

Review for 2nd Midterm

Midterm-2

- **Wednesday October 29 at 6pm**
 - Section 1 – N100 BCC (Business College)
 - Section 2 – 158 NR (Natural Resources)
- Allowed one sheet of notes (both sides) and calculator
- Covers Chapters 27-31 and homework sets #5-8
- Send an email to your professor if you have a class conflict and need a make-up exam
- **Review in class on Tuesday, October 28th**

Current and Resistance

- Current $i = \frac{dq}{dt}$

- Current density $i = \int \vec{J} \cdot d\vec{A}$

- If J is uniform and parallel to dA

$$i = JA$$

Current and Resistance

- Ohm's law

$$V = iR$$

- Power lost to heat energy in a resistor

$$P = iV$$

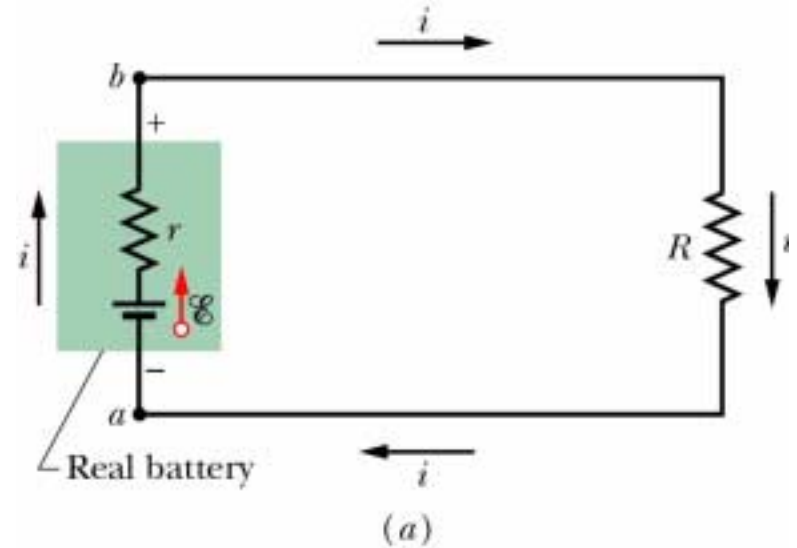
$$P = i^2 R$$

$$P = \frac{V^2}{R}$$

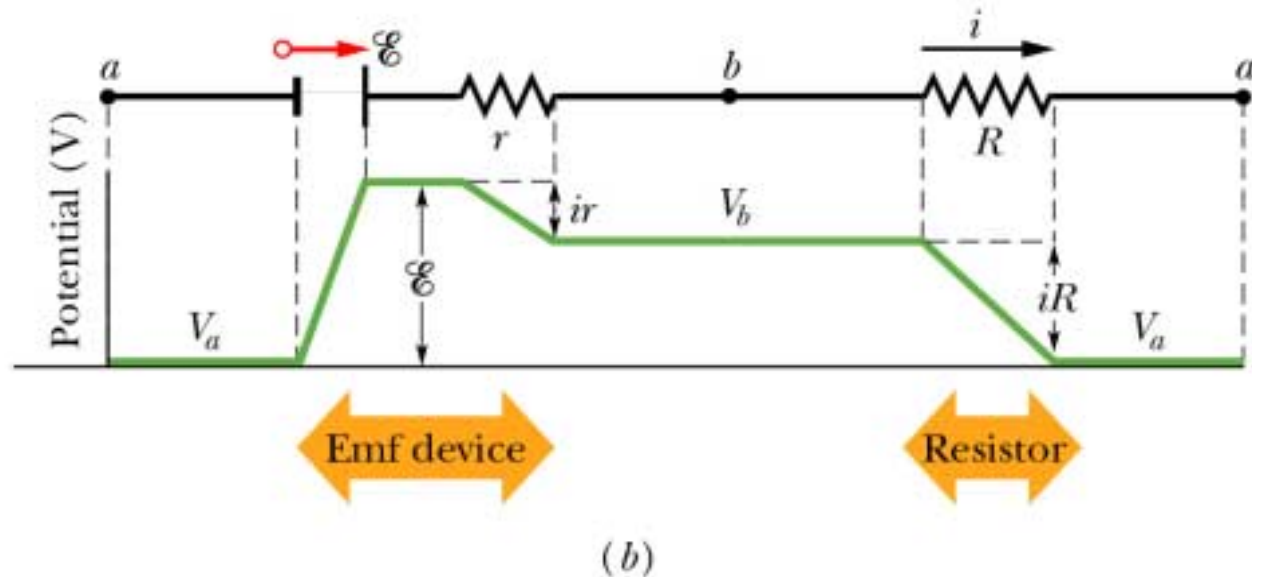
Loop Rule

$$\mathcal{E} - ir - iR = 0$$

$$i = \frac{\mathcal{E}}{r + R}$$

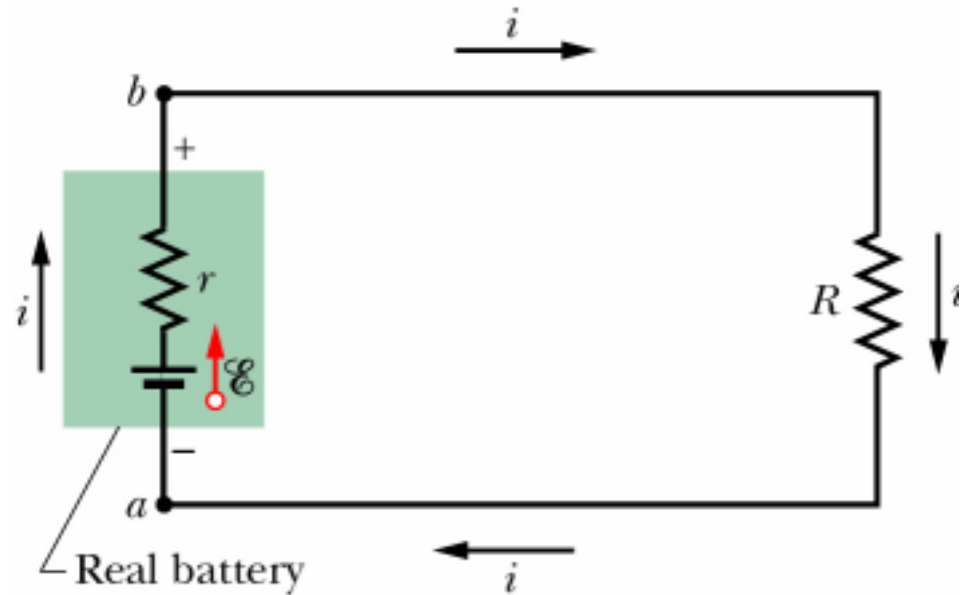


- Graphical representation



Circuits

$$V_b - V_a = iR$$



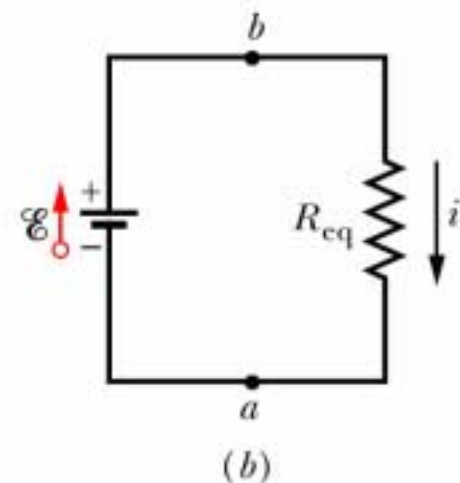
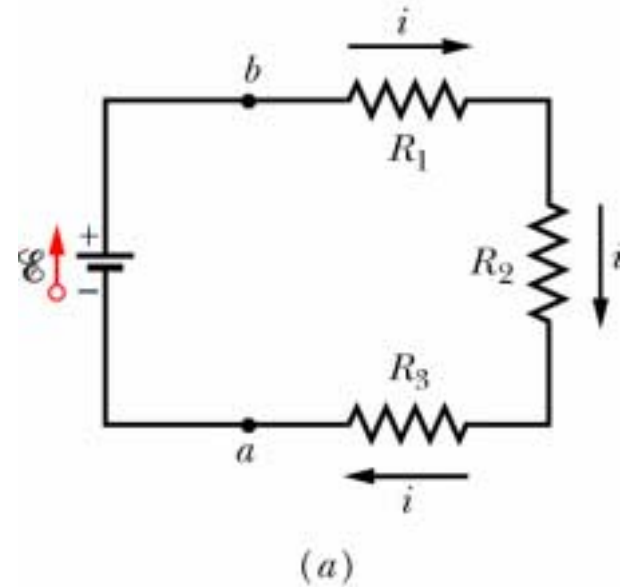
- Substituting for i gives

$$V_b - V_a = \mathcal{E} \frac{R}{R + r}$$

Circuits

- Resistors in series
- Resistors have identical currents, i
- Sum of V 's across resistors = applied $V = E$.
- R_{eq} is sum of all resistors

