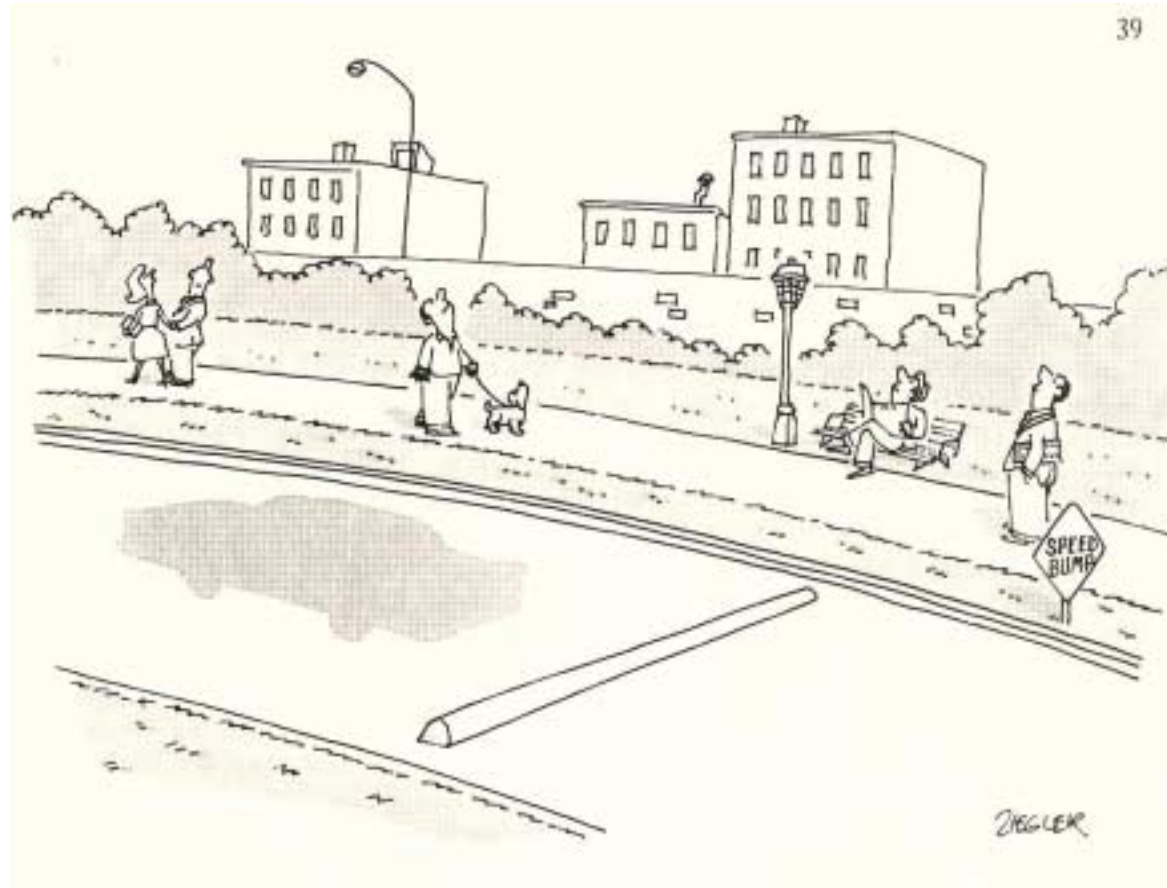


# Review for final



# Schedule

- Dec. 3-5 (Wed-Fri) – Review for final
- Dec. 3 (Wed) – HW set #12 due at 11pm
- Dec. 8 (Mon) – Corrections #3 due at 7am
- Dec. 8 (Mon) – **Final Exam 5:45-7:45pm**
  - Section 1 – N130 BCC (Business College)
  - Section 2 – 158 NR (Natural Resources)

# Final Exam

- If  $>2$  finals on Mon. may take make-up final exam on **Tues. from 8-10am**
  - **Email Prof. Tollefson with list of other finals**
- Allowed 3 sheets of notes (both sides) and calculator
- Covers Chapters 22-37, HW sets 1-12
- Exam will have 20 questions
- Need photo ID

# Electric Fields

- Point charge  $q$

$$F = k \frac{|q||q_0|}{r^2}$$

$$E = k \frac{q}{r^2}$$

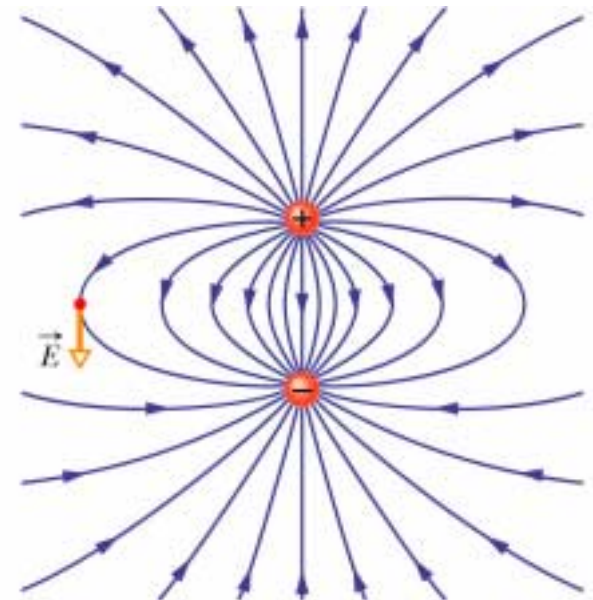
$$V = k \frac{q}{r}$$

- $E$  points away from positive charges and towards negative charges
- Superposition principle (many charges)

