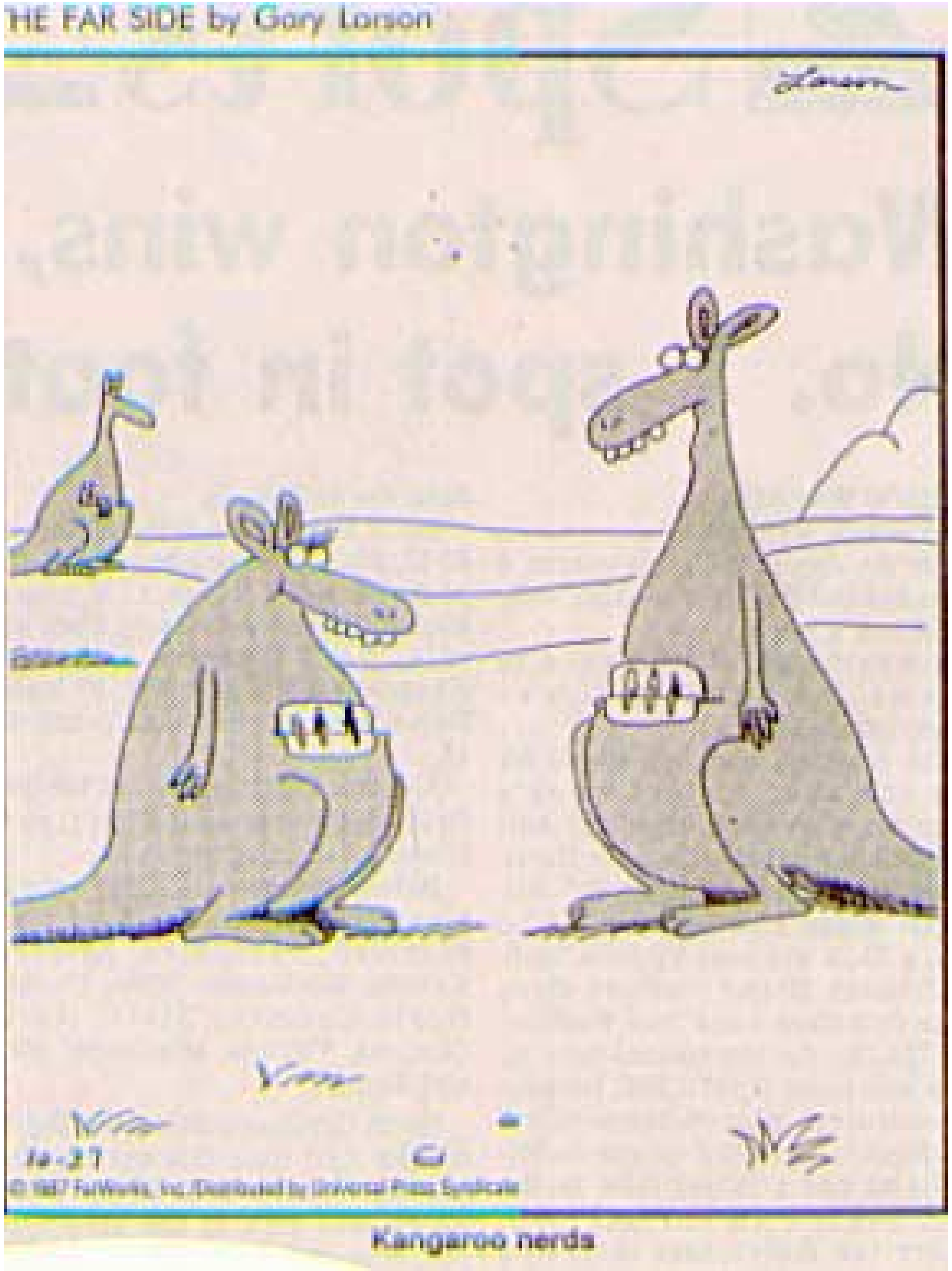
October 22th

Induction and Inductance

Chapter 31



Midterm-2

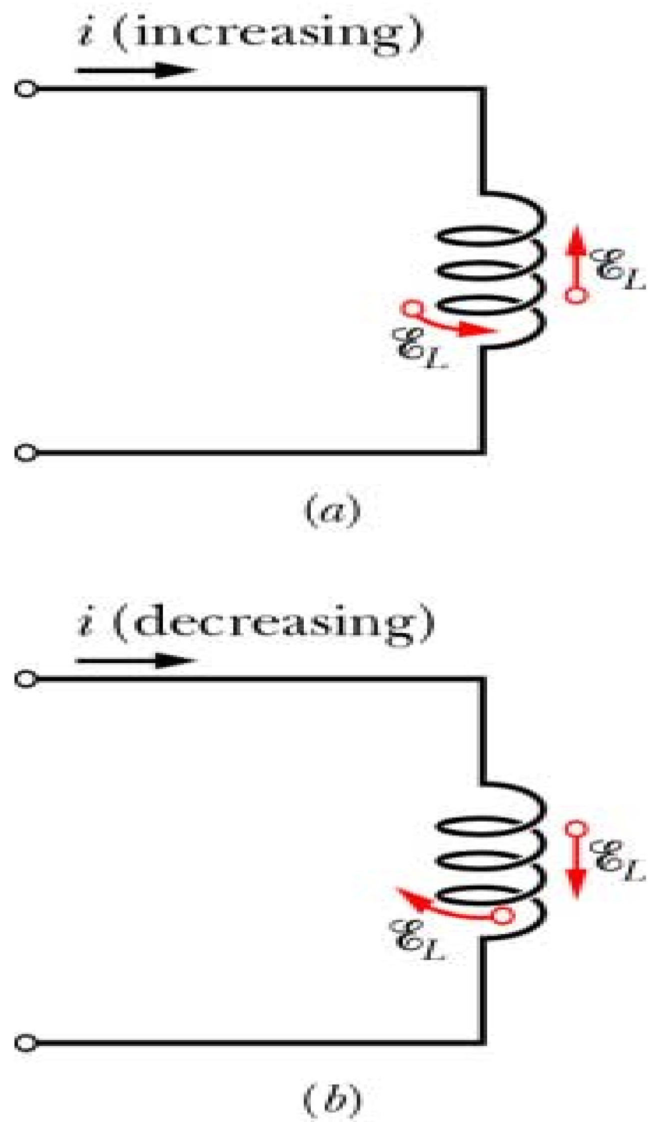
- Wednesday October 29 at 6pm
  - Sec 1 – N100 BCC (Business College)
  - Sec 2 – 158 NR (Natural Resources)
- Allowed one sheet of notes (both sides) and calculator
- Covers Chapters 27-31 and homework sets #5-8
- Send an email to your professor if you have a class conflict and need a make-up exam