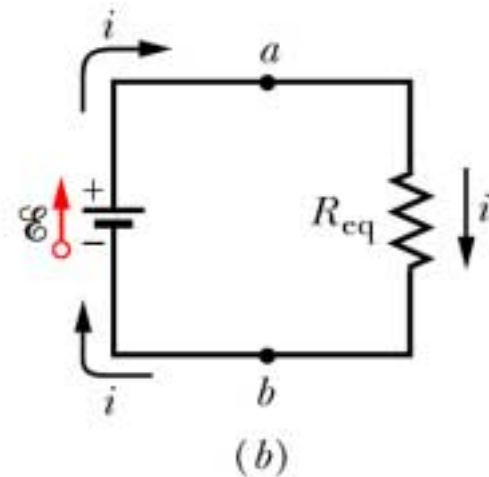
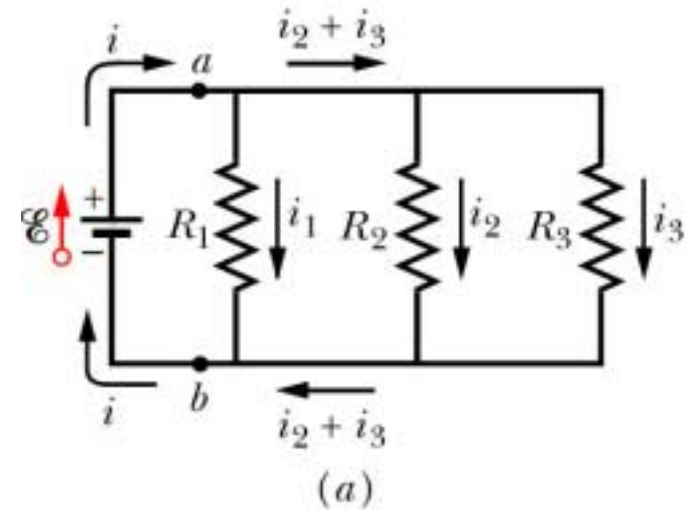
$$R_{eq} = \sum_{j=1}^n R_j$$



Circuits

- Resistors in parallel
- Resistors have identical $V = E$
- $i_1 = V/R_1$ etc
- R_{eq} given by

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$



Junction Rule

- Arbitrarily label currents, using different subscript for each branch
- Using conservation of charge at each junction