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

$$V = \sum_{i=1}^n V_i$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$$



# Electric Potential

- Electric potential from field

$$\Delta V = - \int_i^f \vec{E} \cdot d\vec{s}$$

- Constant field over distance  $d$

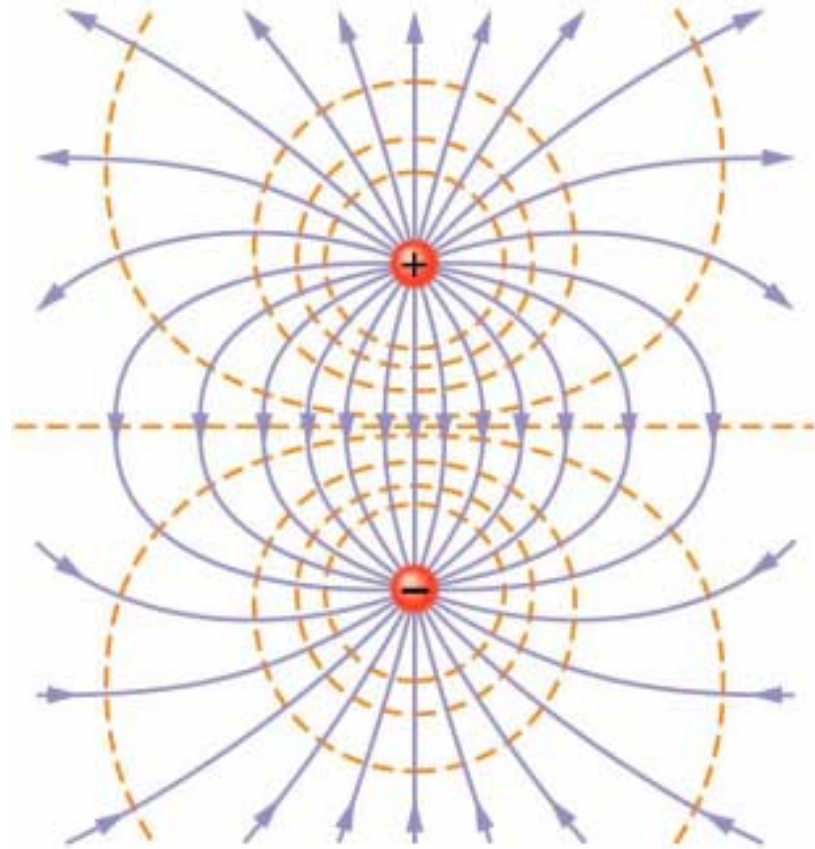
$$\Delta V = -Ed$$

- Work done moving charge  $q_0$

$$W = q_0 \Delta V$$

# Electric Potential

- Blue lines are the electric field
- Dashed lines are equipotential surfaces where all points are at the same potential
- $V$  decreases by constant intervals from the positive charge to the negative charge



(c)

# Guass' Law

- Electric flux

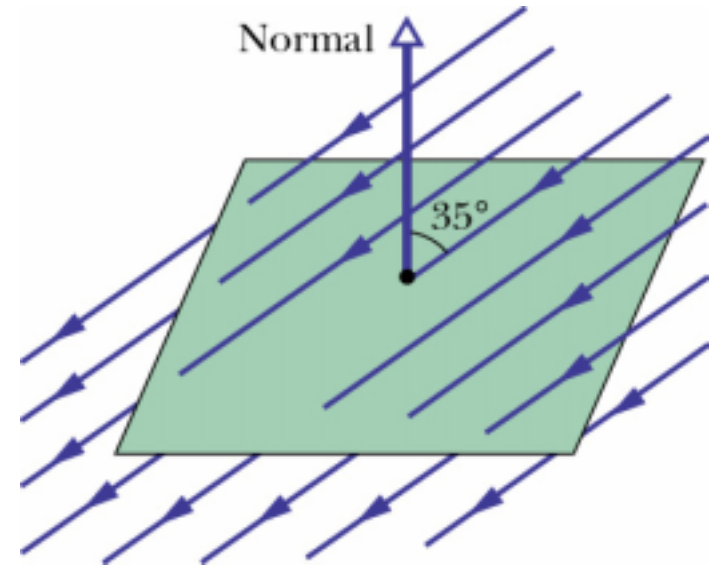
$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$\vec{E} \cdot d\vec{A} = E dA \cos \theta$$

- Gauss' Law

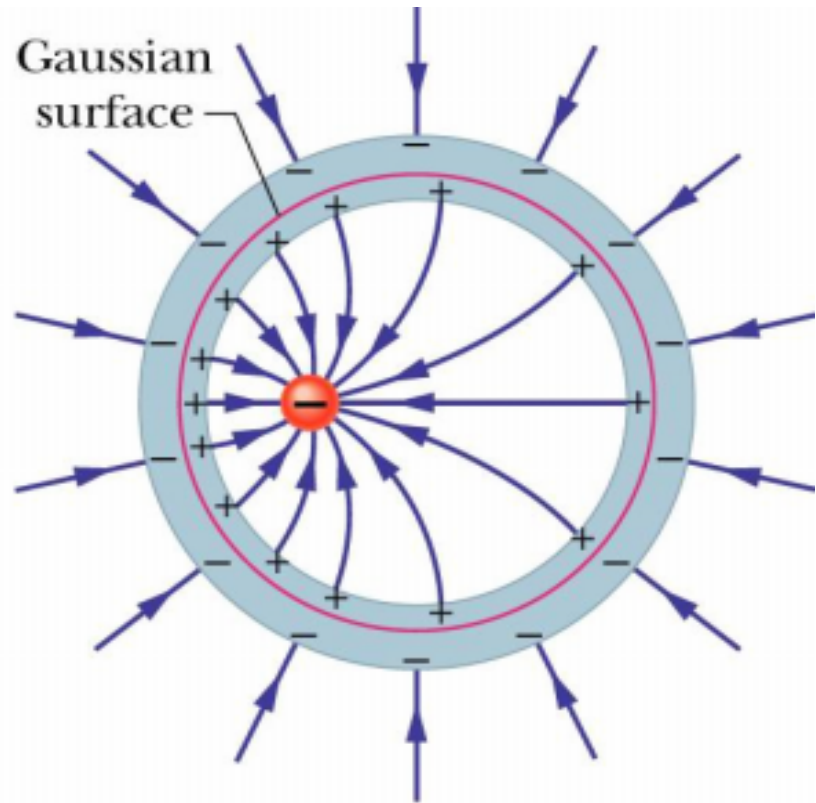
$$\epsilon_0 \Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

- Net charge  $q_{enc}$  is sum of all enclosed charges and may be +, -, or zero



# Conductors (example)

- Charge  $q_1$  inside
- $E=0$  inside conductor
- Thus  $\Phi=0$  for Gaussian surface (red line)
- So **net** charge enclosed must be 0
- Induced charge of  $q_2 = -q_1$  lies on inner wall of conductor
- Shell is neutral so charge of  $q_3 = -q_2$  on outer wall





