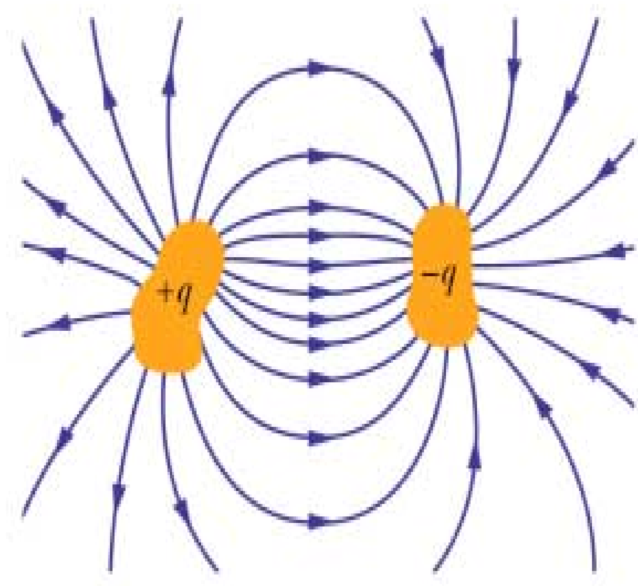
September 15th

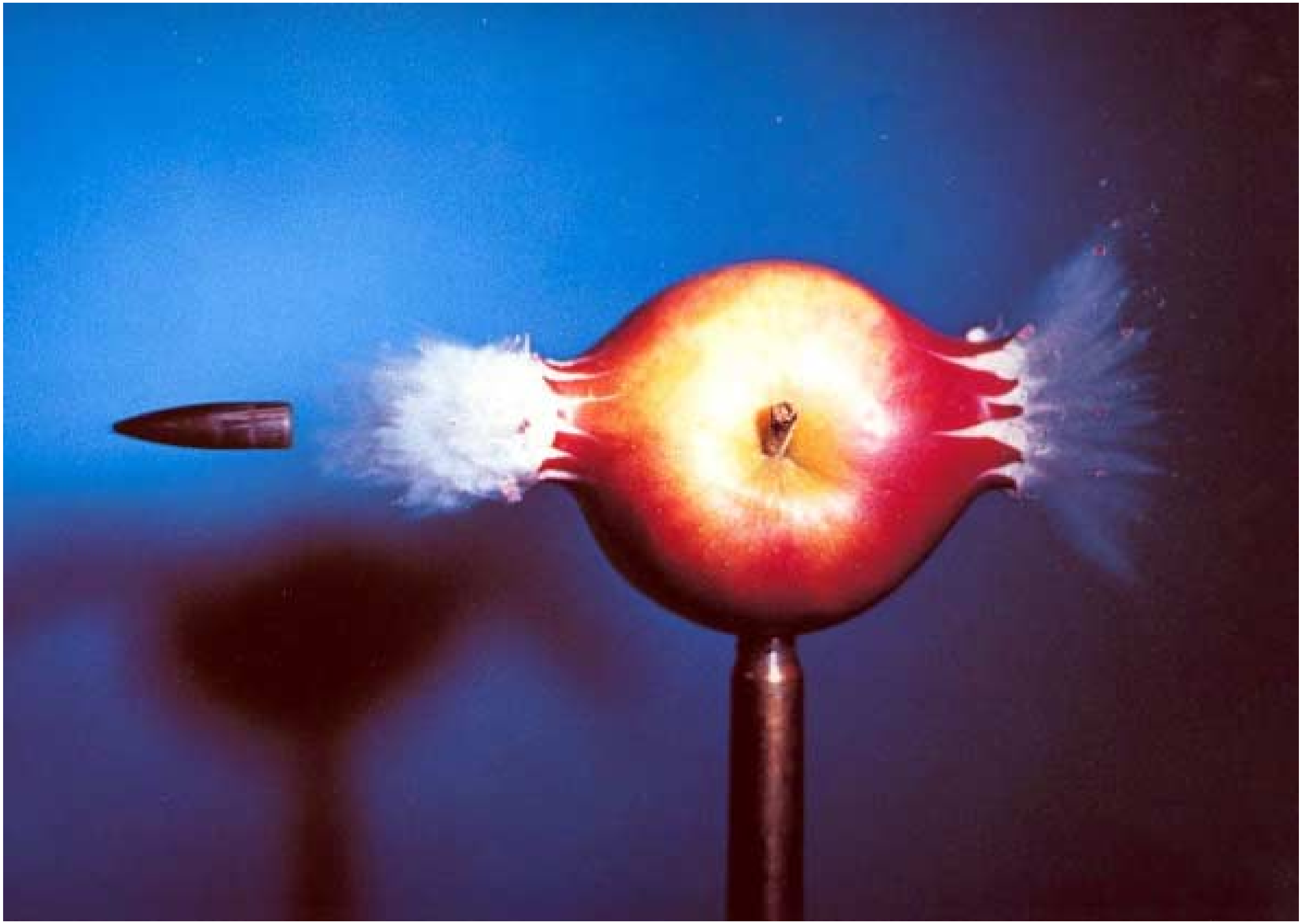
Chapter 26 Capacitance

Capacitance (Fig. 26-2)

- Capacitor – device used to store potential energy from an *E* field
- The *E* field comes from stored charge
- This energy might be stored slowly, but can be released quickly – photoflash, heart defibrillator
- A capacitor is formed from two isolated conductors - equipotentials
- When capacitor is charged, plates have equal but opposite charges +*q* and -*q*

