September 3rd

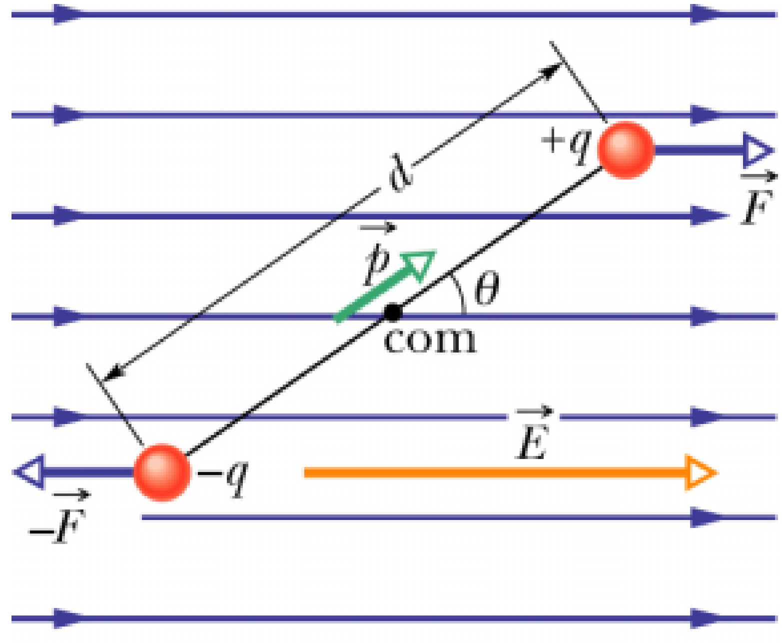
Chapters 23 & 24

Electric Dipole

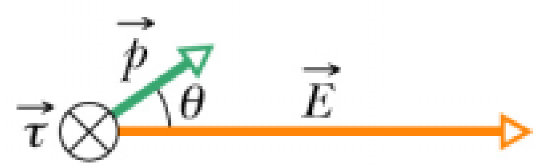
$$\vec{\tau} = \vec{p} \times \vec{E}$$

- Torque acting on a dipole tends to rotate *p* into the direction of *E*
- Work done by *E* field on dipole when rotated

$$W = \int \tau d\theta$$



(a)



(b)

Electric Dipole

- Potential energy, *U*, related to work, *W* by

$$U = -W = -\int \tau d\theta$$

- Potential energy related to torque

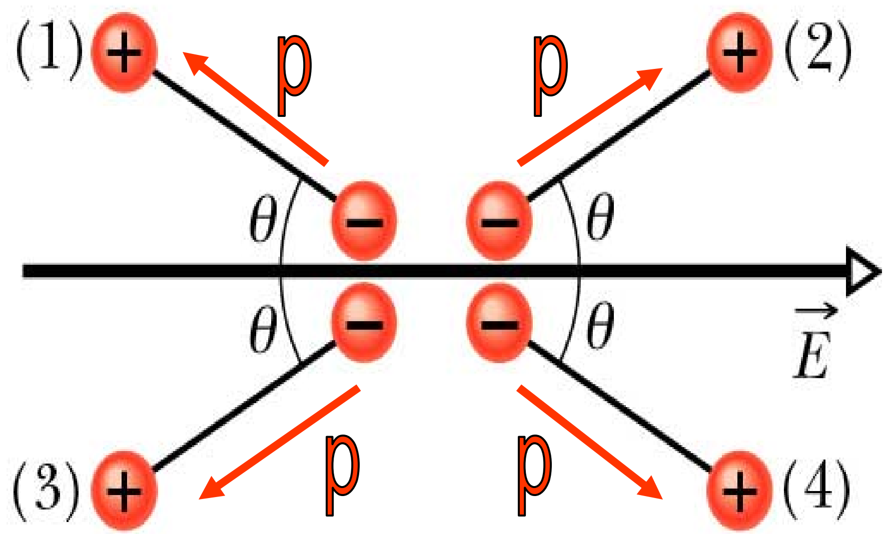
$$U = -\vec{p} \bullet \vec{E} = -pE\cos\theta$$