Review - Self Inductance

- Self-induce emf, ε_L appears in any coil in which the current is changing

$$\varepsilon_L = -L\frac{di}{dt}$$

- Direction of ε_L follows Lenz's law and opposes the change in current



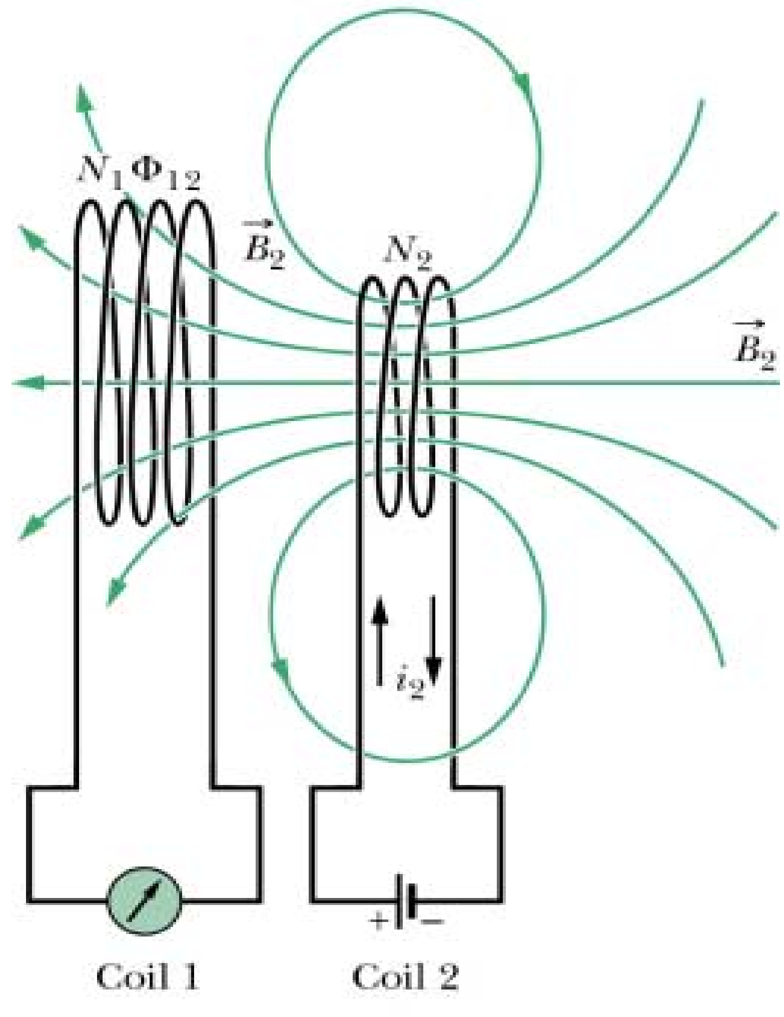
Review - Mutual Inductance

- What is induced emf in coil 1 from a changing current in coil 2?

$$\mathcal{E}_1 = -M_{12}\frac{di_2}{dt}$$

where

$$M_{12} = \frac{N_1\Phi_{12}}{i_2}$$

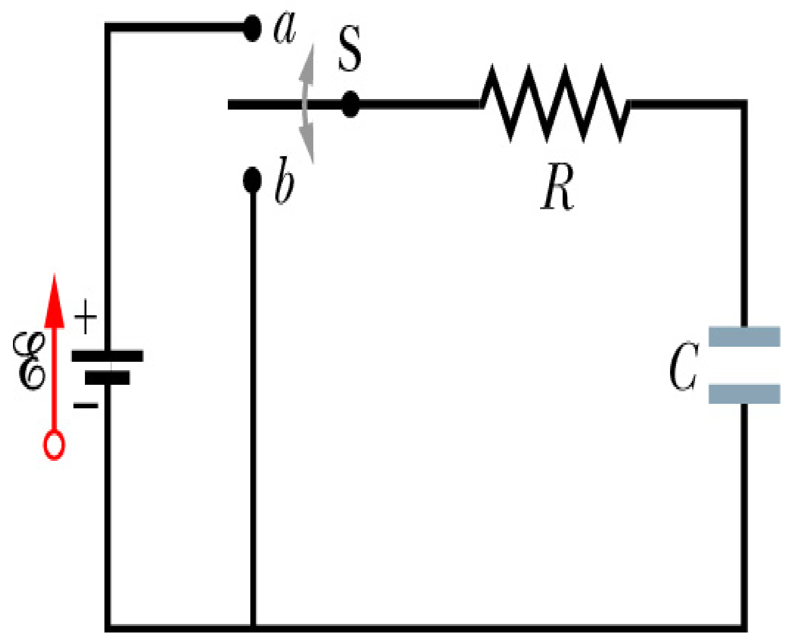


Review RC circuit

- RC circuit is a resistor and capacitor in series
  - Charging up a capacitor (switch at a)

$$q = C\mathcal{E}(1 - e^{-t/\tau_c})$$

  - Discharging capacitor (switch at b)