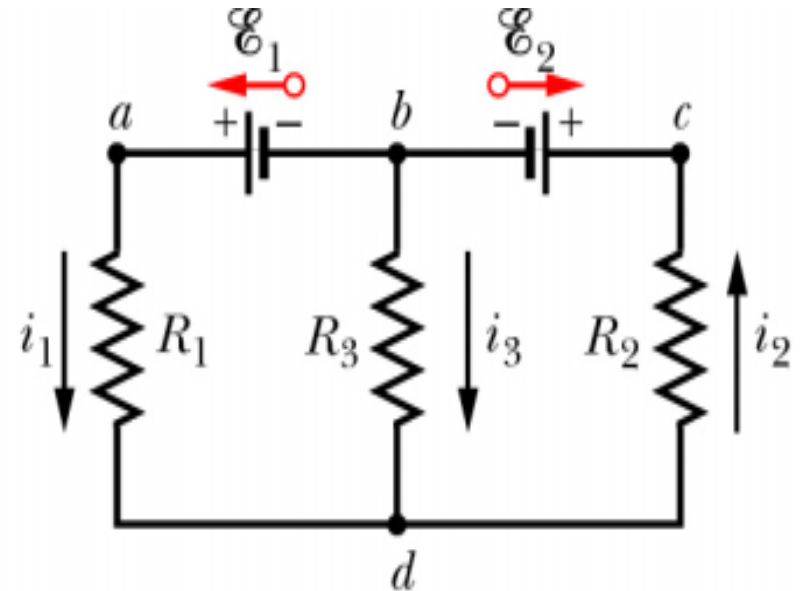
$$i_{in} = i_{out}$$

- At point d

$$i_1 + i_3 = i_2$$

- At point b

$$i_1 + i_3 = i_2$$



- At point a

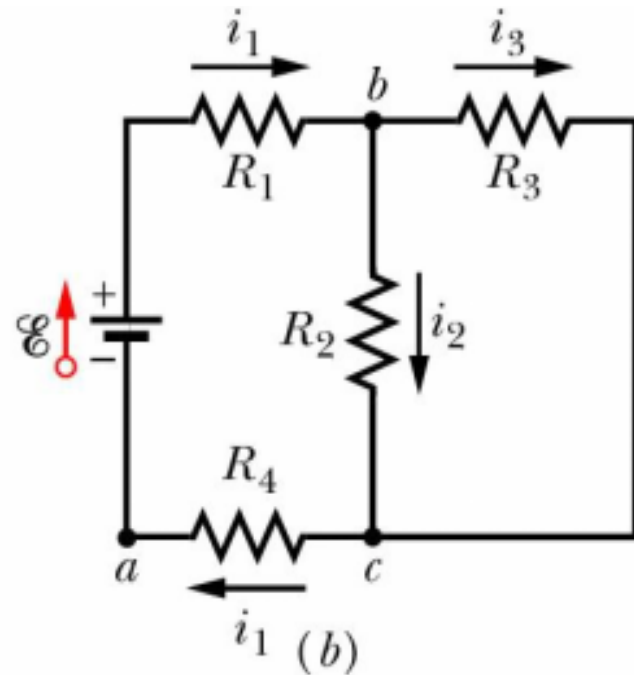
$$i_1 = i_1$$

- At point c

$$i_2 = i_2$$

Circuits

- What is i_1 ?
- $R_1=R_2=R_3=R_4=2$ ohm
- $E=5$ V



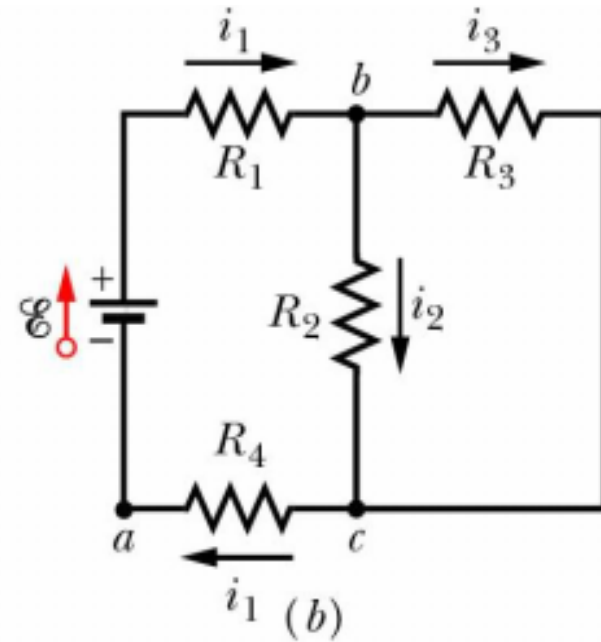
Circuits

- What is i_2 ?
- Three unknowns so we need three equations

$$E - i_1 R_1 - i_2 R_2 - i_1 R_4 = 0$$

$$-i_3 R_3 + i_2 R_2 = 0$$

$$i_1 = i_2 + i_3$$



$$i_2 = -\frac{i_1 R_3}{(R_3 + R_2)}$$

Motion in a B Field

- Force on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- F_B does not change the speed (magnitude of v) or kinetic energy of particle

- Charged particles moving with $v \perp$ to a B field move in a **circular path** with radius, r

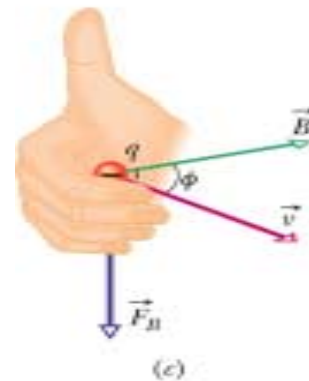
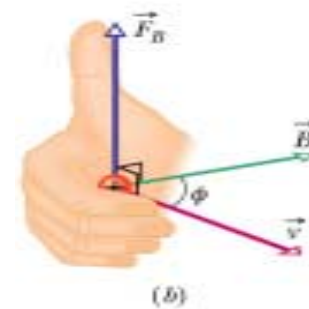
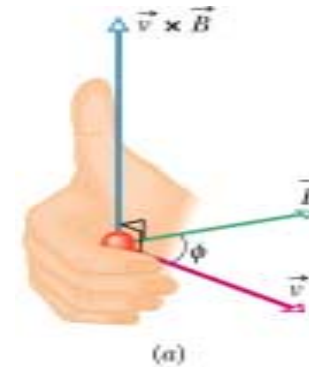
$$r = \frac{mv}{qB}$$

