stationary metal brushes are in contact with slip rings and connected to external circuit
  - emf and current in circuit alternate in time





Inductance



- Calculate emf for generator with N turns of area A and rotating with constant angular velocity, ω
- Magnetic flux is

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

- Relate angular dis-placement to angular velocity

$$\theta = \omega t$$

- Flux through one loop is

$$\Phi_B = BA\cos\omega t$$

Inductance

- Faraday's law says

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

- Substitute

$$\Phi_B = BA\cos\omega t$$

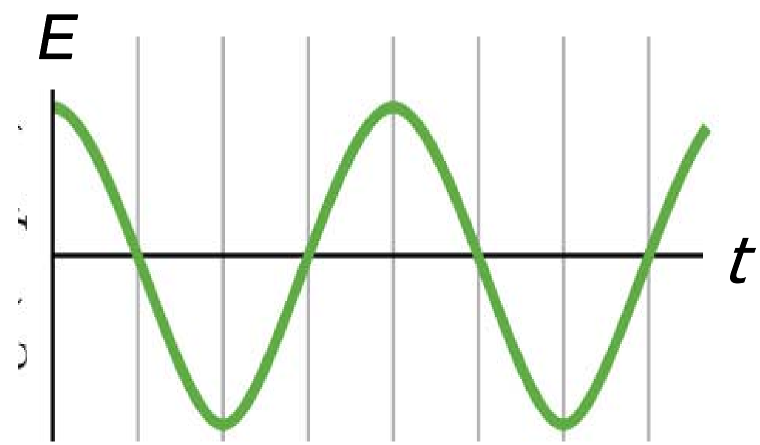
$$\mathcal{E} = -NBA\frac{d}{dt}(\cos\omega t)$$

$$\mathcal{E} = NBA\omega\sin\omega t$$

- Maximum emf is when ωt = 90 or 270 degrees

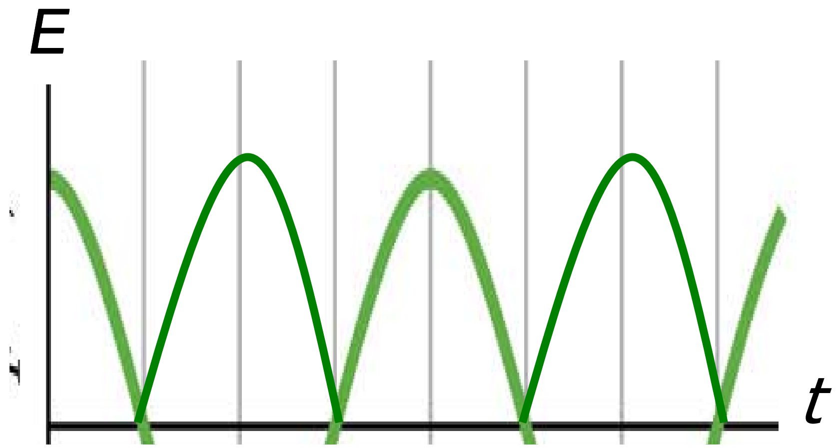
$$\mathcal{E}_{\max} = NBA\omega$$

- Emf is 0 when ωt = 0 or 180 degrees



Inductance

- Direct current (dc) generator
  - Ends of loop are connected to a single split ring
  - Metal brush contacts to split ring reverse their roles every half cycle
  - Polarity of induced emf reverses but polarity of split ring remains the same



- Not suitable for most applications
  - Can use to charge batteries
- Commercial dc gen. use out of phase coils

Inductance

- Motors – converts electrical energy to mechanical energy
  - Generator run in reverse
  - Current is supplied to loop and the torque acting on the current-carrying loop causes it to rotate
  - Do mechanical work by using the rotating armature
  - As loop rotates, changing B field induces an emf
  - Induced emf (back emf) reduces the current in the loop – remember Lenz's law
  - Power requirements are greater for starting a motor and for running it under heavy loads