$$q = q_0 e^{-t/\tau_c}$$

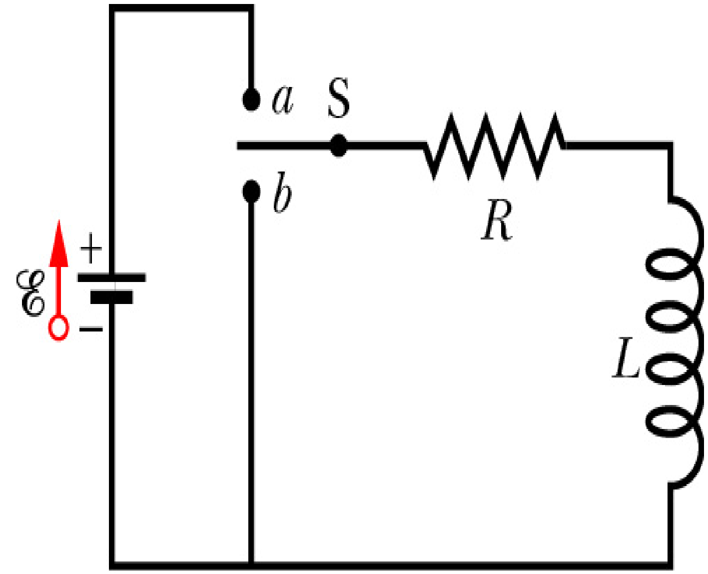
where

$$\tau_C = RC$$

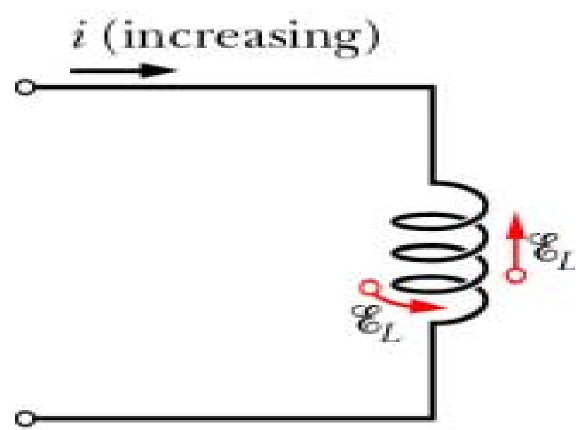
Inductance

- RL circuit is a resistor and inductor in series
- Close switch to point a
  - Initially *i* is increasing through inductor so ε_L opposes rise and *i* through *R* will be

$$i < \mathcal{E}/R$$



$$\mathcal{E}_L = -L\frac{di}{dt}$$



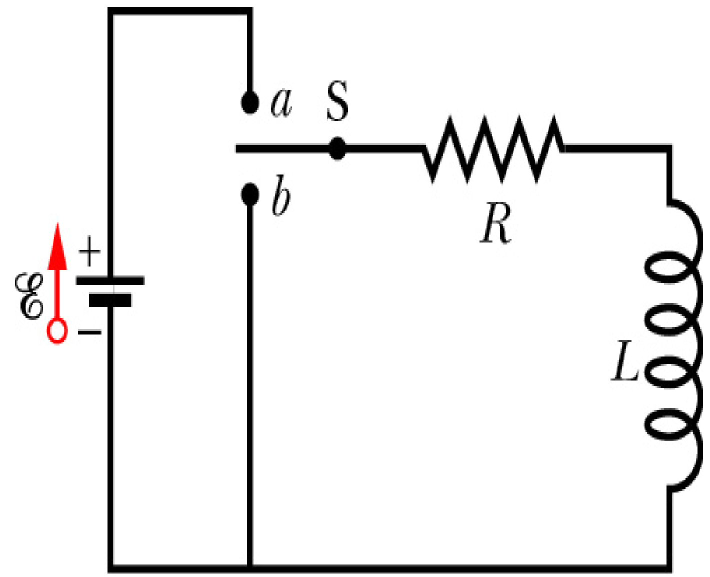
  - Long time later, *i* is constant so ε_L=0 and *i* in circuit is

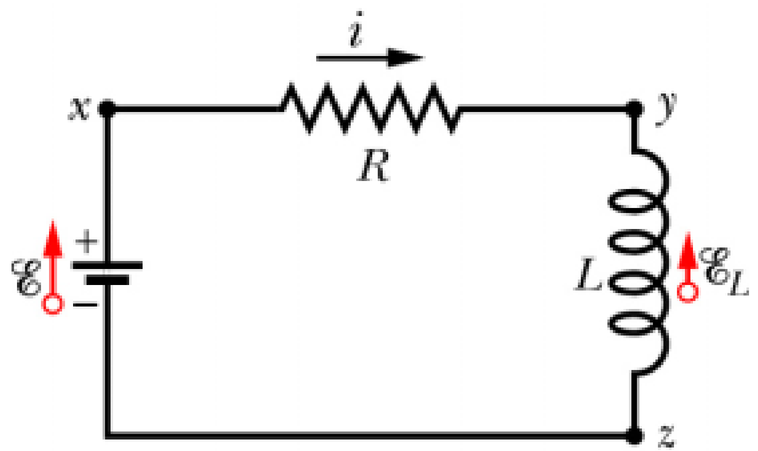
$$i = \mathcal{E}/R$$

Inductance

- Initially an inductor acts to oppose changes in current through it
- Long time later inductor acts like ordinary conducting wire
- Apply loop rule right after switch has been closed at a