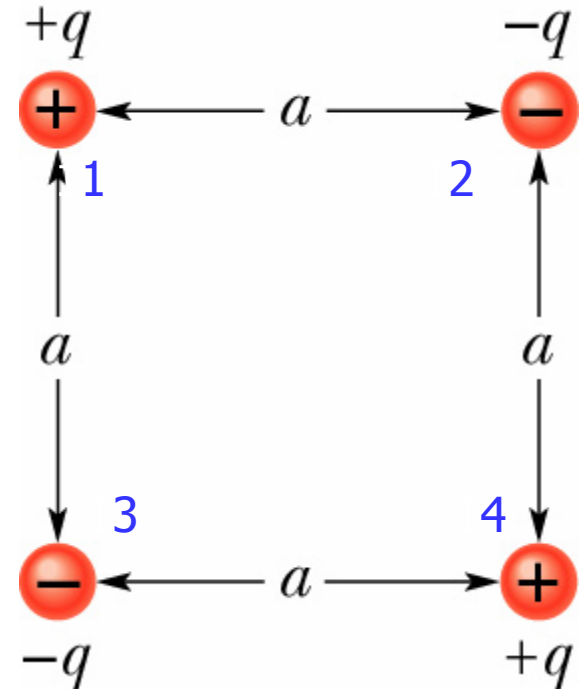
# Electric Potential Energy

- Work required to assemble the charges

$$U = U_{12} + U_{13} + U_{14} \\ + U_{23} + U_{24} + U_{34}$$

- where  
etc

$$U_{12} = k \frac{q_1 q_2}{d}$$



# Motion in a B Field

- Force on a charged particle due to a magnetic field is

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- $F_B$  does not change the speed (magnitude of  $v$ ) or kinetic energy of particle

- Charged particles moving with  $v \perp$  to a  $B$  field move in a **circular path** with radius,  $r$

$$r = \frac{mv}{qB}$$

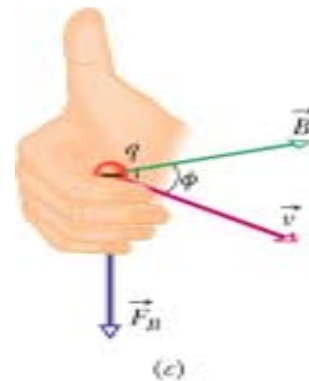
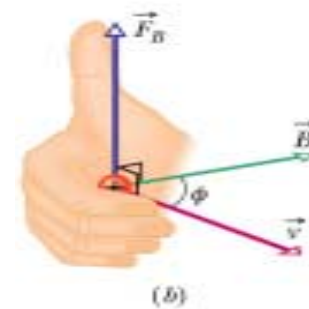
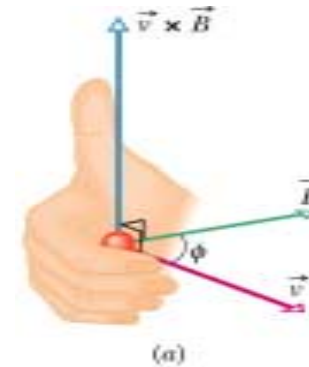
- Force on a current carrying wire due to a magnetic field is

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

# Motion in a B Field

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- **Right-hand rule** – For positive charges - when the fingers sweep  $\vec{v}$  into  $\vec{B}$  through the smaller angle  $\phi$  the thumb will be pointing in the direction of  $F_B$
- For negative charges  $F_B$  points in opposite direction



# Motion in a B Field

- Circular motion

$$r = \frac{mv}{qB}$$

- Period (time for one revolution)

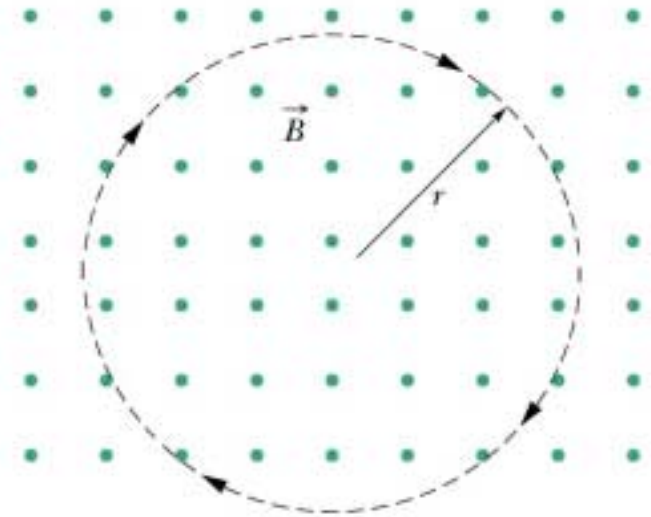
$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Frequency (the number of revolutions per unit time)

$$f = \frac{1}{T}$$

- Angular frequency:

$$\omega = 2\pi f$$

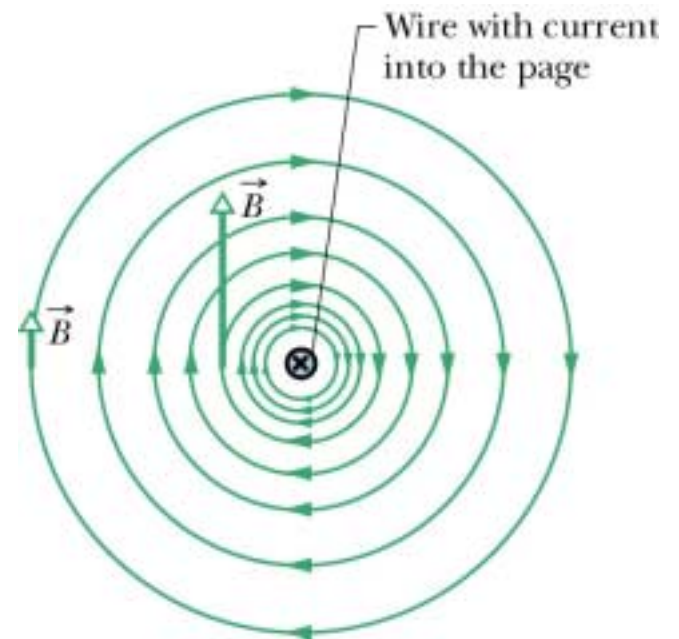


# B Fields from Currents

- $B$  field a distance  $R$  from a long straight wire carrying current  $i$

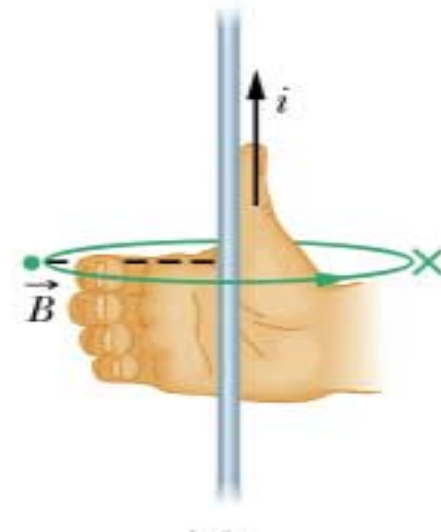
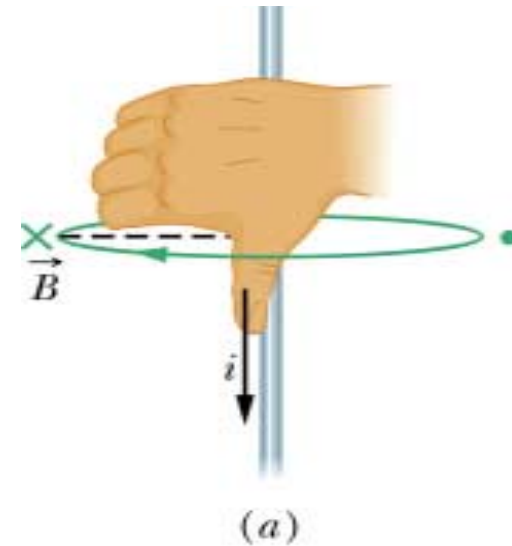
$$B = \frac{\mu_0 i}{2\pi R}$$

- $B$  field is tangent to magnetic field lines



# B Fields from Currents

- right-hand rule
- Point thumb in direction of current flow
- Fingers will curl in the direction of the magnetic field lines due to current

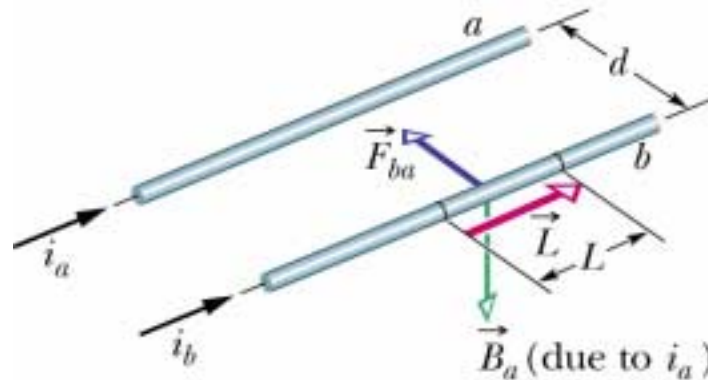


# B Fields from Currents

- Force on a wire carrying current,  $i_1$ , due to  $B$  of another parallel wire with current  $i_2$

$$F = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

- Force is attractive if current in both wires are in the same directions
- Force is repulsive if current in both wires are in the opposite directions

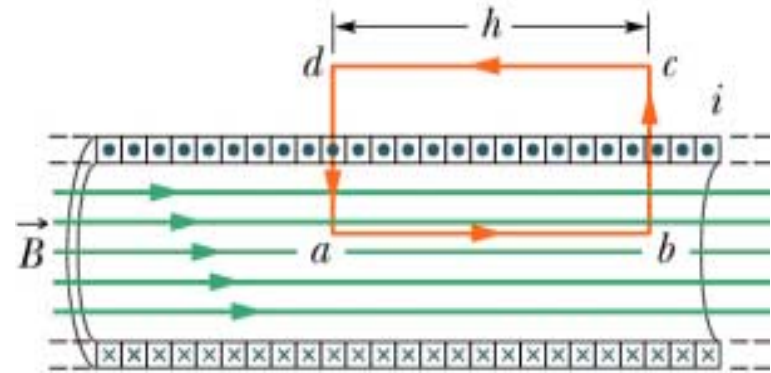


# B Fields from Currents

- For ideal solenoid:

$$B = \mu_0 i n$$

$n$  is the number of turns/length





# Faraday's Law

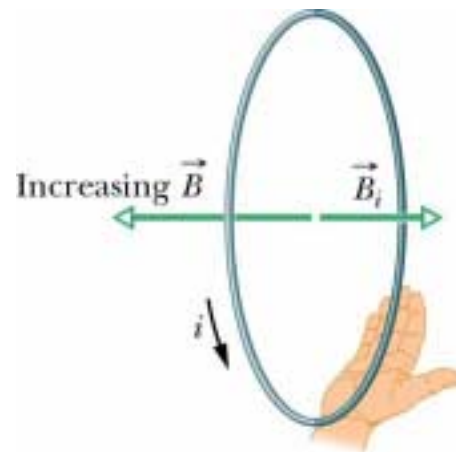
- Magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta$$

- Faraday's law (N loops)

$$E = -N \frac{d\Phi_B}{dt}$$

- Lenz's law – induced emf gives rise to a current whose  $B$  field opposes the change in flux that produced it



# Faraday's law

- We can change the magnetic flux through a loop (or coil) by:
  - Changing **magnitude of  $B$  field** within coil
  - Changing **area of coil**, or portion of area within  $B$  field
  - Changing **angle** between  $B$  field and area of coil (e.g. rotating coil)

$$E = -NA \cos \theta \frac{dB}{dt}$$

$$E = -NB \cos \theta \frac{dA}{dt}$$

$$E = -NBA \frac{d(\cos \theta)}{dt}$$

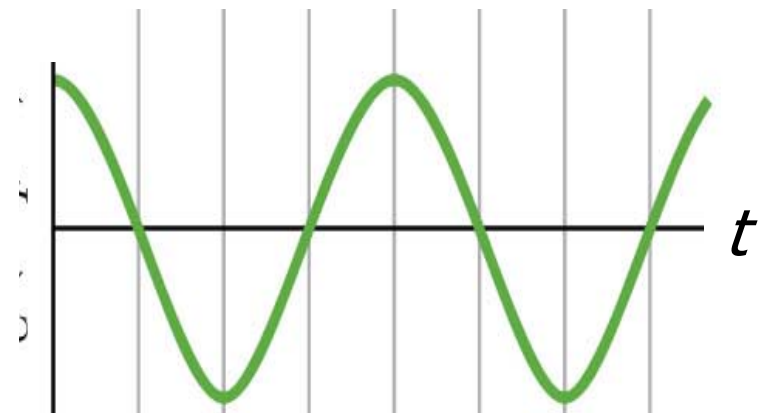
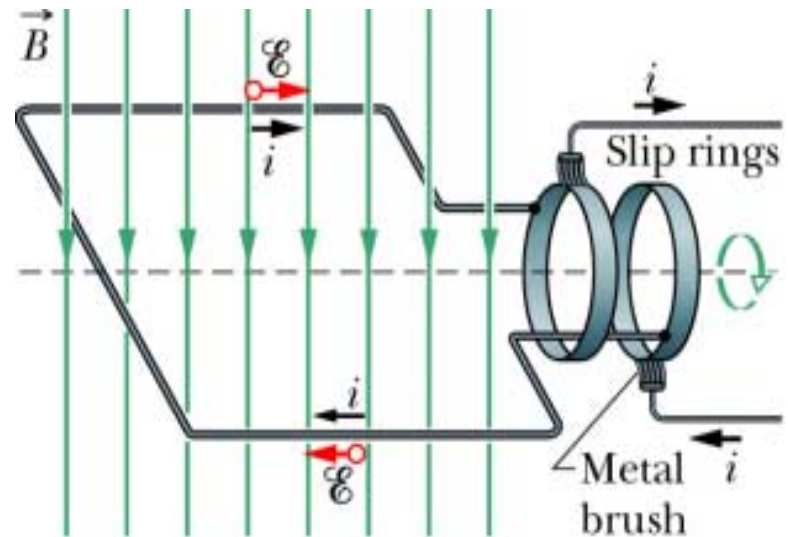
# Generators