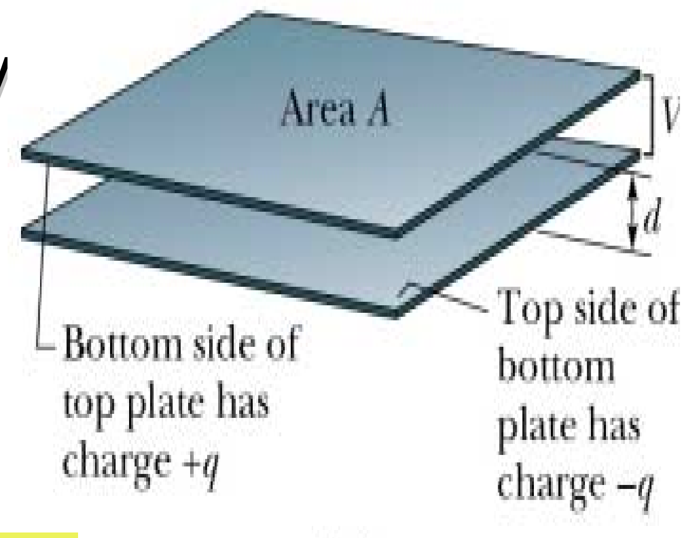


Harold Edgerton - Stroboscope



Capacitance (Fig. 26-3)

- Capacitance is a proportionality constant relating *q* and *V*
  - *q* is the absolute value of the charge on one plate.
  - *V* is the potential difference between plates.



$$q = CV$$ or $$C = q/V$$

- *C* depends only on geometry of plates, not on their *q* or *V*

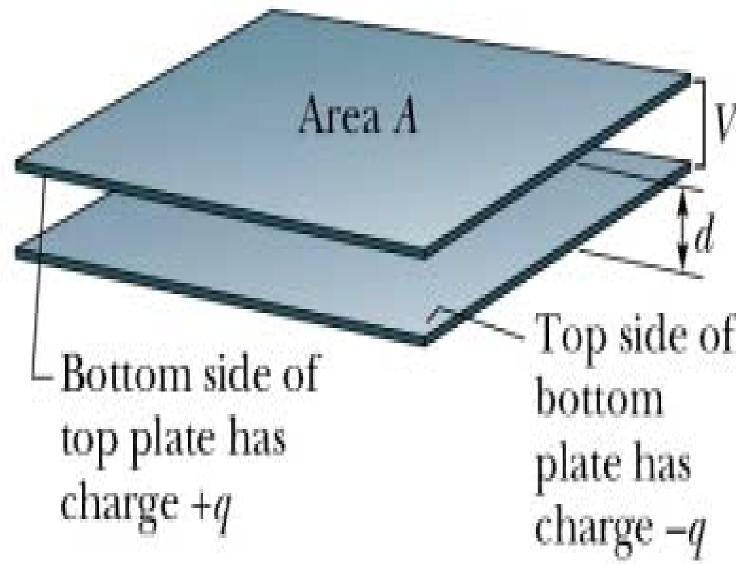
Fig. 26-1 An assortment of capacitors.