- Force on a current carrying wire due to a magnetic field is

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

Motion in a B Field

- **Right-hand rule** – For positive charges - when the fingers sweep \vec{v} into \vec{B} through the smaller angle ϕ the thumb will be pointing in the direction of \vec{F}_B
- For negative charges \vec{F}_B points in opposite direction



Motion in a B Field

- Circular motion

$$r = \frac{mv}{qB}$$

- Period (time for one revolution)

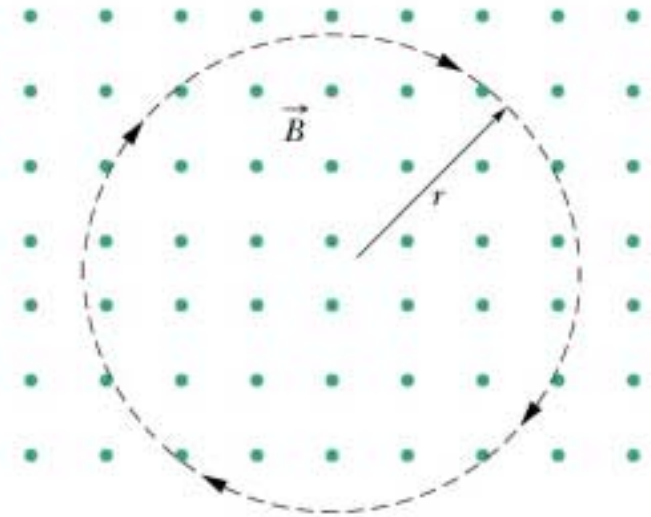
$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Frequency (the number of revolutions per unit time)

$$f = \frac{1}{T}$$

- Angular frequency:

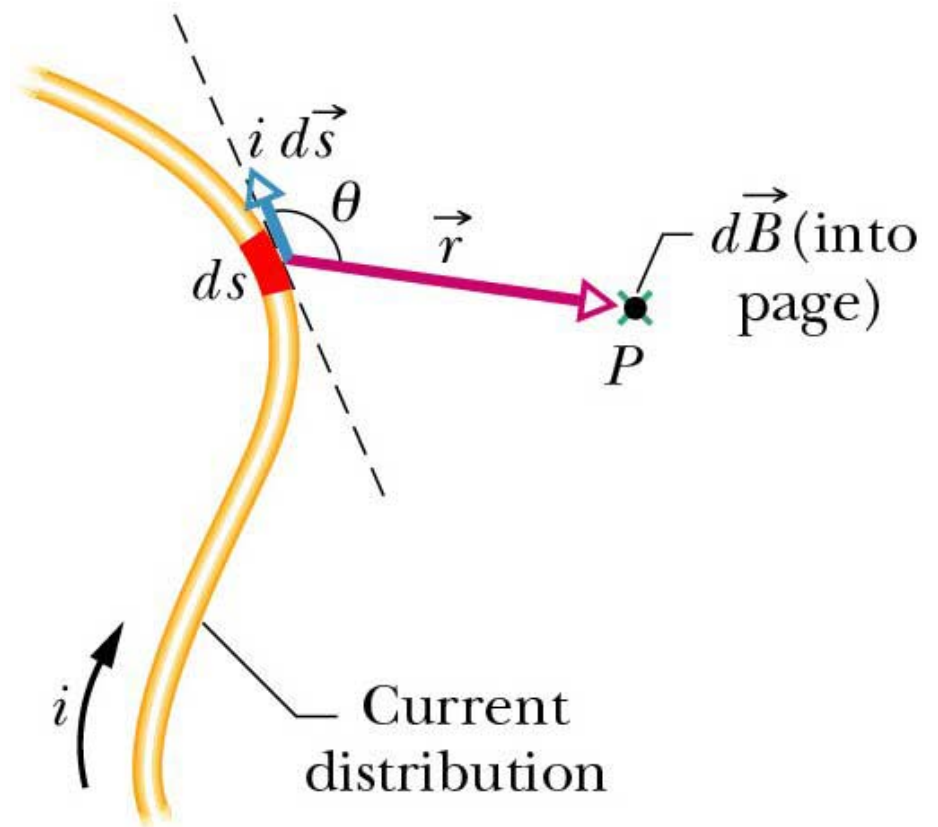
$$\omega = 2\pi f$$



B Fields from Currents

- Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$$

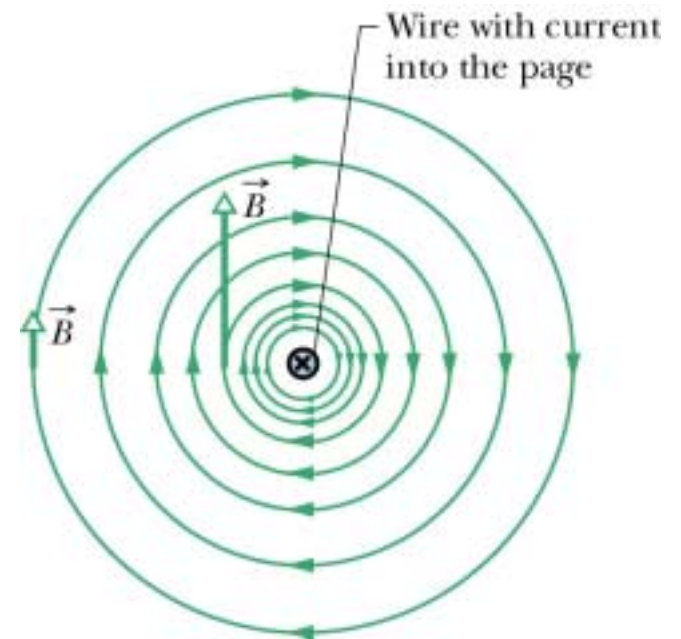


B Fields from Currents

- B field a distance R from a long straight wire carrying current i

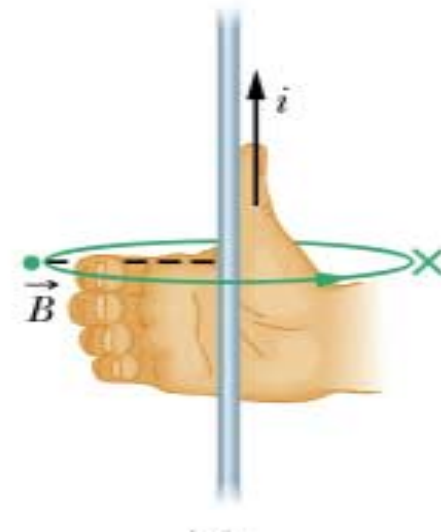
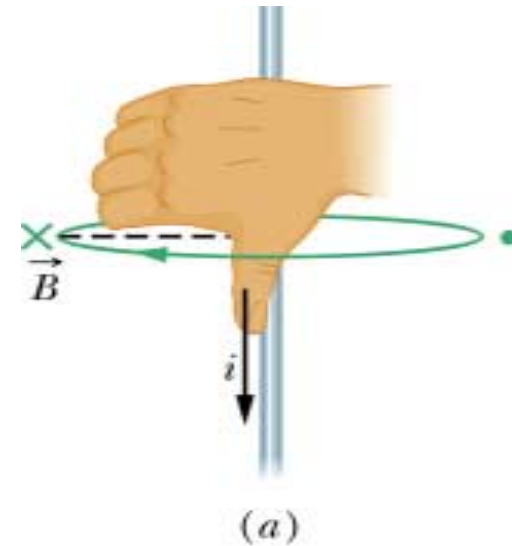
$$B = \frac{\mu_0 i}{2\pi R}$$

- B field is tangent to magnetic field lines



B Fields from Currents

- right-hand rule
- Point thumb in direction of current flow
- Fingers will curl in the direction of the magnetic field lines due to current



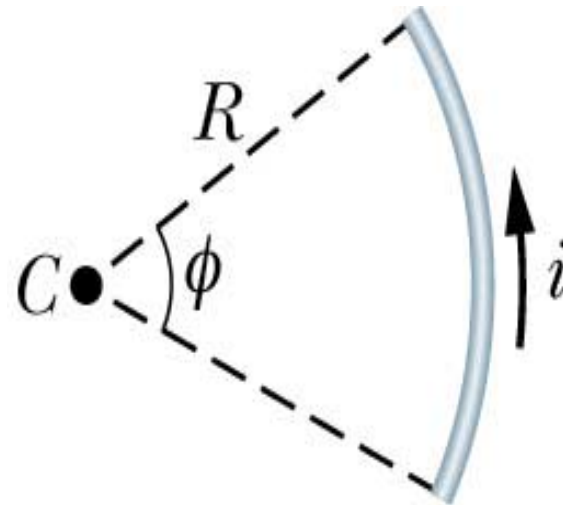
B Fields from Currents

- B field at the center of an arc is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- Express ϕ in radians
- For a complete loop ($\phi = 2\pi$) then B is