- Generator with  $N$  turns of area  $A$  and rotating with constant angular velocity,  $\omega$
- Magnetic flux is

$$\Phi_B = BA \cos \omega t$$

- Emf is

$$E = NBA \omega \sin \omega t$$



# Circuits

- Current  $i = \frac{dq}{dt}$

- Resistors

- Ohm's law  $V = iR$

- Power lost  $P = iV = i^2 R = \frac{V^2}{R}$

# Circuits

- Definition of capacitance:  $C = \frac{q}{V}$
- Parallel plates of area  $A$  and separation  $d$

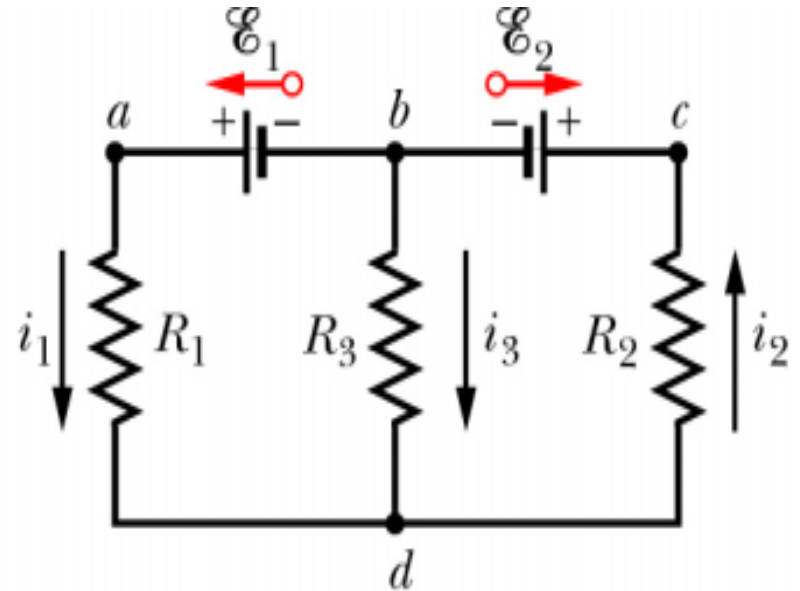
$$C = \frac{\epsilon_0 A}{d}$$

# Circuits

- Junction rule:

$$i_{in} = i_{out}$$

- Loop rule: sum of potential changes around each loop is zero



# Circuits

## Parallel

## Series

- Capacitors

$$C_{eq} = \sum_i^n C_i$$

- $V$  same on each
- Total  $q$  is sum

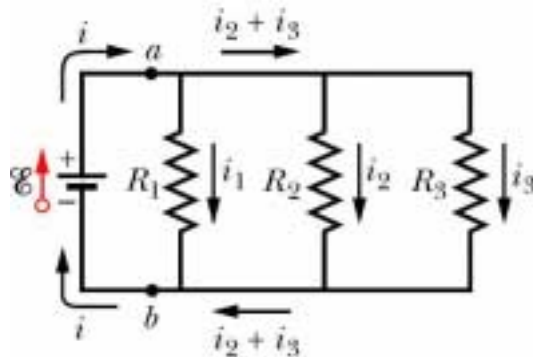
$$\frac{1}{C_{eq}} = \sum_i^n \frac{1}{C_i}$$

- $q$  same on each
- Total  $V$  is sum

- Resistors

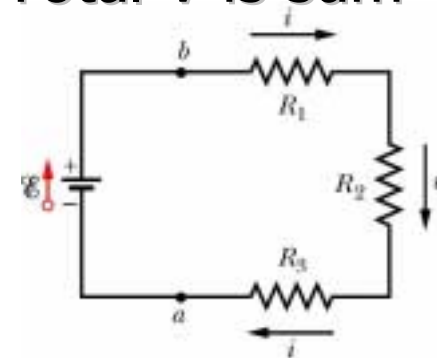
$$\frac{1}{R_{eq}} = \sum_i^n \frac{1}{R_i}$$

