October 9/10th

Magnetic Fields Due to Currents

Chapter 30

Review – Chap. 29

- Force on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- Charged particles moving with *v* ⊥ to a *B* field move in a circular path with radius, *r*

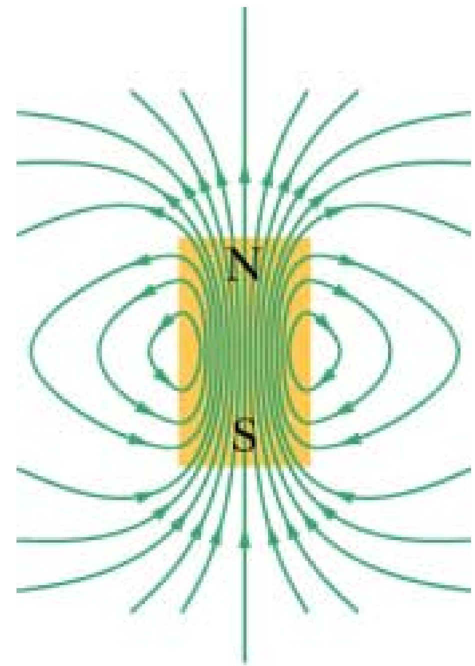
$$r = \frac{mv}{qB}$$

- Force on a current carrying wire due to a magnetic field is

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

Review

- Set up a *B* field in two ways:
- Intrinsic magnetic field
  - Magnetic field of electrons in a material add together to give a net magnetic field around the material – i.e. permanent magnet
- Electrically charged particles which are moving – i.e. current in a wire

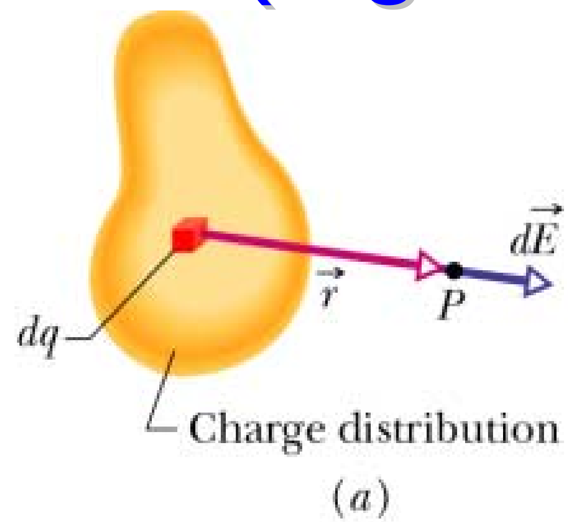


(a)

B Fields from Currents (Fig. 30-1)

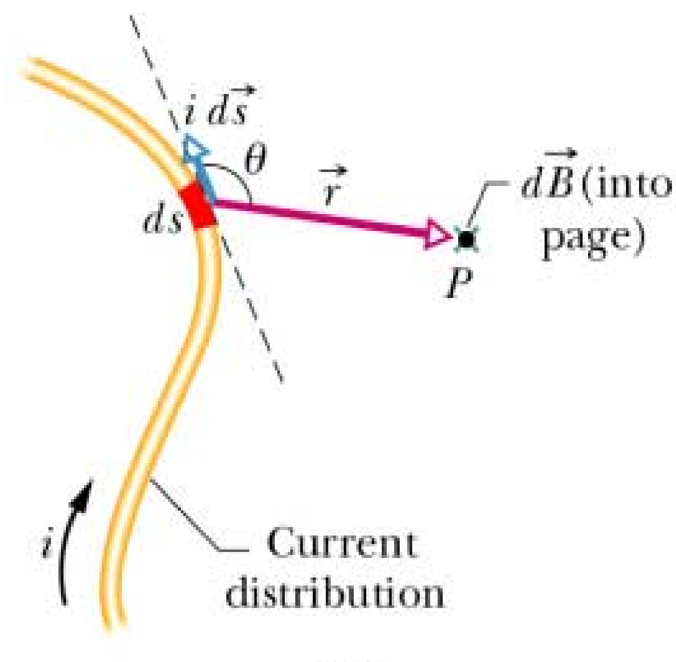
- *E* field produced by a distribution of charges