Capacitance (Fig. 26-3)

- Capacitance is a measure of how much *q* is needed on plates to get *V* between them
  - Greater *C*, more *q* required

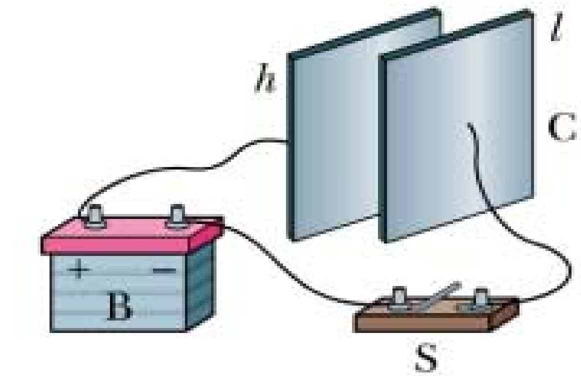
- SI unit for *C* is Farad

$$1F = 1C/V$$

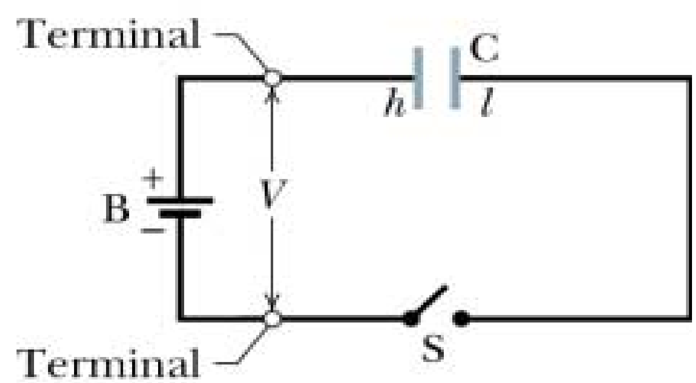


Capacitance (Fig. 26-4)

- Can charge a capacitor using a battery
- Battery – device maintains certain *V* between its terminals by internal electrochemical reactions
- Initially *V* on plates is 0
- Close switch, plates gradually charge up to *V* of battery through flow of electrons