- *U* related to the orientation of dipole in *E* field
  - Least when *p* and *E* are in same directions
  - Greatest when *p* and *E* are in opposite directions

Checkpoint #5

- Rank a) magnitude of torque and b) *U*, greatest to least

$$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$$



- Magnitudes are same
- a) All tie

$$U = -\vec{p} \bullet \vec{E} = -pE\cos\theta$$

- *U* greatest at θ = 180
- b) 1 & 3 tie, then 2 &4

Charge distributions

- Calculate *E* field from a continuous line or region of charge - Use calculus and a charge density

- Linear charge density $$\lambda = Q / Length$$

- Surface charge density $$\sigma = Q / Area$$

- Volume charge density $$\rho = Q / Volume$$

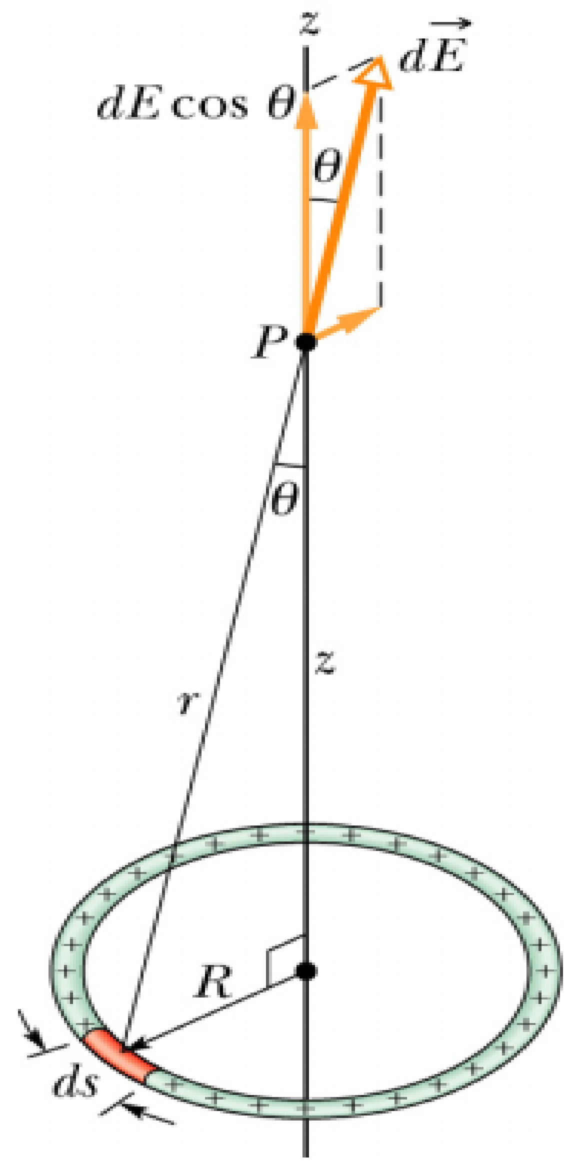