Review for Inductance

- Inductor is a device used to produce and store a desired *B* field (e.g. solenoid)
- A current *i* in an inductor with *N* turns produces a magnetic flux, Φ_B, in its central region
- Inductance, *L* is defined as

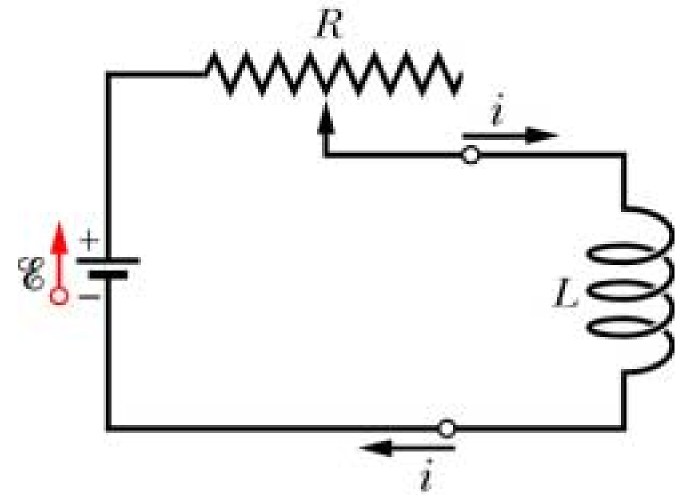
$$L = \frac{N\Phi_B}{i}$$

- Inductance per unit length of a solenoid
  - Depends only on geometry of device (like capacitor)

$$\frac{L}{l} = \mu_0 n^2 A$$

Inductance

- A changing current in a coil generates a self-induced emf, ε_L in the coil
- Process is called self-induction
- Change current in coil using a variable resistor, ε_L will appear in coil only while the current is changing



$$L = \frac{N\Phi_B}{i}$$

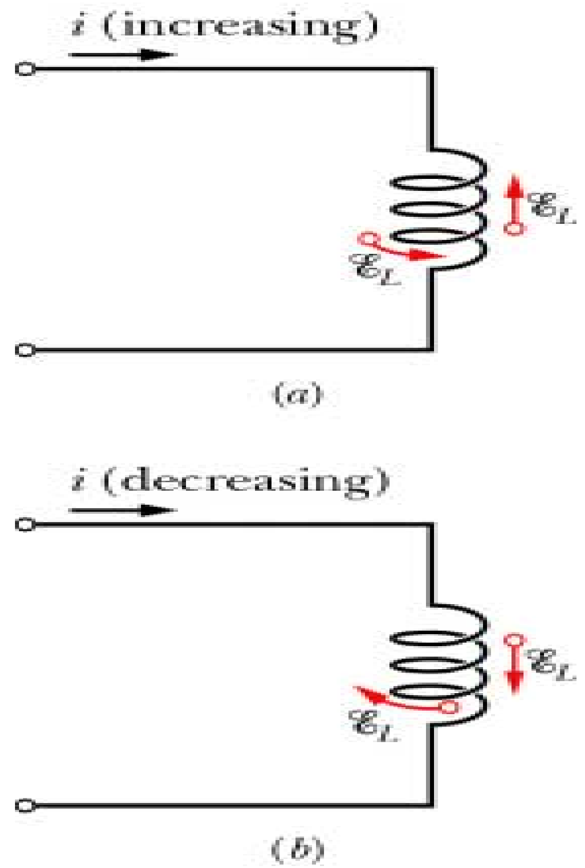
$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

Inductance

- Induced emf only depends on rate of change of current, not its magnitude
- Direction of ε_L follows Lenz's law and opposes the change in current
- Self-induced V_L across inductor
  - Ideal inductor $$V_L = \mathcal{E}_L$$
  - Real inductor (like real battery) has some internal resistance