- $V$  same on each



$$R_{eq} = \sum_i^n R_i$$

- $i$  same in each
- Total  $V$  is sum



# Elements of RLC circuits

Resistor

Inductor

Capacitor

- Energy stored

$$U = 0$$

$$U_B = \frac{1}{2} Li^2$$

$$U_E = \frac{1}{2} \frac{q^2}{C}$$

- Voltage change

$$V = (-)iR$$

$$V = -L \frac{di}{dt}$$

$$V = (-) \frac{q}{C}$$

(-) means sign relative to the direction of current flow

- Power lost

$$P = i^2 R$$

$$P = 0$$

$$P = 0$$



# LC Circuits

- Charge

$$q = Q \cos(\omega t)$$

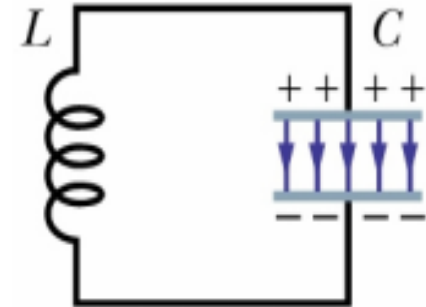
- Current

$$i = \frac{dq}{dt} = -Q\omega \sin(\omega t)$$

- Angular frequency

$$\omega = \sqrt{\frac{1}{LC}}$$

- No power loss



# RLC Circuits

- Charge on capacitor

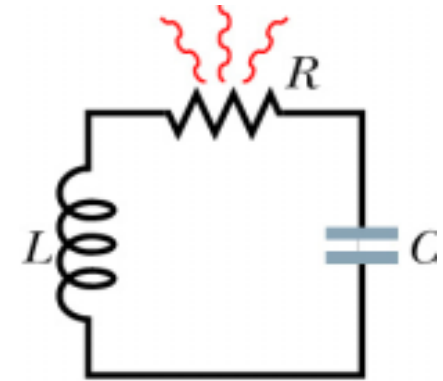
$$q = Qe^{-Rt/2L} \cos(\omega't)$$

- Angular frequency

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

- Natural frequency

$$\omega = \sqrt{\frac{1}{LC}}$$

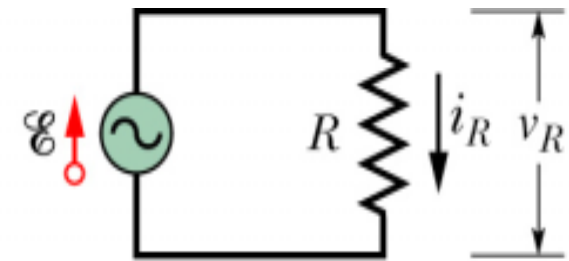


# AC Circuits

$$E = E_m \sin \omega_d t, \quad \omega_d = \text{driving frequency}$$

Resistive load

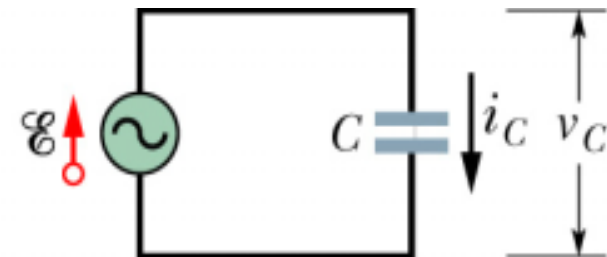
$$I_R = \frac{V_R}{R}$$



Capacitive load

$$I_C = \frac{V_C}{X_C}$$

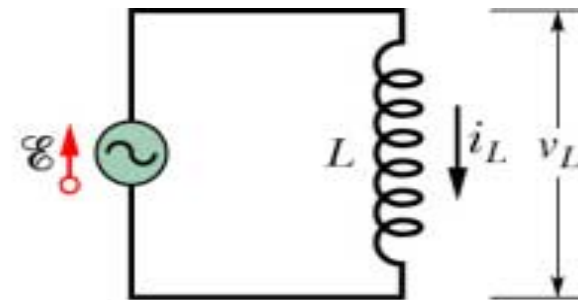
$$X_C = \frac{1}{\omega_d C}$$



Inductive load

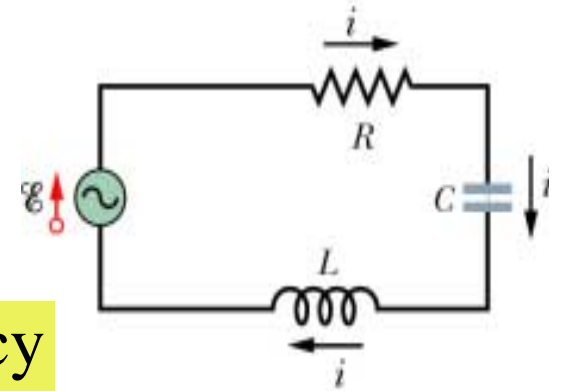
$$I_L = \frac{V_L}{X_L}$$

$$X_L = \omega_d L$$



$X$  is the reactance

# AC Circuits



- Input  $E = E_m \sin \omega_d t$ ,  $\omega_d =$  driving frequency

- Current (same everywhere)  $i = I \sin(\omega_d t - \phi)$

- Solution  $I = \frac{E_m}{Z}$   $\tan \phi = \frac{X_L - X_C}{R}$   $\frac{X_L}{X_C} = \frac{(\omega_d)^2}{\omega^2}$

- $Z$  is the impedance  $Z = \sqrt{R^2 + (X_L - X_C)^2}$

- $I$  is maximum on resonance where

$$X_L = X_C$$

$$Z = R$$

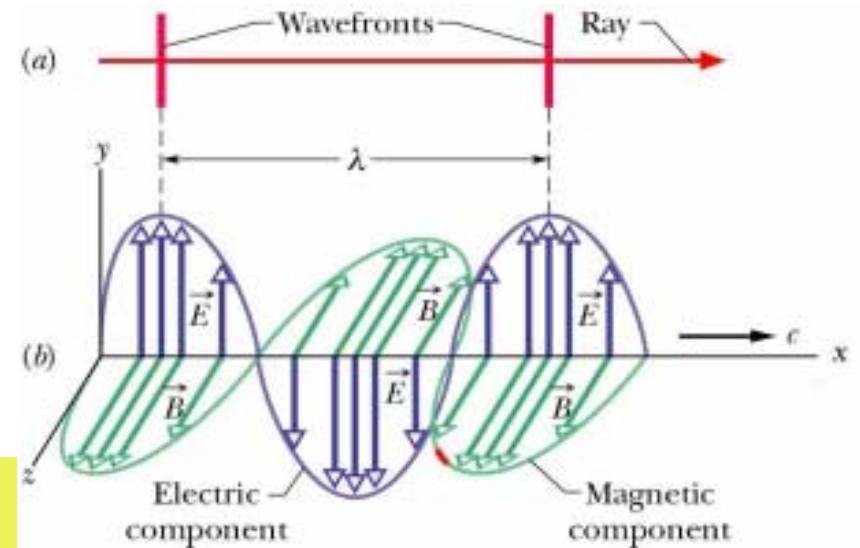
$$\omega_d = \omega$$

# EM Waves

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

$$v = c = f\lambda = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$



Direction and power per unit area

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Intensity

$$I = \frac{1}{2\mu_0 c} E_m^2$$

Pressure

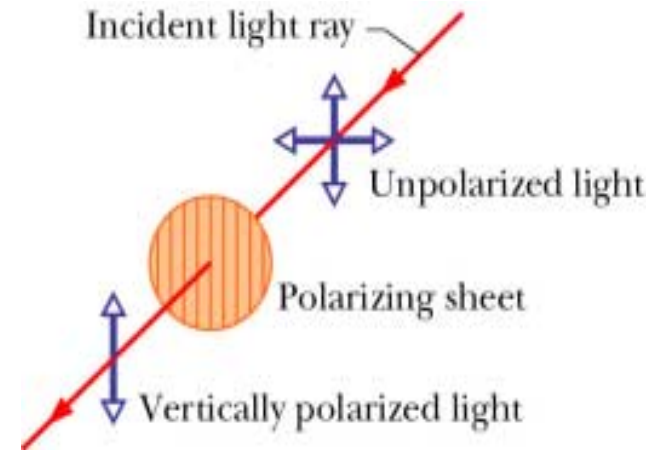
$$p_r = \frac{I}{c}$$

$$p_r = \frac{2I}{c}$$

absorption reflection

# Polarization

- Polarization is the direction of the  $E$  field
- Intensity of **unpolarized** light with intensity  $I_0$  after hitting a polarizing sheet
- Intensity of **polarized** light with intensity  $I_0$  after hitting a polarizing sheet