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$



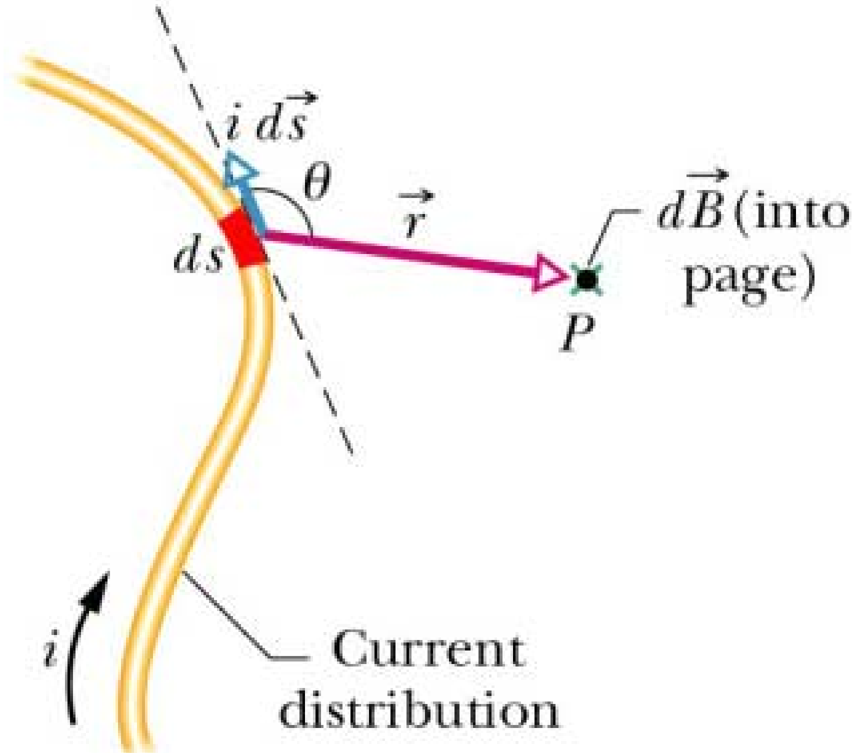
- *B* field produced by distribution of currents

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$



B Fields from Currents (Fig. 30-1b)

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$$



- Current-length element,

$$i\, d\vec{s}$$

is product of a scalar and a vector.

- Find net B field by integrating.
- A new constant - Permeability constant, μ_0

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m / A$$

B Fields from Currents (Fig. 30-1b)

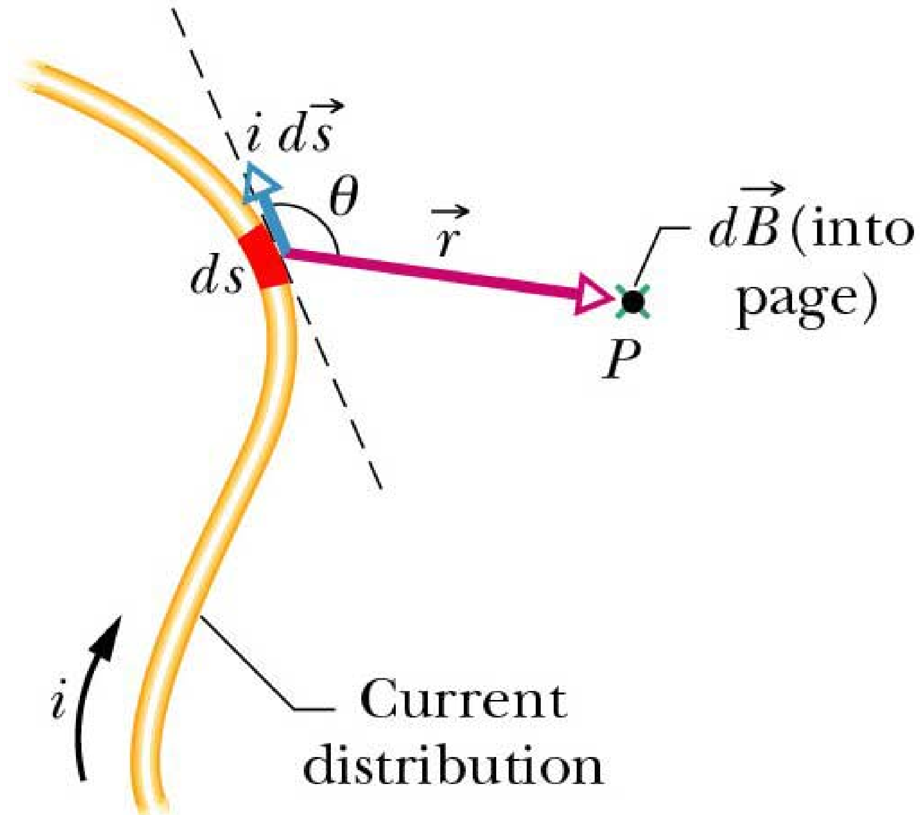
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