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

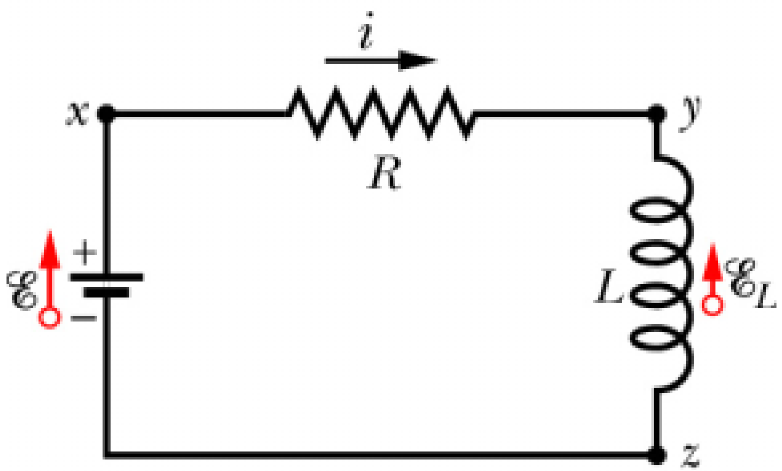




Inductance

- Differential equation similar to capacitors

$$\mathcal{E} = iR + L\frac{di}{dt}$$



- Solution is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

- Inductive time constant is

$$\tau_L = \frac{L}{R}$$

- Satisfies conditions:
  - At t=0, $i = 0$
  - At t=∞, $i = \mathcal{E}/R$

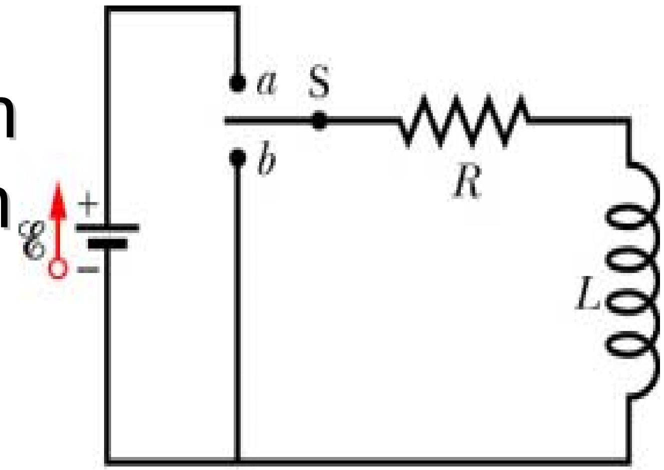
Inductance

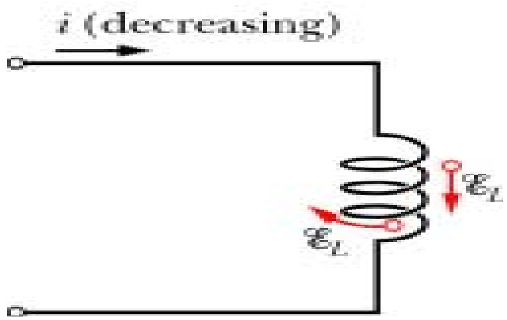
- Now move switch to position b so battery is out of system
- Current will decrease with time and loop rule gives

$$iR + L\frac{di}{dt} = 0$$

- Solution is

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$





- Satisfies conditions
  - At t=0, $i = i_0 = \varepsilon / R$
  - At t=∞, $i = 0$

RL circuits Summary

- Circuit is closed (switch to “a”)

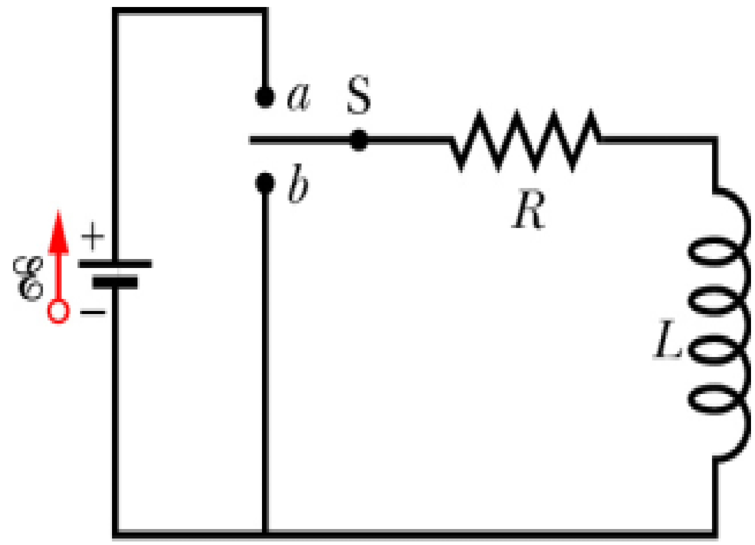
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

- Circuit is opened (switch to “b”)