$$B = \frac{\mu_0 i}{2R}$$

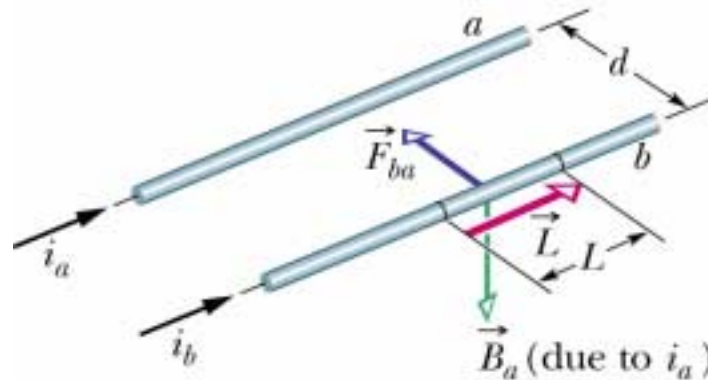


B Fields from Currents

- Force on a wire carrying current, i_1 , due to B of another parallel wire with current i_2

$$F = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

- Force is attractive if current in both wires are in the same directions
- Force is repulsive if current in both wires are in the opposite directions

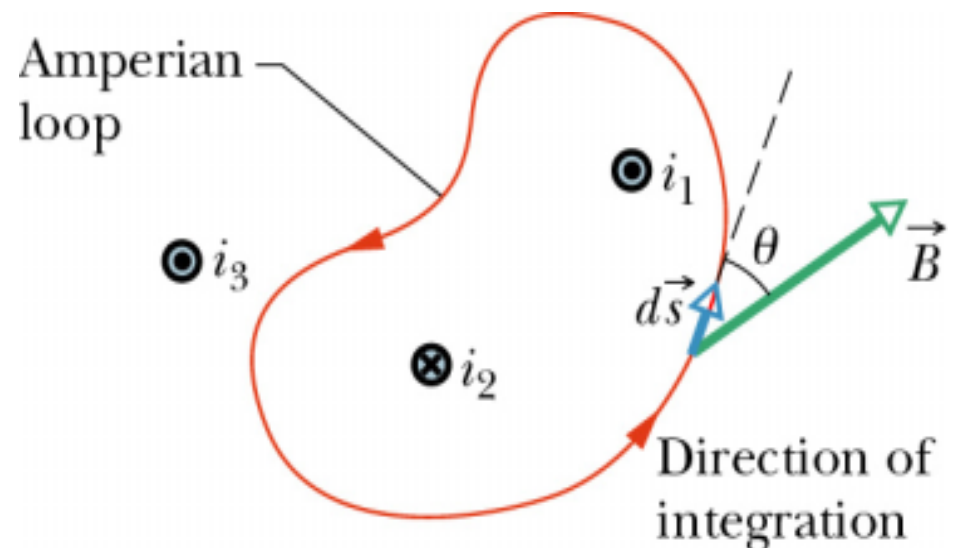


B Fields from Currents

- For symmetric distributions of charge use **Ampere's law** to calculate B field

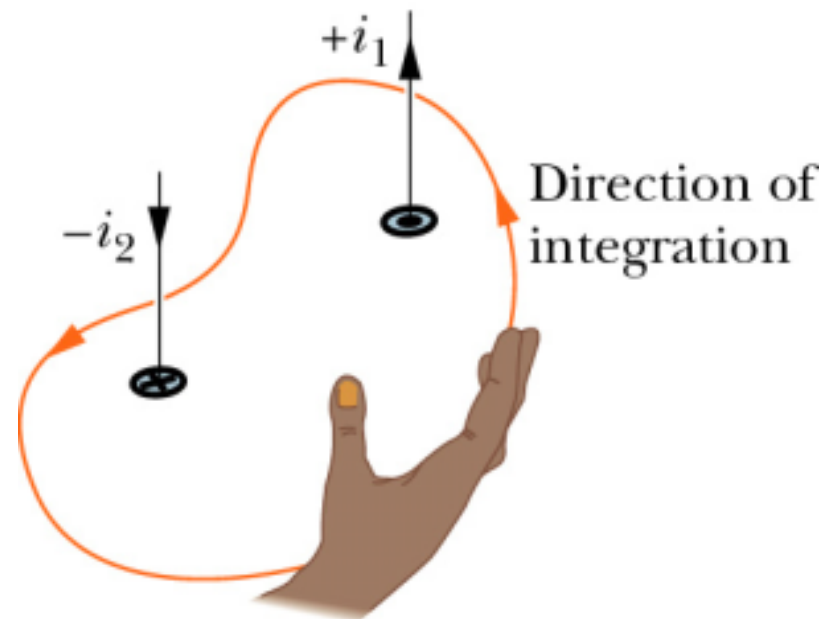
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- Integral around closed loop called **Amperian loop**



B Fields from Currents

- Use the **right-hand rule** to determine the signs for the currents encircled by the Amperian loop
- Curl right hand around Amperian loop with fingers pointing in direction of integration
- Current going through loop in the same direction as thumb is positive.
- Current going in the opposite direction is negative.