- Rewrite in vector form

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}$$

- Known as the Biot-Savart Law

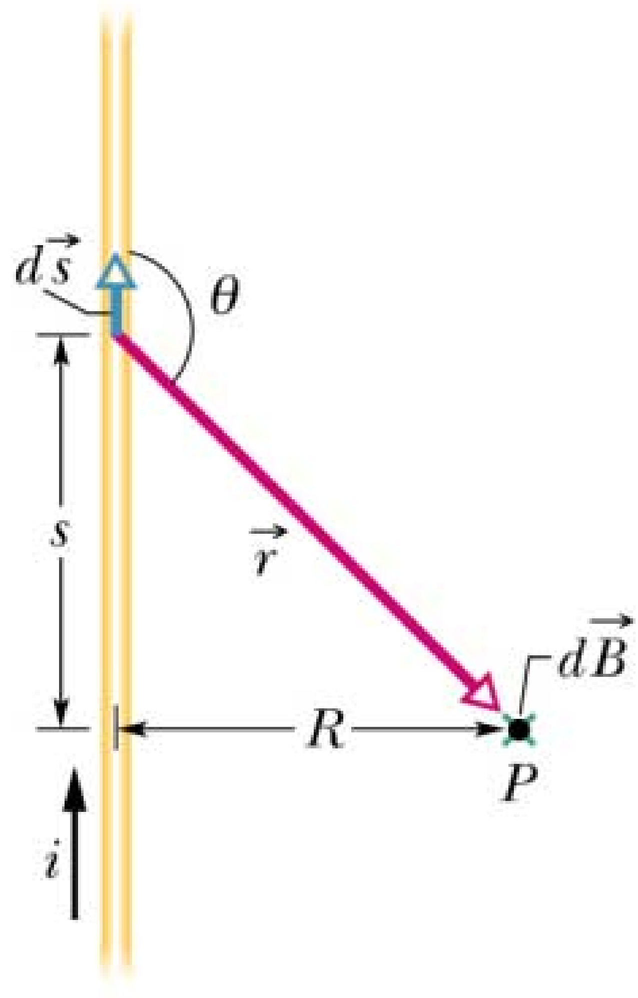


B Fields from Currents (Fig. 30-5)

- Use Biot-Savart Law to calculate B field produced by an infinitely long straight wire with current, i

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

- Direction of dB at point P is into the page for all ds from -∞ to +∞



B Fields from Currents (Fig. 30-5)

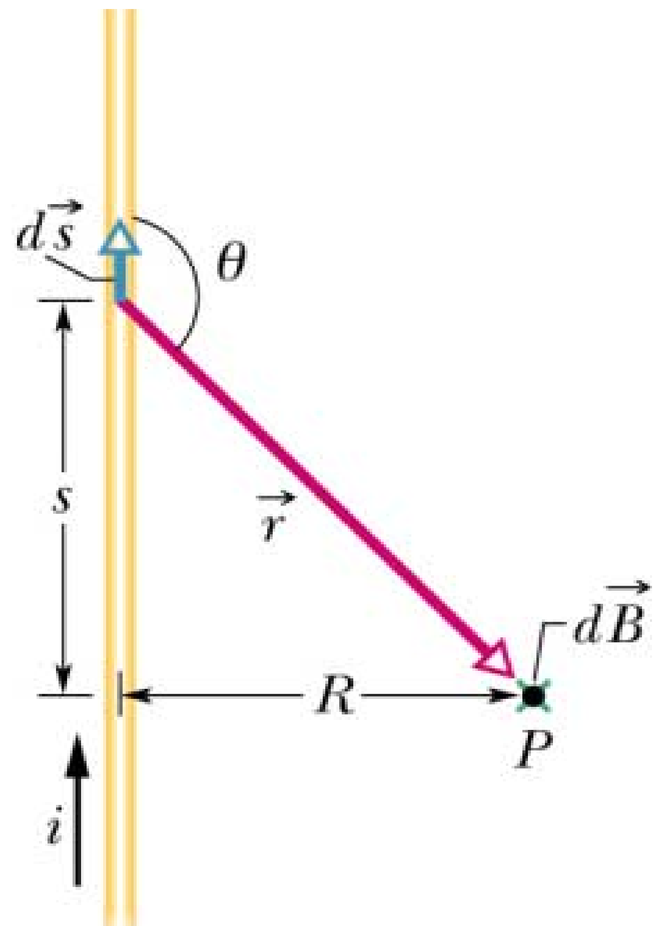
$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$$