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

- Time constant is

$$\tau_L = \frac{L}{R}$$

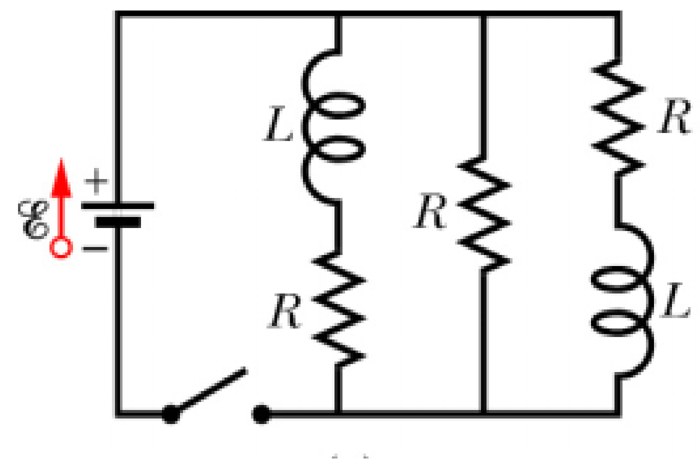


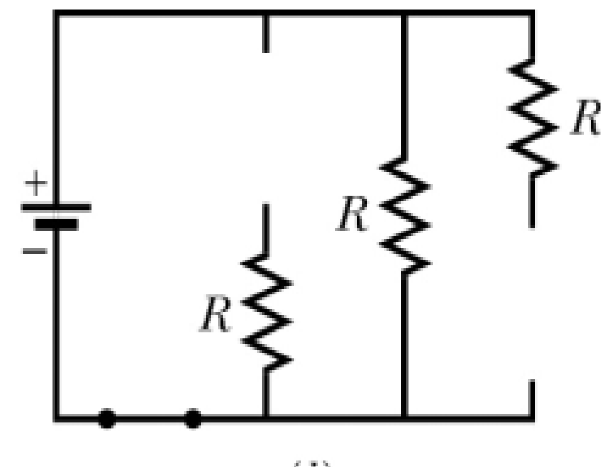
Switch at a, current through inductor is:

- Initially i = 0 (acts like broken wire)
- Long time later $i = \mathcal{E}/R$ (acts like simple wire)

Inductance (Prob. 31-5)

- Have a circuit with resistors and inductors
- What is the current through the battery just after closing the switch?
- Inductor oppose change in current through it
- Right after switch is closed, current through inductor is 0
- Inductor acts like broken wire

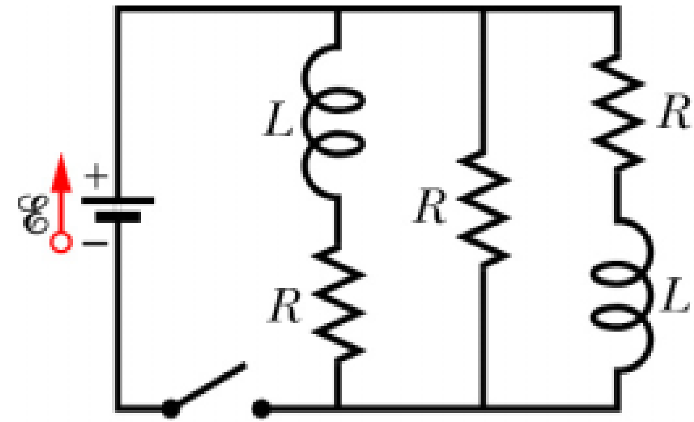




Inductance (Prob. 31-5)