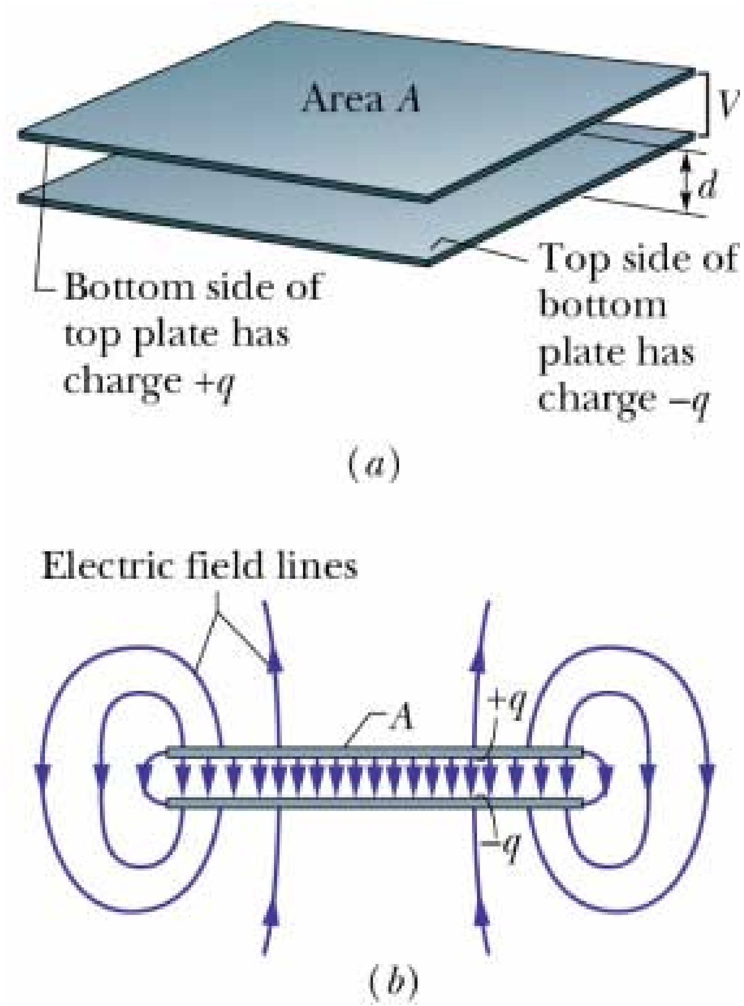


(a)



(b)

Capacitance (Fig. 26-3)



We ignore these (edge) fringe fields

Capacitance (Checkpoint #1)

- Does the C of a capacitor increase, decrease or remain the same when
  - A) charge, q, on it is doubled
  - B) V across it is tripled

Remember C of capacitor only depends on its geometry so C is the same for A and B

Capacitance

- Calculate *C* of a capacitor from its geometry using steps:
- 1) Assume charge, *q*, on the plates
- 2) Find *E* between plates using *q* and Gauss' law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- 3) Find *V* from *E* using

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

- 4) Get *C* using

$$C = \frac{q}{V}$$

Capacitance (Fig. 26-5)

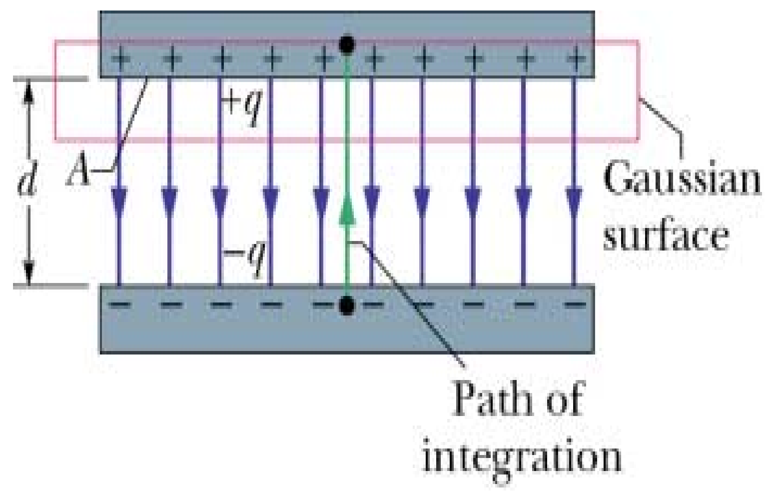
- Simplify Gauss' law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- 1) Pick Gaussian surface to enclose charge on + plate and *E* and *dA* to be parallel

$$\vec{E} \bullet d\vec{A} = EA$$

$$q = \varepsilon_0 EA$$



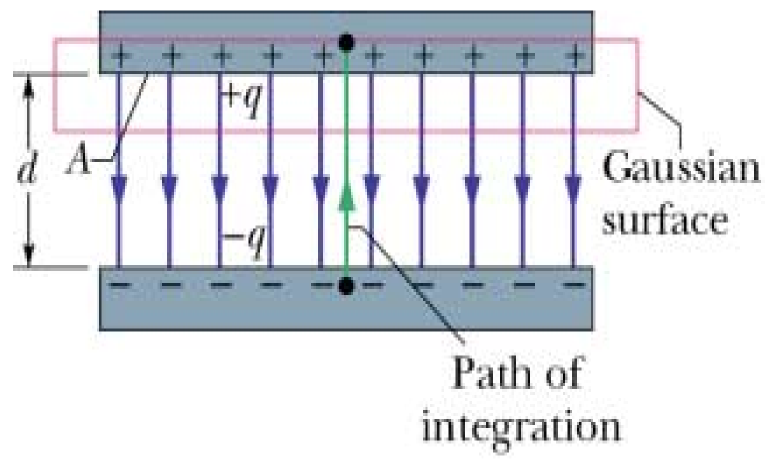
Capacitance (Fig. 26-5)