Find total *B* field by integrating from 0 to +∞ and multiplying by 2

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin\theta}{r^2} ds = \frac{\mu_0 i I}{2\pi}$$

Where

$$I = \int_0^\infty \frac{\sin\theta}{r^2} ds$$



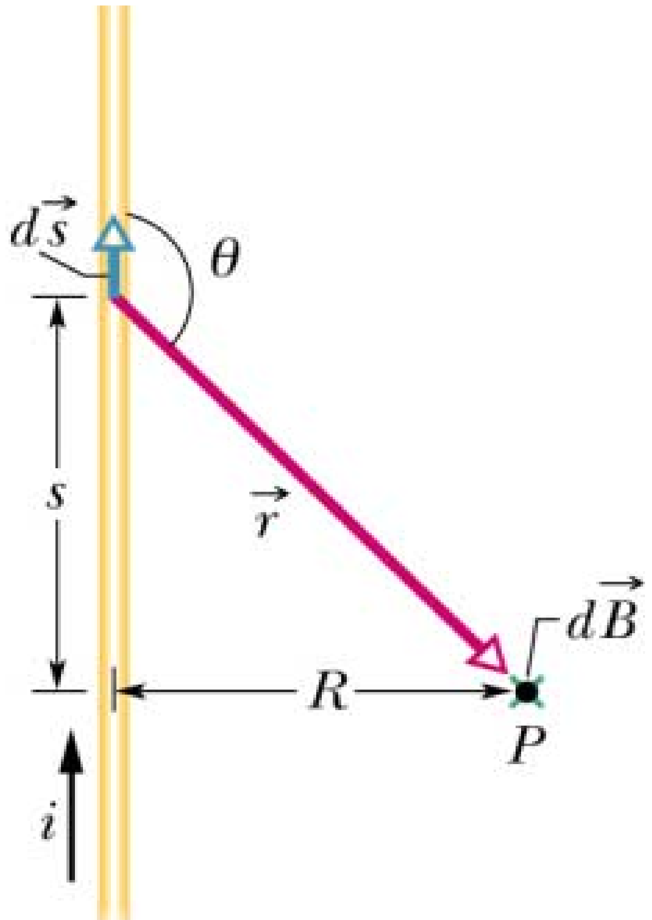
B Fields from Currents (Fig. 30-5)

- Variables *r*, *s* and θ are related by

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r}$$

$$\sin\theta = \frac{R}{\sqrt{s^2 + R^2}}$$



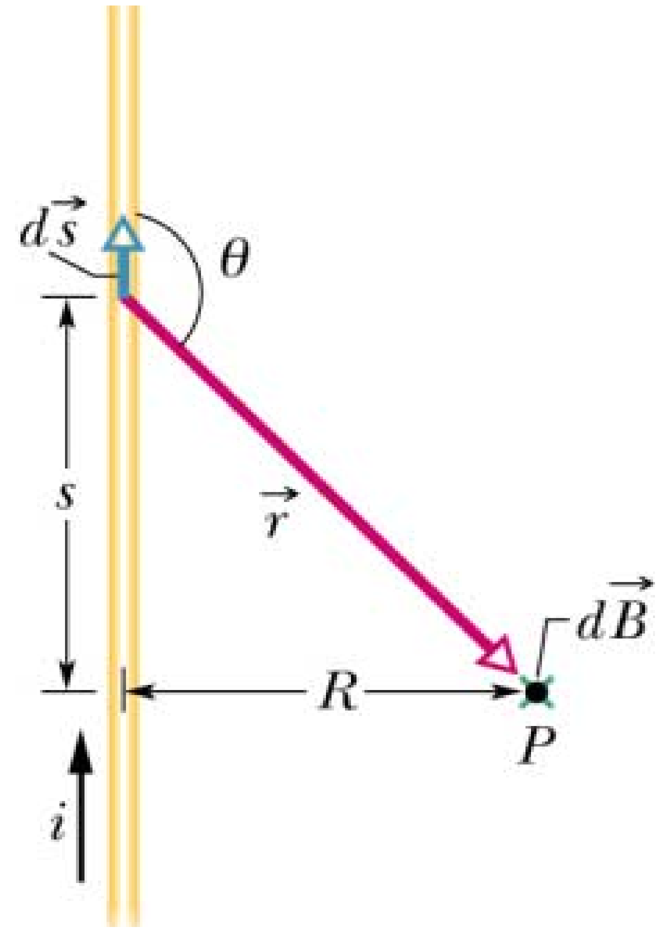
B Fields from Currents (Fig. 30-5)

