Review for Midterm-1
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- **Wednesday Sept. 24th at 6pm**
  - Section 1 (the 4:10pm class) exam in – BCC N130 (Business College)
  - Section 2 (the 6:00pm class) exam in – NR 158 (Natural Resources)

- **Allowed one sheet of notes (both sides) and calculator**

- **Need photo ID**

- **Send Prof. Tollefson email if you need to take the make-up exam and explain why (tollefson@pa.msu.edu)**
  - Make-up exam is at 8am Thursday (meet at 3234 BPS by 7:55am)

- **Use the help-room to prepare**

- **Review in class on Tuesday**
Electric Force

- The magnitude of the electrostatic force, $F$, between 2 charged particles with charges $q_1$ and $q_2$, respectively, and separated by a distance $r$ is defined as

$$F = \frac{k|q_1||q_2|}{r^2}$$

- This is Coulomb’s law where $k$ is a constant
- The forces on 2 point charges are equal and opposite, pointing to (away from) the other particle for unlike (like) charges

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$
Electric Field

- Electric field, $E$, is the force per unit positive test charge

\[ E = \frac{F}{q_0} \]

- For a point charge

\[ F = k \frac{|q_0||q|}{r^2} \]

so

\[ E = k \frac{|q|}{r^2} \]
Electric Field

- \( E \) points towards a negative point charge and away from a positive point charge.

- Superposition principle

\[
\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n
\]

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n
\]

- Given the \( E \) field we can find the force on charge \( q \)

\[
\vec{F} = q\vec{E}
\]
If the vector addition gives zero you do not need to calculate each one.

For example, in the figure below, if $q_1=q_2$ then $\vec{E}_1 + \vec{E}_2 = 0$ at the origin and the field comes only from $q_3$. 
Flux

- Calculate flux of uniform $E$ through cylinder

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

- 3 surfaces - a, b, and c

$$\Phi = \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}$$
Flux

$\vec{E} \cdot d\vec{A} = E \ dA \cos \theta$
Gauss’ Law

- Gauss’ Law

\[ \varepsilon_0 \Phi = q_{enc} \]

- Also write it as

\[ \varepsilon_0 \int \vec{E} \cdot d\vec{A} = q_{enc} \]

- Net charge \( q_{enc} \) is sum of all enclosed charges and may be +, -, or zero
Example for Gauss’ Law

- Charge $q_1$ inside
- $E=0$ inside conductor
- Thus $\Phi=0$ for Gaussian surface (red line)
- So net charge enclosed must be 0
- Induced charge of $q_2 = -q_1$ lies on inner wall of sphere
- Shell is neutral so charge of $q_3 = -q_2$ on outer wall
Charge distributions

- $E$ field from a continuous line or region of charge
- Use calculus and a charge density instead of total charge, $Q$

- Linear charge density
  \[ \lambda = \frac{Q}{\text{Length}} \]

- Surface charge density
  \[ \sigma = \frac{Q}{\text{Area}} \]

- Volume charge density
  \[ \rho = \frac{Q}{\text{Volume}} \]
Gauss’ Law (Fig. 24-15)

- Non-conducting sheet of charge $\sigma$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\varepsilon_0 (EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$
Electric Potential

- Electric potential energy $U$ for a constant $E$ and work done by the field

\[ \Delta U = U_f - U_i = -W \]

\[ \Delta U = -Fd = -qEd \]

- Electric potential for a constant $E$

\[ \Delta V = \frac{\Delta U}{q} = -Ed \]
Electric Potential (Fig. 25-5)

- Work done by field
  \[ W = q_0 \int_{i}^{f} \vec{E} \cdot d\vec{s} \]

- Used to find
  \[ \Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]

- Potential decreases if path is in the direction of the electric field
1) Suppose we generate an electric field of

\[ \vec{E} = 200.0 \ (V/m) \ \hat{i} \]

What is the change in the electric potential, measured in Volts, associated with a moving charge of 1.4 C from (0,0) m to (2,2) m?

\[ \Delta V = -\int_{i}^{f} \vec{E} \cdot d\vec{s} \]

A) -400,  B) -280,  C) 600,  D) -800,  E) 1000
2) Suppose we generate an electric field of

\[ \vec{E} = 1.0 \ (V / m) \ \hat{i} + 2.0 \ (V / m) \ \hat{j} \]

What is the work done (in J) by an external agent \( W^* \) to move a charge of 6.0 C from (0,0) m to (2,2) m?

\[ W^* = -W = -q_0 \int_{i}^{f} \vec{E} \cdot d\vec{s} \]

A) -6,  B) 6,  C) -36,  D) 70,  E) -24
Electric Potential

Summary for a point change

\[ F = k \frac{|q| |q_0|}{r^2} \]

\[ E = k \frac{q}{r^2} \]

\[ V = k \frac{q}{r} \]
Electric Potential (Fig. 25-3)

- Dashed lines are the edge of equipotential surfaces where all points are at the same potential.

- Equipotential surfaces are always $\perp$ to electric field lines and to $E$.

- In this example $V$ decreases by constant intervals from the positive charge to the negative charge.
Electric Potential

- Use superposition principle to find the potential due to $n$ point charges

\[ V = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_i} \]

- This is an algebraic sum, not a vector sum
- Include the sign of the charge
Electric Potential (Mathematica)
Electric Field from Potential

- Take $s$ axis to be $x$, $y$, or $z$ axes

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

- If $E$ is uniform and $s$ is $\perp$ to equipotential surface

$$E = -\frac{\Delta V}{\Delta s}$$
Total potential energy for a collection of charges is the scalar sum of individual potential energies - work required to assemble the charges

\[ U = U_{12} + U_{13} + U_{14} \]
\[ + U_{23} + U_{24} + U_{34} \]

where

\[ U_{12} = k \frac{q_1 q_2}{d} \]

etc
Capacitance

- Calculate $C$ of a capacitor from its geometry using steps:
- 1) Assume charge, $q$, on the capacitor
- 2) Find $E$ between using $q$ and Gauss’ law

$$\varepsilon_0 \int \vec{E} \cdot d\vec{A} = q_{enc}$$

- 3) Find $V$ from $E$ using

$$\Delta V = -\int_{i}^{f} \vec{E} \cdot d\vec{s}$$

- 4) Get $C$ using

$$C = \frac{q}{V}$$
Capacitance (Fig. 26-5)

- Parallel-plate capacitor

\[ C = \frac{\varepsilon_0 A}{d} = \frac{A}{4\pi k d} \]

- Only depends on area \( A \) of plates and separation \( d \)

- \( C \) increases if we increase \( A \) or decrease \( d \)
Energy in a Capacitor

- Work required from 0 to total charge $q$ is
  \[ W = \frac{1}{C} \int_{0}^{q} q' \, dq' = \frac{q^2}{2C} \]

- Potential energy = work
  \[ U = \frac{q^2}{2C} \]

- Or, use
  \[ q = CV \]
  \[ U = \frac{1}{2} CV^2 \]
Capacitance

- Capacitors in parallel
  - $V$ across each is equal
  - Total $q$ is sum

- Capacitors in series
  - $q$ is equal on each
  - Total $V$ is sum

$$C_{eq} = \sum_{i}^{n} C_i$$

$$\frac{1}{C_{eq}} = \sum_{i}^{n} \frac{1}{C_i}$$
Capacitance

- Place a dielectric in capacitor its capacitance increases by numerical factor.
- Called dielectric constant, $\kappa$

\[
C_{\text{dielectric}} = \kappa C_{\text{air}}
\]