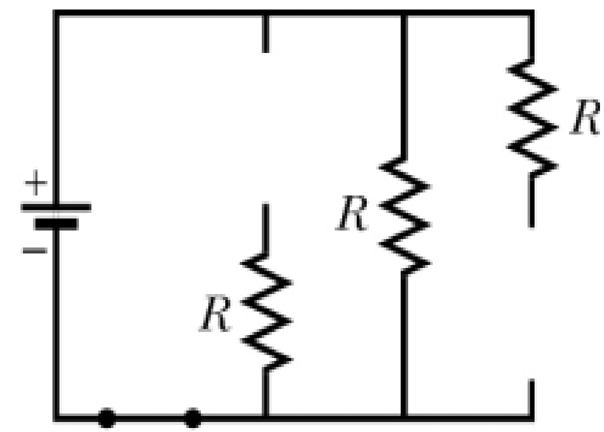
- Apply loop rule

$$E - iR = 0$$



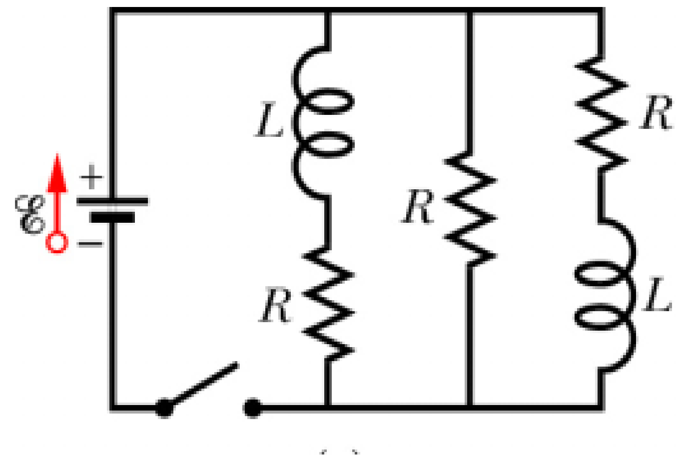
- Immediately after switch closed, current through the battery is

$$i = \frac{E}{R}$$



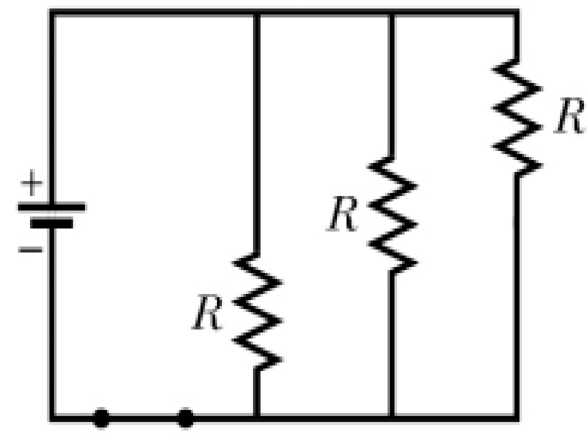
Inductance (Prob. 31-5)

- What is the current through the battery a long time after the switch has been closed?
- Currents in circuit have reached equilibrium so inductor acts like simple wire
- Circuit is 3 resistors in parallel



$$i = \frac{E}{R_{eq}}$$

$$R_{eq} = \frac{R}{3}$$



(c)

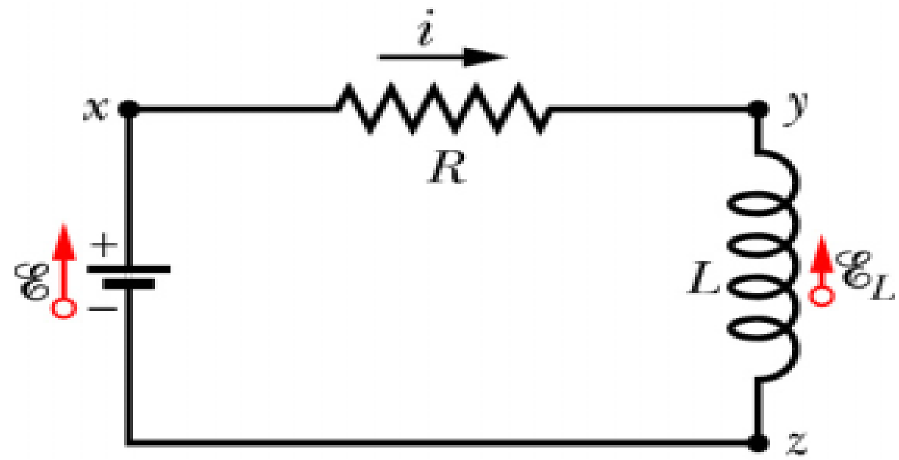
Inductance

- How much energy is stored in a *B* field?
- Conservation of energy expressed in loop rule

$$\mathcal{E} = L\frac{di}{dt} + iR$$

- Multiply each side by *i*

$$\mathcal{E}i = Li\frac{di}{dt} + i^2R$$



- $P = i\varepsilon$ is the rate at which the battery delivers energy to rest of circuit
- $P = i^2R$ is the rate at which energy appears as thermal energy in resistor