- Substituting $$\sin\theta = \frac{R}{\sqrt{s^2 + R^2}}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta}{r^2}\,ds = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{\left(s^2 + R^2\right)^{3/2}}$$

- Using the integral relation

$$\int \frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{x}{a^2\left(x^2 + a^2\right)^{1/2}}$$



B Fields from Currents (Fig. 30-5)

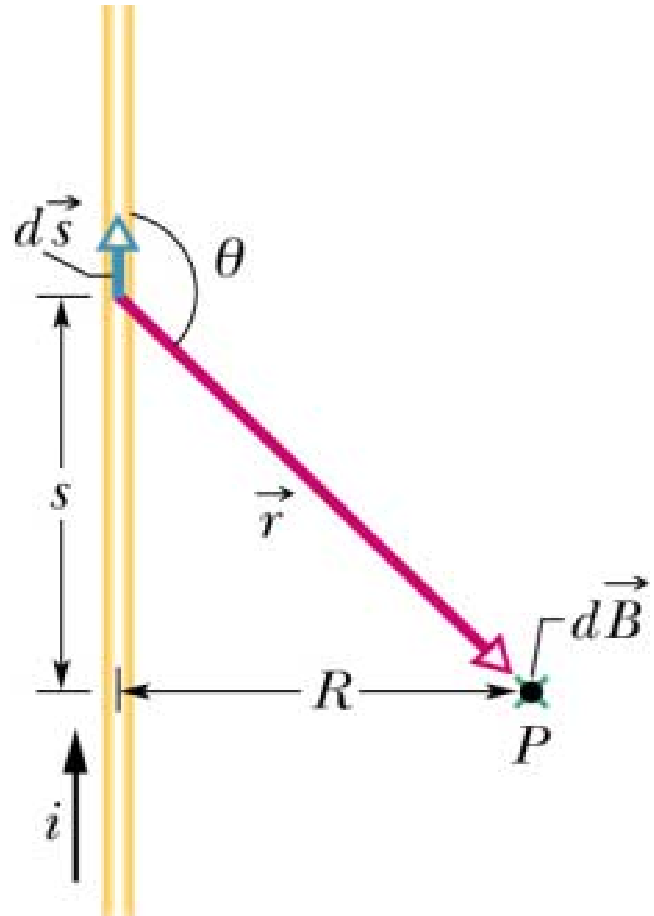
- Evaluating the integral gives

$$B = \frac{\mu_0 i R}{2\pi} \int_0^\infty \frac{ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$B = \frac{\mu_0 i}{2\pi R} \left[\frac{s}{\sqrt{s^2 + R^2}} \right]_0^\infty$$

- For a distance *R* from a long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$