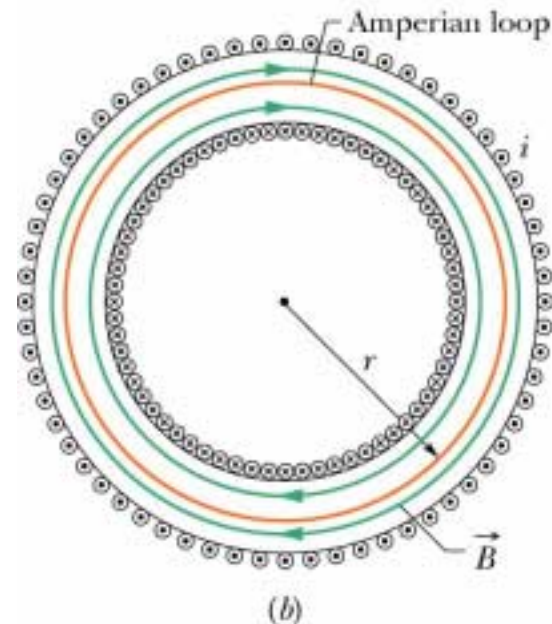
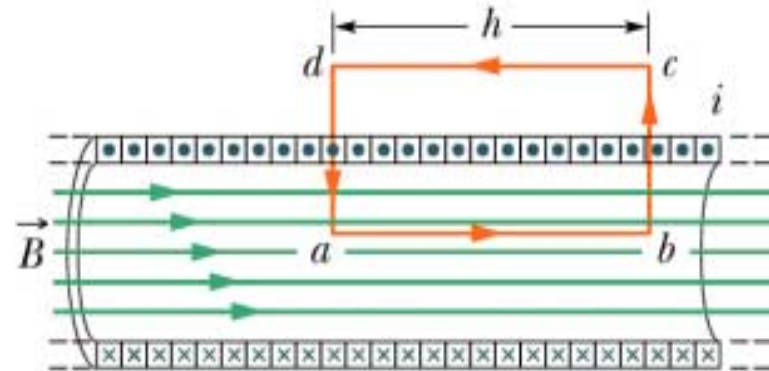
B Fields from Currents

- For ideal solenoid:

$$B = \mu_0 i n$$

- n is # turns/length
- For toroid

$$B = \frac{\mu_0 i N}{2\pi r}$$



Currents from B Fields

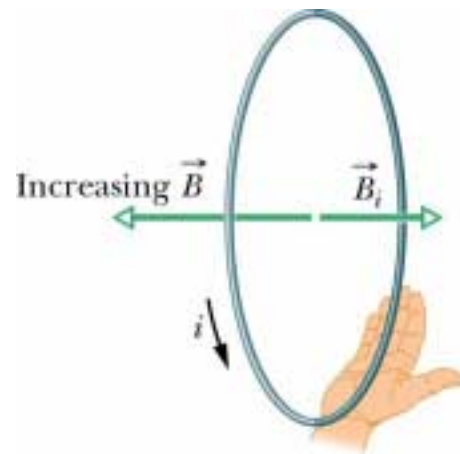
- Magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Faraday's law (N loops)

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- Lenz's law – induced emf gives rise to a current whose B field opposes the change in flux that produced it



Faraday's law

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta$$

- We can change the magnetic flux through a loop (or coil) by:
 - Changing magnitude of B field within coil
 - Changing area of coil, or portion of area within B field
 - Changing angle between B field and area of coil (e.g. rotating coil)

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -NA \cos \theta \frac{dB}{dt}$$

$$\mathcal{E} = -NB \cos \theta \frac{dA}{dt}$$

$$\mathcal{E} = -NBA \frac{d(\cos \theta)}{dt}$$

Generators

- Generator with N turns of area A and rotating with constant angular velocity, ω
- Magnetic flux is

$$\Phi_B = BA \cos \omega t$$

- Emf is

$$\mathcal{E} = NBA \omega \sin \omega t$$