Inductance

- Middle term is rate at which energy dU_B/dt is stored in the B field

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

- Energy stored in magnetic field

$$U_B = \frac{1}{2}Li^2$$

- Similar to energy stored in electric field

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

Inductance

- What is the energy density of a *B* field?
- Energy density, u_B is energy per unit volume

$$u_B = \frac{U_B}{Al}$$

- Magnetic energy density

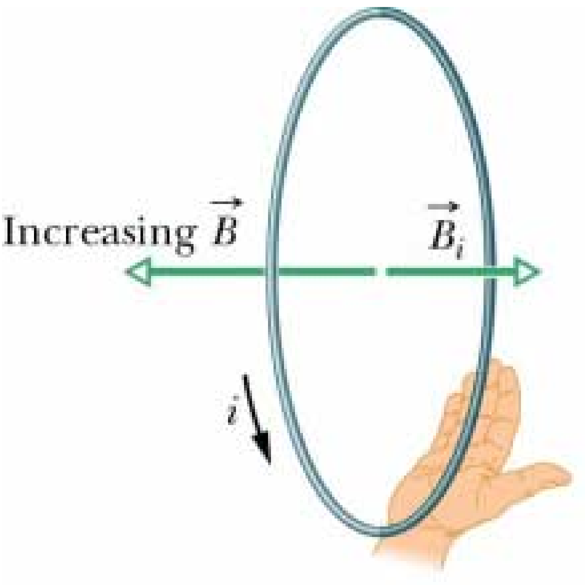
$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

- Similar to electric energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

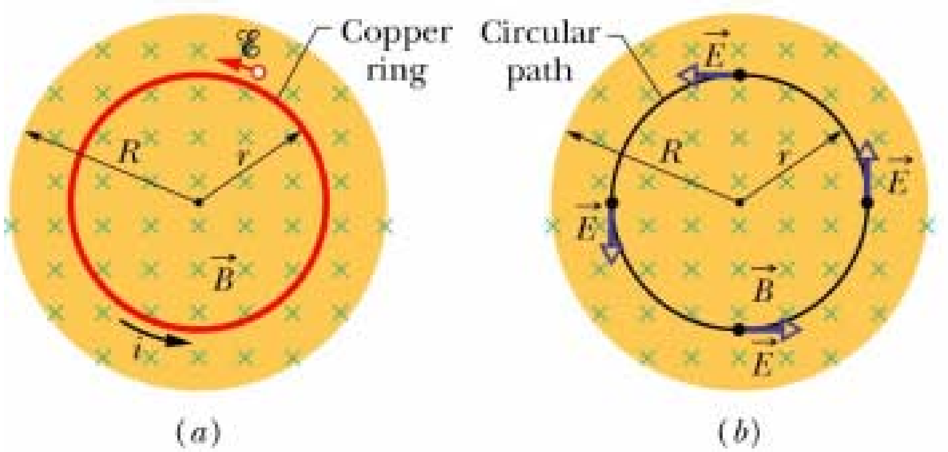
Induced Electric Fields

- Put a copper ring in a uniform *B* field which is increasing in time so the magnetic flux through the copper ring is changing



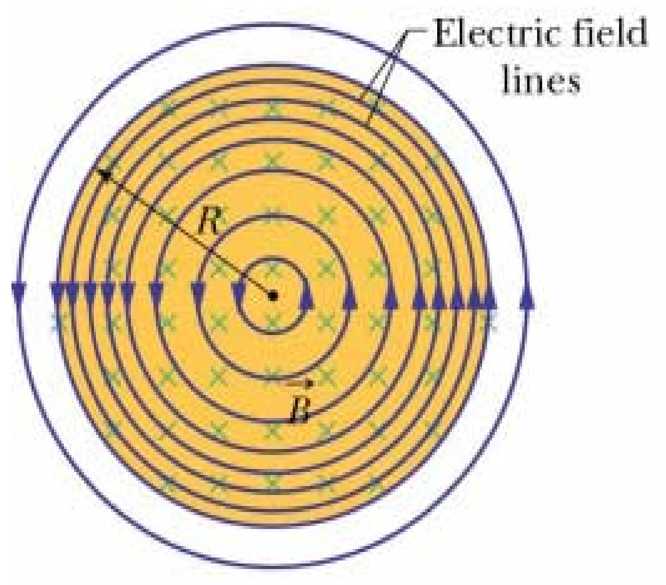
- By Faraday's law an induced emf and current are produced

- If there is a current there must be an *E* field present to move the conduction electrons around ring



Induced Electric Fields (Fig. 31-13)

- Induced *E* field acts the same way as an *E* field produced by static charges, it will exert a force, *F=qE*, on a charged particle
- True even if there is no copper ring (the picture shows a region of magnetic field increasing into the board which produces circular electric field lines).
- Restate Faraday's law – A changing *B* field produces an *E* field given by



$$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$