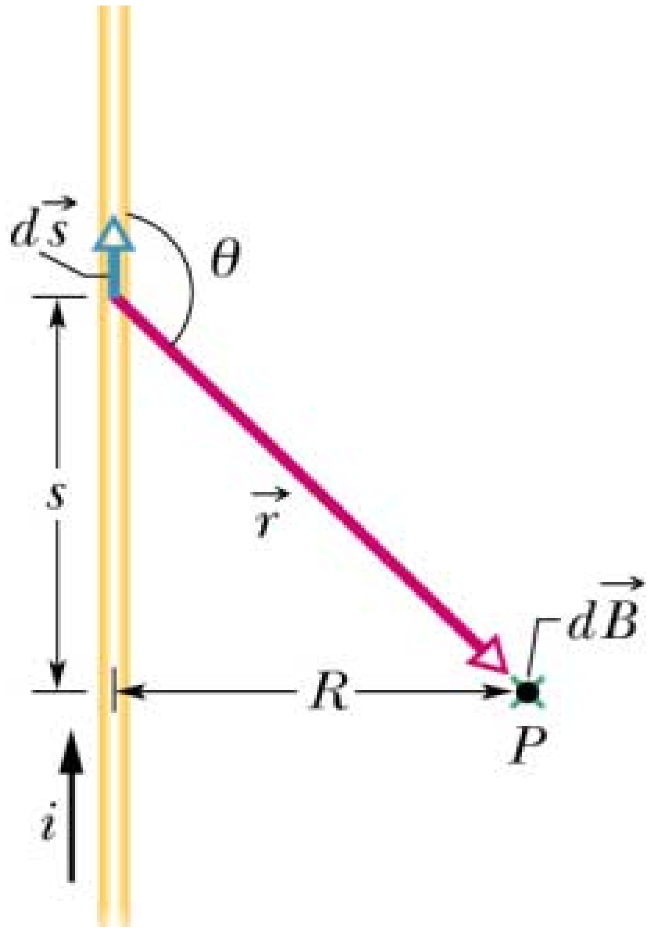
B Fields from Currents (Fig. 30-5)

- Evaluating the integral gives

$$I = \frac{1}{R}$$

- Thus for a distance *R* from a long straight wire

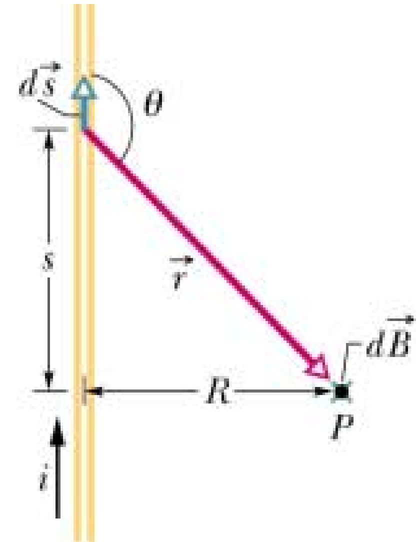
$$B = \frac{\mu_0 i}{2\pi R}$$



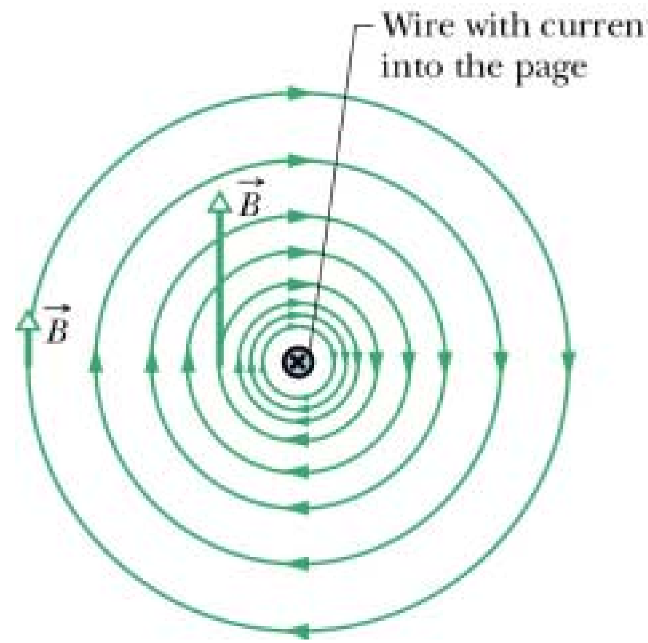
B Fields from Currents (Figs. 30-2,5)

- Notice B field only depends on current, i, and ⊥ distance R from wire

$$B = \frac{\mu_0 i}{2\pi R}$$

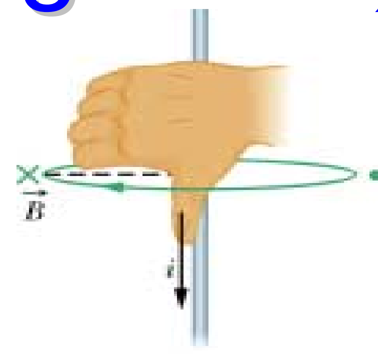


- B field forms concentric rings
- Magnitude of B decreases with distance as $1/R$ (so spacing of the lines deceases)