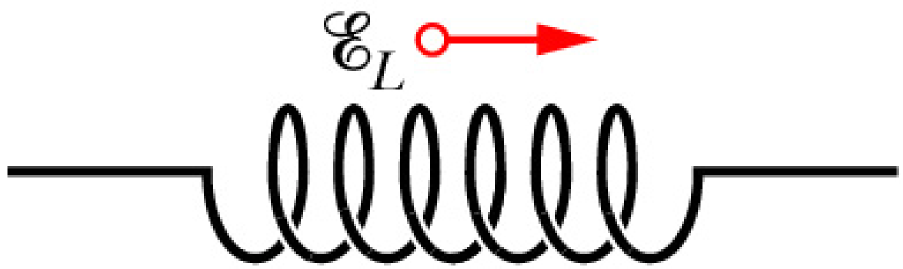
$$V_L = \mathcal{E}_L - iR$$

$$\mathcal{E}_L = -L\frac{di}{dt}$$



Inductance

- Checkpoint #4 – Have an induced emf in a coil. What can we tell about the current through the coil? Is it moving right or left and is it constant, decreasing or increasing?



Only get ε_L if current changing

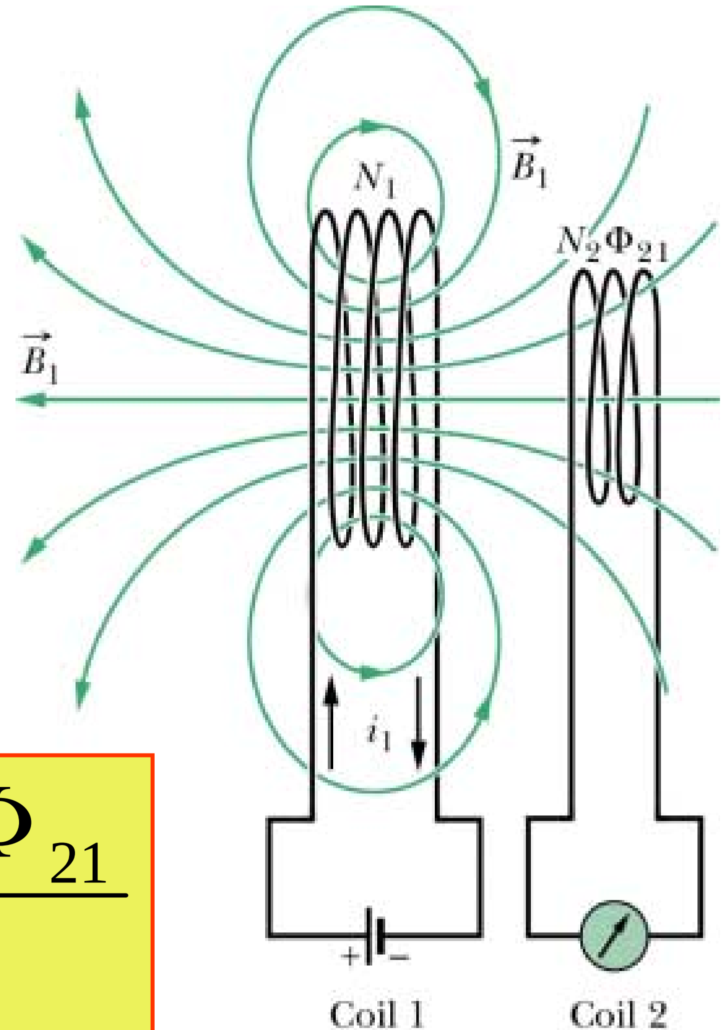
Decreasing and rightward

OR

Increasing and leftward

Inductance

- Mutual induction – current in one coil induces emf in other coil
- Distinguish from self-induction
- Mutual inductance, M_{21} of coil 2 with respect to coil 1 is