$$I = \frac{1}{2} I_0$$

$$I = I_0 \cos^2 \theta$$

# Reflection & Refraction

- Reflection:  $\theta'_1 = \theta_1$
- Refraction (Snell's law)

$$n_2 \sin \theta_2 = n_1 \sin \theta_1$$

- Index of refraction

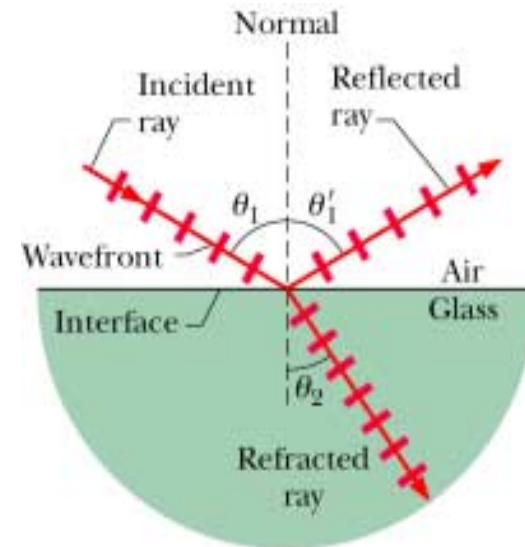
$$n = \frac{\text{speed in vacuum}}{\text{speed in medium}} = \frac{c}{v}$$

$$\omega_1 = \omega_2$$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

- Critical angle (no refracted wave)

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$



# Mirrors

- **Plane** – flat mirror
- **Concave** – caved in towards object
- **Convex** – flexed out away from object

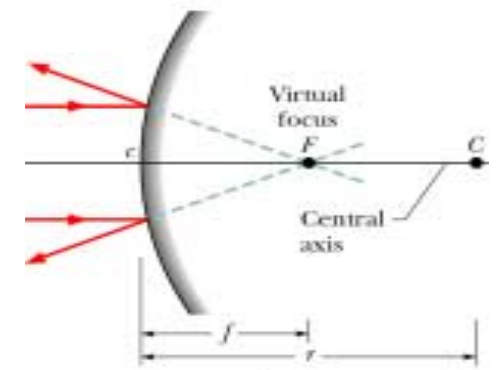
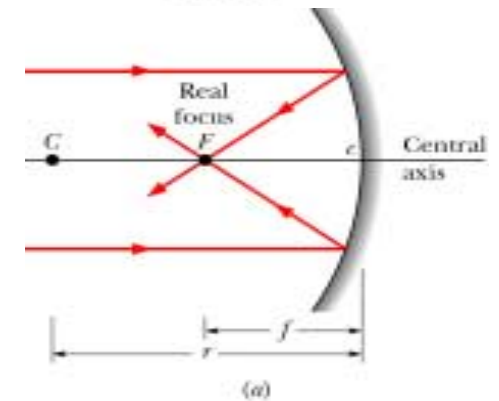
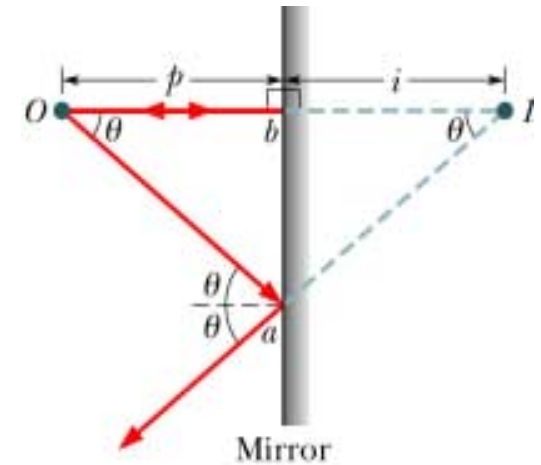
$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$f = \frac{1}{2} r$$

- $r$  = radius of curvature
- $f$  = focal length,  $f > 0$  concave,  $f < 0$  convex
- $p$  = position of object
- $i$  = position of image
- **real images** on side where object is
- **virtual images** on opposite side
- **lateral magnification:**

$$|m| = \frac{h'}{h}$$

$$m = -\frac{i}{p}$$





# Thin Lenses

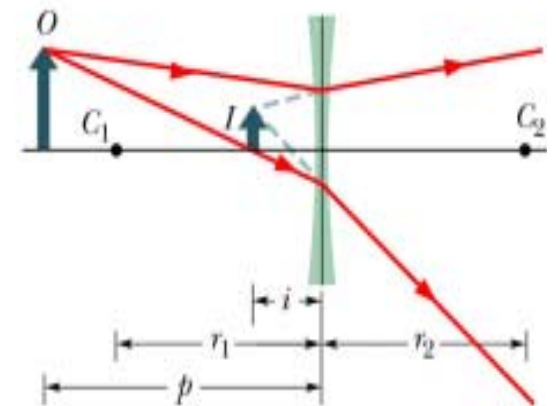
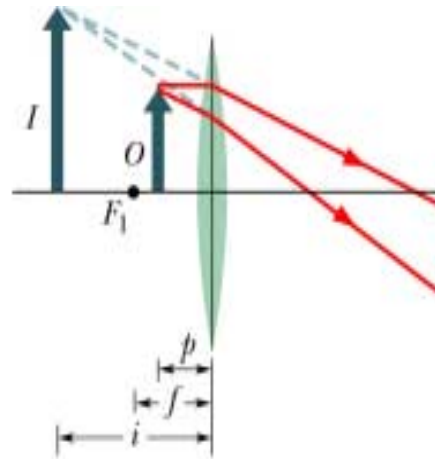
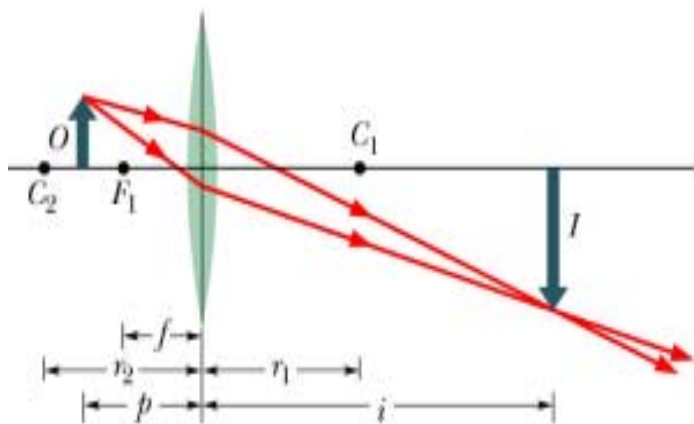
- **Real images:** opposite side - **virtual images:** same side
- **Diverging lens** ( $f < 0$ ): smaller, same orientation, virtual images
- **Converging lens** ( $f > 0$ ): both real and virtual images
- Image position and magnification:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$m = -\frac{i}{p}$$