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{s}$$

- 2) For *V* choose path that follows the *E* field line from – plate to + plate then *E* and *ds* are in opposite directions

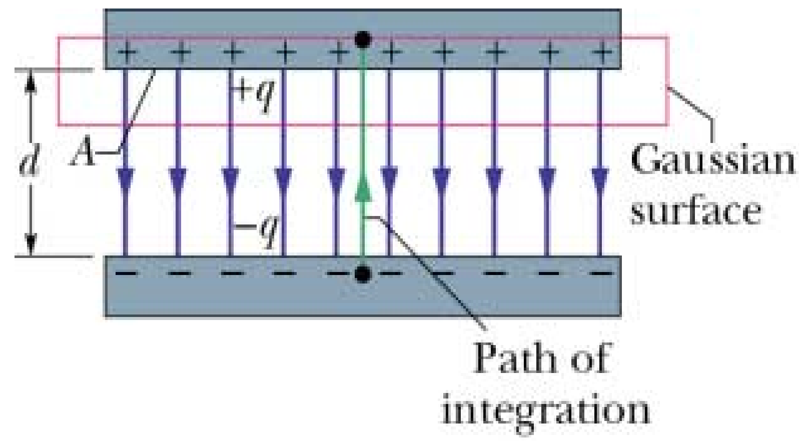
$$\vec{E} \bullet d\vec{s} = -Eds$$

$$V = V_f - V_i = \int_-^+ Eds$$



Capacitance (Fig. 26-5)

- Find *C* for parallel plate capacitor separated by *d*
  - E is constant between plates



$$V = \int_{-}^{+} E ds = E \int_{0}^{d} ds = Ed$$

- *A* is area of plates

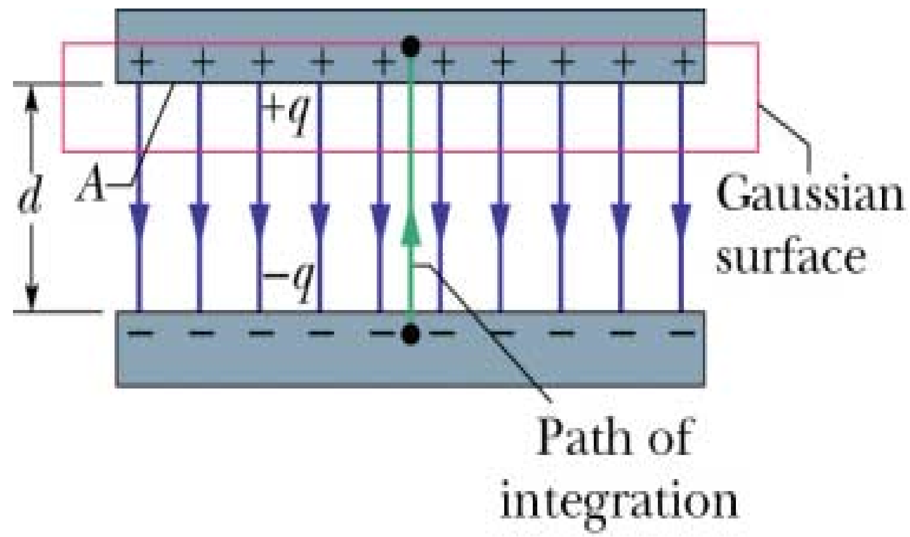
$$q = \varepsilon_0 EA$$

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed}$$

Capacitance (Fig. 26-5)

- Parallel-plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$



- Only depends on area *A* of plates and separation *d*
- *C* increases if we increase *A* or decrease *d*