$$L = \frac{N\Phi_B}{i}$$

$$M_{21} = \frac{N_2\Phi_{21}}{i_1}$$

Inductance

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1}$$

- Rearrange equation

$$M_{21} i_1 = N_2 \Phi_{21}$$

- Vary i_1 with time

$$M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}$$

- Faraday's law

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

- Induced emf in coil 2 due to i in coil 1 is

$$\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$

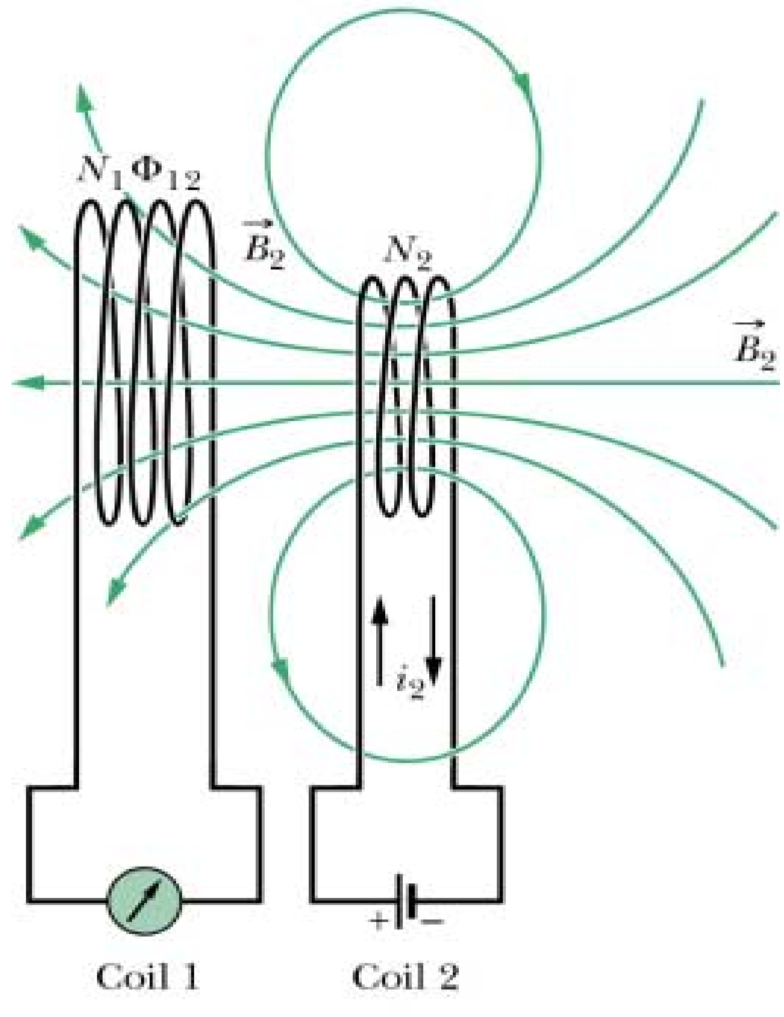
- Obeys Lenz's law (minus sign)

Inductance

- Reverse roles of coils
- What is induced emf in coil 1 from a changing current in coil 2?
- Same game as before

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2}$$

$$\mathcal{E}_1 = -M_{12} \frac{di_2}{dt}$$



Inductance

- The mutual inductance terms are equal

$$M_{12} = \frac{N_1 \Phi_{12}}{i_2} \qquad M_{21} = \frac{N_2 \Phi_{21}}{i_1} \qquad M_{21} = M_{12} = M$$

- Rewrite emfs as

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \qquad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

- Notice same form as self-induced emf

$$\mathcal{E}_L = -L \frac{di}{dt} \qquad L = \frac{N \Phi_B}{i}$$

Faraday's law

- If B is constant within coil

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = BA\cos\theta$$

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- Change the magnetic flux and induce a current and voltage by
  - Changing magnitude of *B* field within coil
  - Changing area of coil, or portion of area within *B* field
  - Changing angle between *B* field and area of coil (e.g. rotating the coil)

$$\mathcal{E} = -NA\cos\theta \frac{dB}{dt}$$

$$\mathcal{E} = -NB\cos\theta \frac{dA}{dt}$$

$$\mathcal{E} = -NBA \frac{d(\cos\theta)}{dt}$$