Review

- Magnetic flux
  \[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

- Faraday’s law (one loop) for emf (E) (induced voltage)
  \[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Faraday’s law (N loops)
  \[ \mathcal{E} = -N \frac{d\Phi_B}{dt} \]

- Lenz’s law – induced emf gives rise to a current whose \( B \) field opposes the change in flux that produced it
Review

- Induced emf of a conductor moving with velocity, \( v \), in a \( \perp B \) field is given by
  \[ \mathcal{E} = BLv \]

- Induced current in loop in a \( B \) field experiences a force
  \[ F_B = iL \times \vec{B} \]

- Found \( F_1 \) opposes your force \( F_{app} \)
  \[ \vec{F}_{app} = -\vec{F}_1 \]

- Work you do in pulling the loop appears as thermal energy in the loop
Inductance

- **Generators** – convert mechanical energy to electrical energy
- External agent rotates loop of wire in $B$ field
  - Hydroelectric plant
  - Coal burning plant
- Changing $\Phi_B$ induces an emf and current in an external circuit
Inductance

- **Alternating current (ac) generator**
  - Ends of wire loop are attached to slip rings which rotate with loop
  - Stationary metal brushes are in contact with slip rings and connected to external circuit
  - emf and current in circuit alternate in time
Inductance

- Calculate emf for generator with N turns of area A and rotating with constant angular velocity, \( \omega \)
- Magnetic flux is
  \[
  \Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta
  \]
- Relate angular displacement to angular velocity
  \[\theta = \omega t\]
- Flux through one loop is
  \[
  \Phi_B = BA \cos \omega t
  \]
Inductance

- Faraday’s law says
  \[ \mathcal{E} = -N \frac{d\Phi_B}{dt} \]

- Substitute
  \[ \Phi_B = BA \cos \omega t \]

  \[ \mathcal{E} = -NBA \frac{d}{dt} (\cos \omega t) \]

  \[ \mathcal{E} = NBA \omega \sin \omega t \]

- Maximum emf is when \( \omega t = 90 \) or 270 degrees
  \[ \mathcal{E}_{\text{max}} = NBA \omega \]

- Emf is 0 when \( \omega t = 0 \) or 180 degrees
Inductance

- **Direct current (dc) generator**
  - Ends of loop are connected to a single split ring
  - Metal brush contacts to split ring reverse their roles every half cycle
  - Polarity of induced emf reverses but polarity of split ring remains the same

- Not suitable for most applications
  - Can use to charge batteries
  - Commercial dc gen. use out of phase coils
Inductance

- **Motors** – converts electrical energy to mechanical energy
  - Generator run in reverse
  - Current is supplied to loop and the torque acting on the current-carrying loop causes it to rotate
  - Do mechanical work by using the rotating armature
  - As loop rotates, changing $B$ field induces an emf
  - Induced emf (back emf) reduces the current in the loop – remember Lenz’s law
  - Power requirements are greater for starting a motor and for running it under heavy loads
Review for Inductance

- **Inductor** is a device used to produce and store a desired $B$ field (e.g. solenoid).
- A current $i$ in an inductor with $N$ turns produces a magnetic flux, $\Phi_B$, in its central region.
- **Inductance**, $L$ is defined as
  \[ L = \frac{N\Phi_B}{i} \]

- Inductance per unit length of a solenoid:
  - Depends only on geometry of device (like capacitor)
  \[ \frac{L}{l} = \mu_0 n^2 A \]
Inductance

- A changing current in a coil generates a self-induced emf, $\varepsilon_L$ in the coil.
- Process is called self-induction.
- Change current in coil using a variable resistor, $\varepsilon_L$ will appear in coil only while the current is changing.

\[ L = \frac{N\Phi_B}{i} \]

\[ \mathcal{E}_L = -N \frac{d\Phi_B}{dt} = - \frac{d(N\Phi_B)}{dt} = - \frac{d(Li)}{dt} = -L \frac{di}{dt} \]
Inductance

- Induced emf only depends on rate of change of current, not its magnitude.
- Direction of $\varepsilon_L$ follows Lenz’s law and opposes the change in current.
- Self-induced $V_L$ across inductor:
  - Ideal inductor: $V_L = \varepsilon_L$
  - Real inductor (like real battery) has some internal resistance: $V_L = \varepsilon_L - iR$
Inductance

- Checkpoint #4 – Have an induced emf in a coil. What can we tell about the current through the coil? Is it moving right or left and is it constant, decreasing or increasing?

\[ \mathcal{E}_L \]

Only get \( \mathcal{E}_L \) if current changing

Decreasing and rightward OR
Increasing and leftward
Inductance

- **Mutual induction** – current in one coil induces emf in other coil
- Distinguish from **self-induction**
- **Mutual inductance**, $M_{21}$ of coil 2 with respect to coil 1 is

\[
L = \frac{N \Phi_B}{i} \quad \quad M_{21} = \frac{N_2 \Phi_{21}}{i_1}
\]
Inductance

- Faraday’s law
\[ \mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt} \]

- Rearrange equation
\[ M_{21} = \frac{N_2 \Phi_{21}}{i_1} \]

- Induced emf in coil 2 due to \( i \) in coil 1 is
\[ \mathcal{E}_2 = -M_{21} \frac{di_1}{dt} \]

- Vary \( i \) with time
\[ M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt} \]

- Obeys Lenz’s law (minus sign)
Inductance

- Reverse roles of coils
- What is induced emf in coil 1 from a changing current in coil 2?
- Same game as before

\[ M_{12} = \frac{N_1 \Phi_{12}}{i_2} \]

\[ \mathcal{E}_1 = -M_{12} \frac{di_2}{dt} \]
Inductance

- The mutual inductance terms are equal

\[ M_{12} = \frac{N_1 \Phi_{12}}{i_2} \quad M_{21} = \frac{N_2 \Phi_{21}}{i_1} \quad M_{21} = M_{12} = M \]

- Rewrite emfs as

\[ \mathcal{E}_2 = -M \frac{di_1}{dt} \quad \mathcal{E}_1 = -M \frac{di_2}{dt} \]

- Notice same form as self-induced emf

\[ \mathcal{E}_L = -L \frac{di}{dt} \quad L = \frac{N \Phi_B}{i} \]
Faraday’s law

- If $B$ is constant within coil
  \[ \Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta \]

- Change the magnetic flux and induce a current and voltage by
  - Changing magnitude of $B$ field within coil
  \[ \mathcal{E} = -N \frac{d\Phi_B}{dt} \]
  - Changing area of coil, or portion of area within $B$ field
  \[ \mathcal{E} = -N A \cos \theta \frac{dB}{dt} \]
  - Changing angle between $B$ field and area of coil (e.g. rotating the coil)
  \[ \mathcal{E} = -NBA \frac{d(cos \theta)}{dt} \]