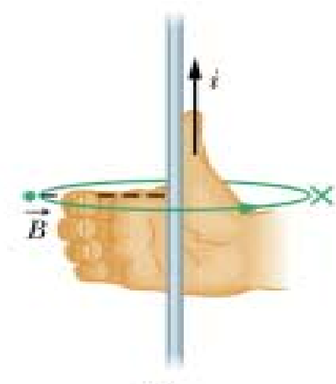


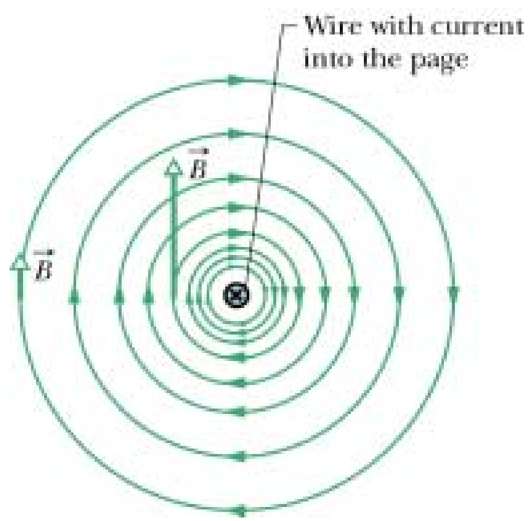
B Fields from Currents (Fig. 30-5)

- New use for right-hand rule
- Point thumb in direction of current flow
- Fingers will curl in the direction of the magnetic field lines due to current
- *B* field is tangent to magnetic field line



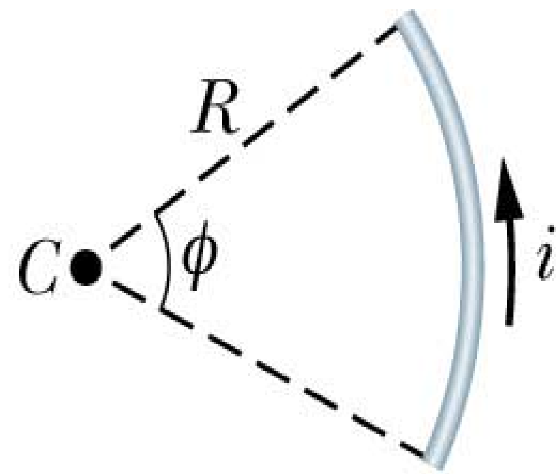
(a)



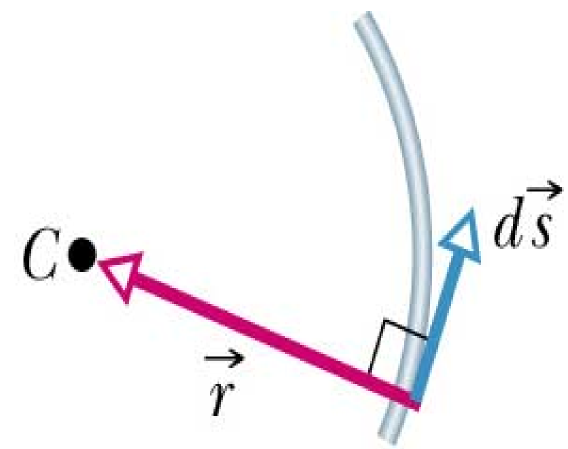


B Fields from Currents (Fig. 30-6)

- What is *B* field due to circular arc of wire?
- Simplify problem by finding *B* at center of arc, point *C*
- Using Biot-Savart and the fact that *r* and *ds* are ⊥ ($\theta = 90^0$)



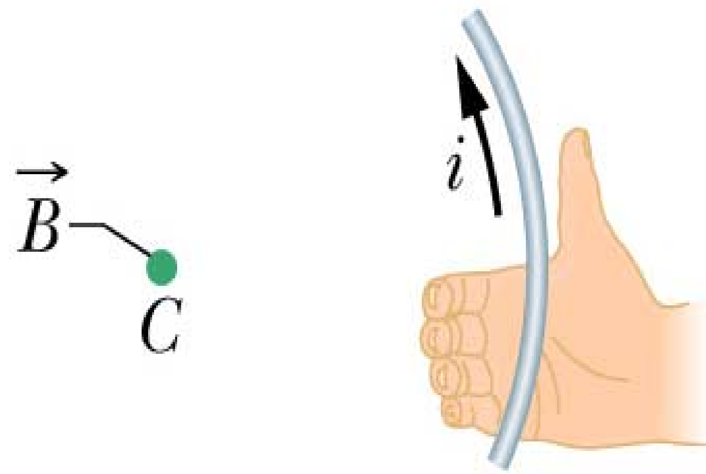
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$