Derived in Section 26-3

- Cylindrical capacitor

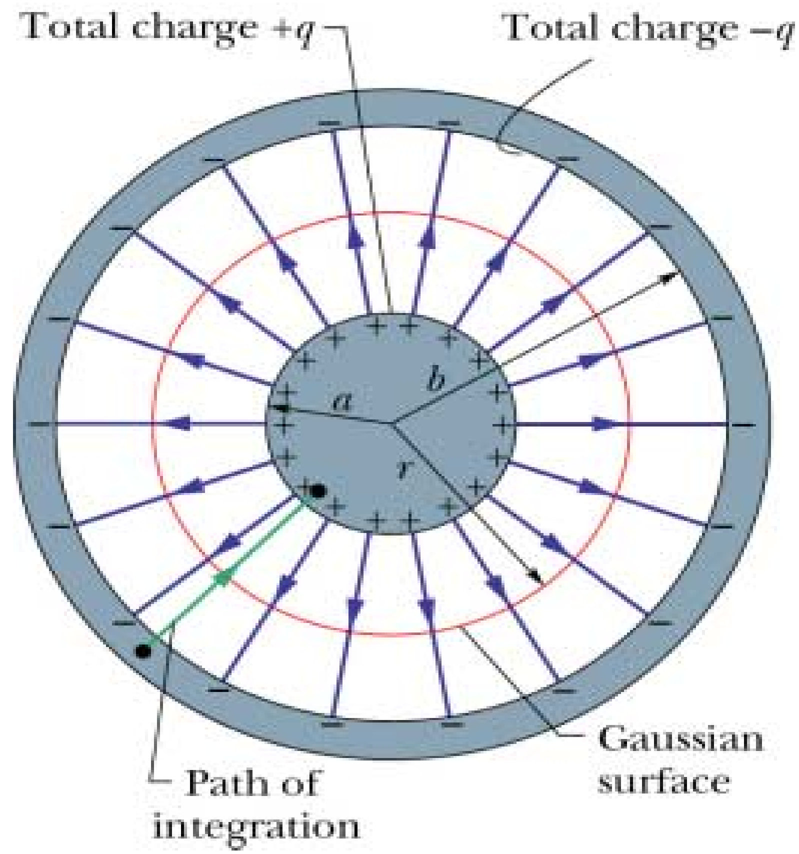
$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

- Spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

- Isolated Sphere

$$C = 4\pi\varepsilon_0 r$$



Capacitance (Checkpoint #2)

- For capacitors charged by same battery, does q stored by capacitor increase, decrease or remain same if plate separation of parallel-plate capacitor is increased.

$$q = CV$$

- All capacitors have same potential V from battery and so q increases (decreases) with C