- **Lens maker's equation:**

$$\frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$



# Interference and Diffraction

- Diffraction **minima** given by  
( $a$ =slit width)

$$a \sin \theta' = m' \lambda$$

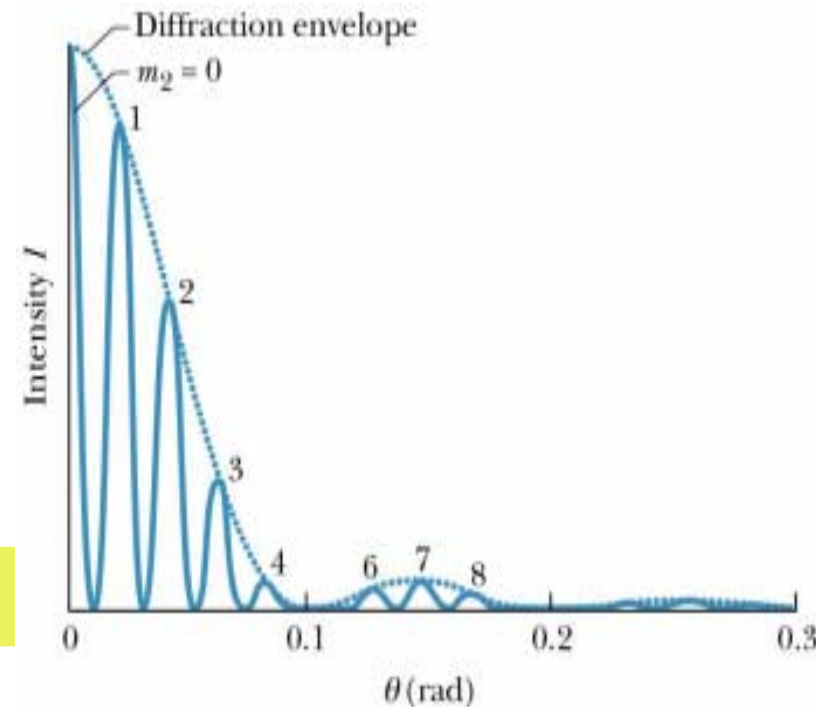
$$y' = \frac{m' \lambda D}{a}, \quad m' = 1, 2, \dots$$

- Two-slit **maxima** given by  
( $d$ =slit separation)

$$d \sin \theta = m \lambda$$

$$y = \frac{m \lambda D}{d}, \quad m = 0, 1, 2, \dots$$

- Small angles  $\sin \theta = \theta$  ( $\theta$  in radians)



# Thin Films

- Phase change at interface
  - Refraction at interface (the transmitted wave) never changes phase
  - Reflection gives a phase change

$$\frac{1}{2}\lambda \text{ if } n_1 < n_2$$

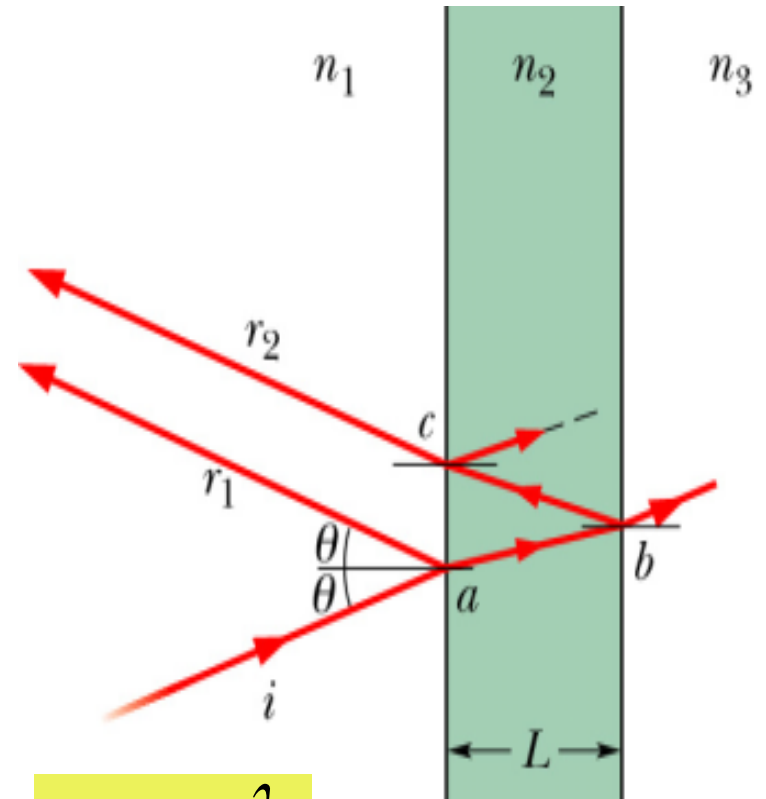
- Phase change due to path a-b-c in material  $n_2$  over a total length  $2L$

- 1 and 2 are in phase if

$$2L = m\lambda_{n_2}, \quad m = 0, 1, 2, \dots$$

- 1 and 2 are out of phase if

$$2L = \left(m + \frac{1}{2}\right)\lambda_{n_2}, \quad m = 0, 1, 2, \dots$$



$$\lambda_{n_2} = \frac{\lambda}{n_2}$$