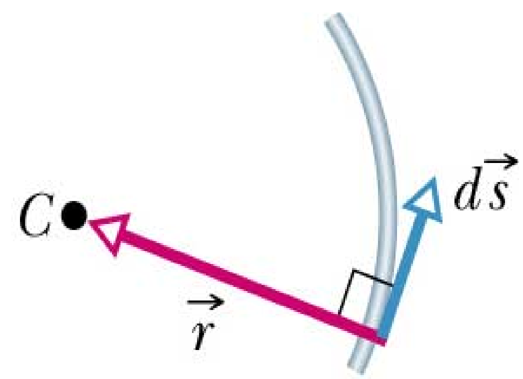


B Fields from Currents (Fig. 30-6)

- Use right-hand rule to find direction of *B* field at C
- Every *ds* gives *dB* directed out of page so get net B by integrating over whole arc

$$B = \int dB = \frac{\mu_0}{4\pi}\int \frac{i\,ds}{R^2} = \frac{\mu_0\, i}{4\pi R^2}\int ds$$





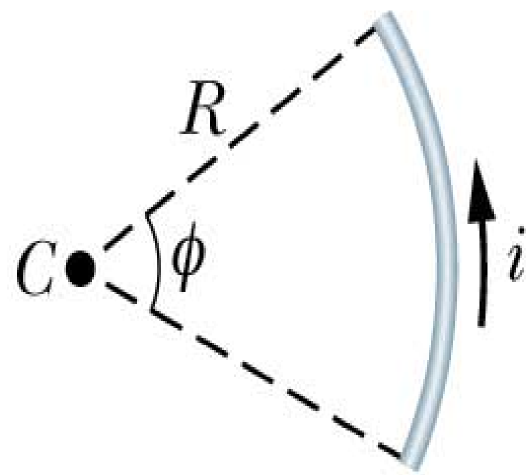
B Fields from Currents (Fig. 30-6)

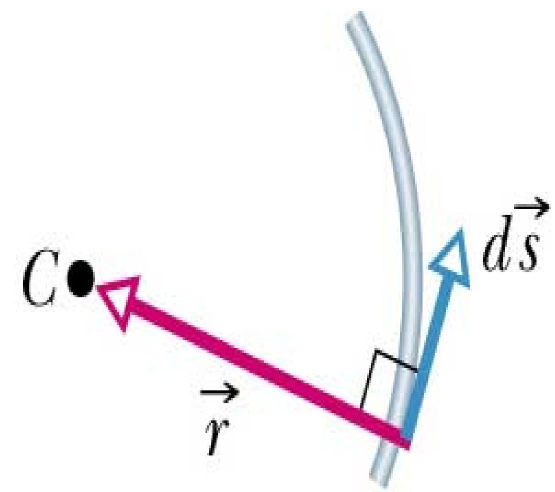
$$\int ds = s$$

- Where s is the arc length related to the angle in radians by

$$\frac{s}{2\pi R} = \frac{\phi}{2\pi}$$

- Thus $$\int ds = \phi R$$

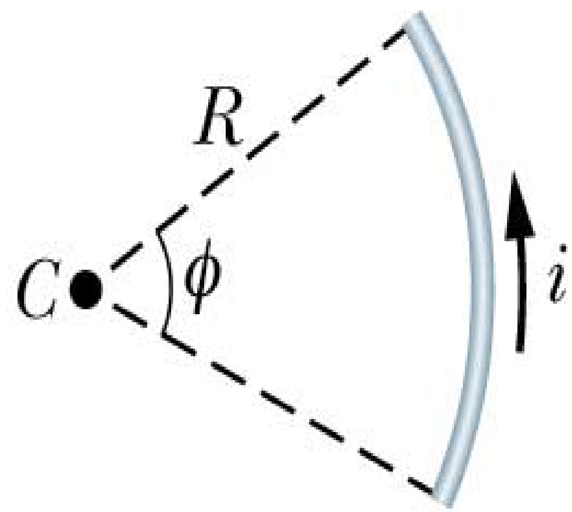




B Fields from Currents (Fig. 30-6)

- *B* field at the center of an arc is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



- Express ϕ in radians not in degrees
- For a complete loop ($\phi = 2\pi$) then *B* is

$$B = \frac{\mu_0 i}{2R}$$