Capacitance

- If plate separation (*d*) of parallel plate capacitor is increased,

$$C = \frac{\varepsilon_0 A}{d}$$

- *d* increases so *C* decreases

- *C* decreases so *q* decreases

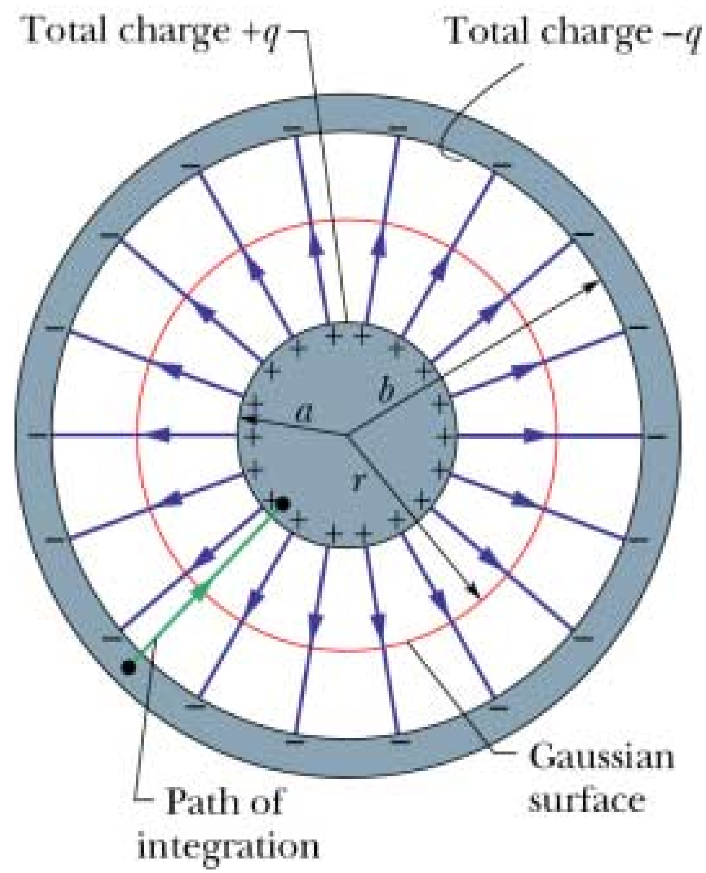
$$q = CV$$

Capacitance (Fig. 26-6)

- Spherical capacitor
- Gaussian sphere between shells, Gauss' law gives

$$q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$$

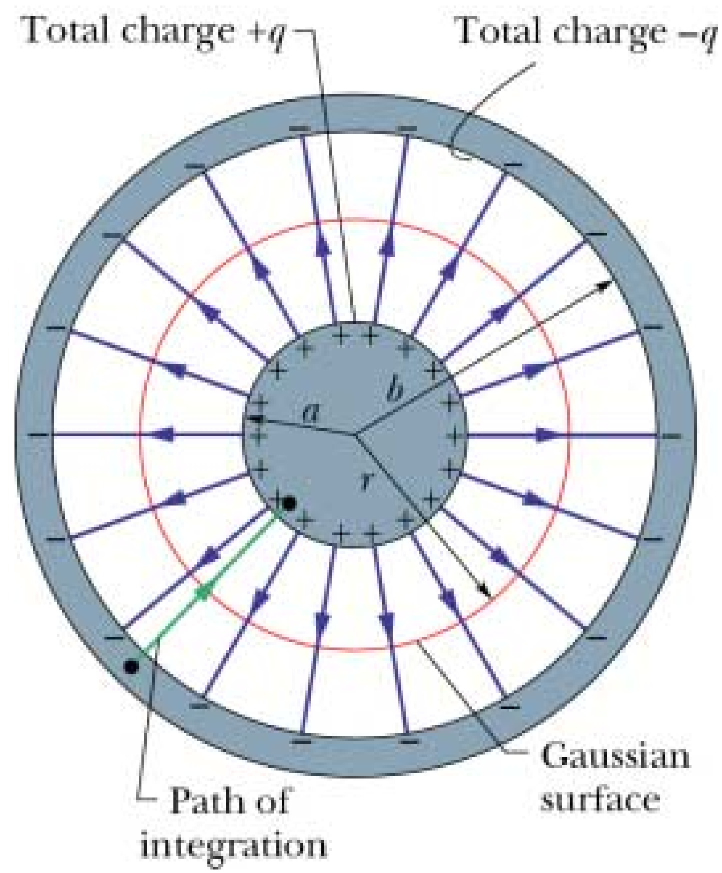


Capacitance (Fig. 26-6)

- Substitute *E* into equation for *V* and replace *ds* with radial *dr*
  - Integrate from – to + plate inward so

$$ds = -dr$$

$$V = \int_{-}^{+} E\,ds = -kq\int_{b}^{a} \frac{dr}{r^2}$$



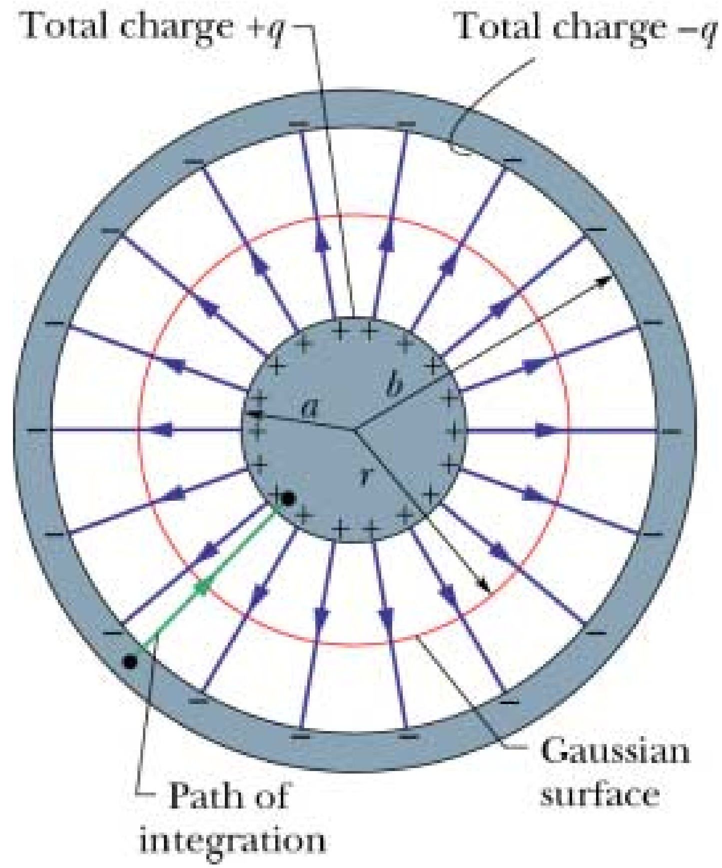
Capacitance (Fig. 26-6)

- Solve for *V*

$$V = -kq\int_b^a \frac{dr}{r^2} = kq\left(\frac{1}{a} - \frac{1}{b}\right)$$

- Substitute into

$$C = \frac{q}{V} = \frac{q}{qk\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{k}\frac{ab}{b-a}$$



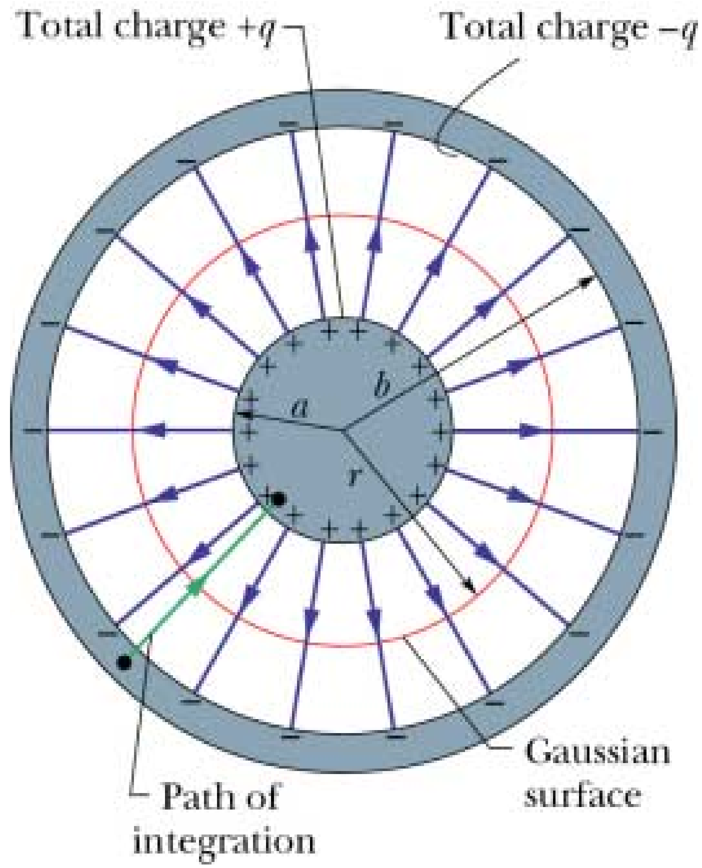
Capacitance (Fig. 26-6)

- For spherical capacitor

$$C = \frac{ab}{k(b-a)}$$

- Rewrite

$$C = \frac{a}{k\left(1 - \frac{a}{b}\right)}$$



Capacitance (Fig. 26-6)

- Capacitance of isolated sphere of radius *R*
- Outer shell moves to ∞ then $b \to \infty$ and let radius $a = R$

$$C = 4\pi\varepsilon_0 R$$