Charge distributions

- Ring of radius *R* and positive charge density λ

$$E = k\frac{q}{r^2}$$

- Divide ring into diff. elements of charge so

$$dq = \lambda ds$$



Electric Field of Charged Ring

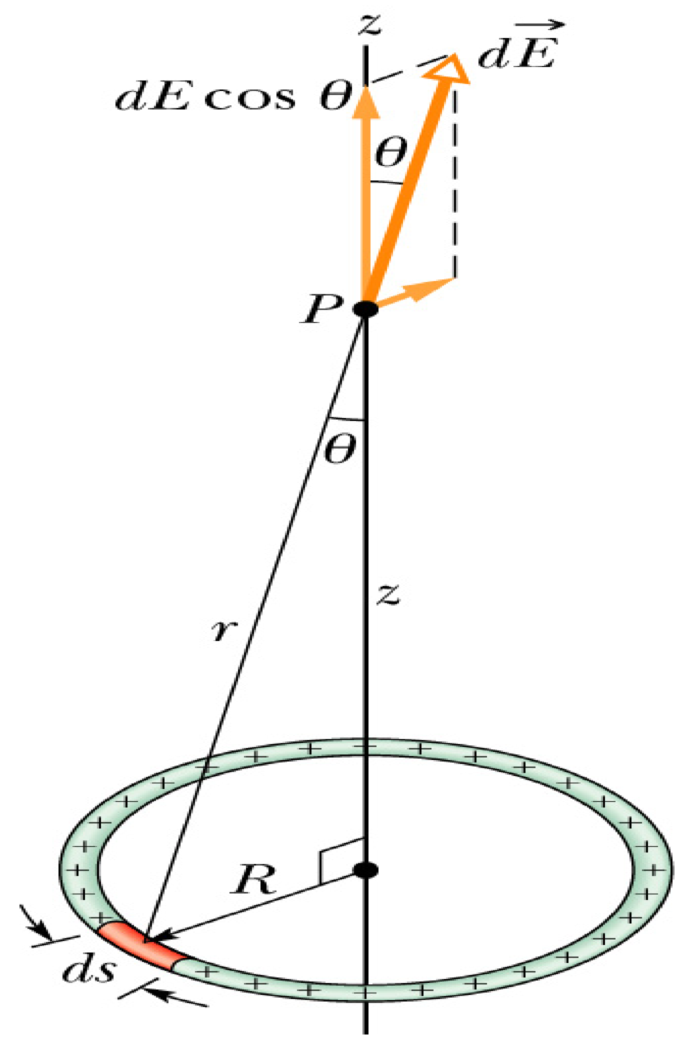
- Charge ring has E of

$$E = \frac{kqz}{\left(z^2 + R^2\right)^{3/2}}$$

- Check $z >> R$ then

$$E = \frac{kq}{z^2}$$

- From far away ring looks like point charge



Electric Field of Charged Disk

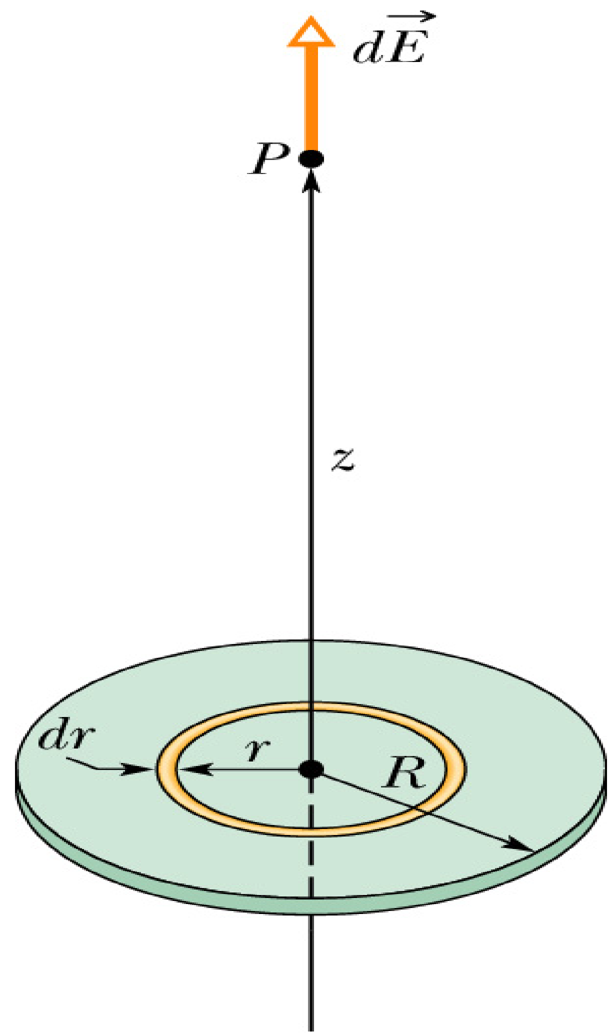
- Charge disk of radius *R*

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- Let R→∞ then get

$$E = \frac{\sigma}{2\varepsilon_0}$$

- Acts as infinite sheet of a non-conductor with uniform charge



Gauss' Law

- Easier way to calculate *E* fields –

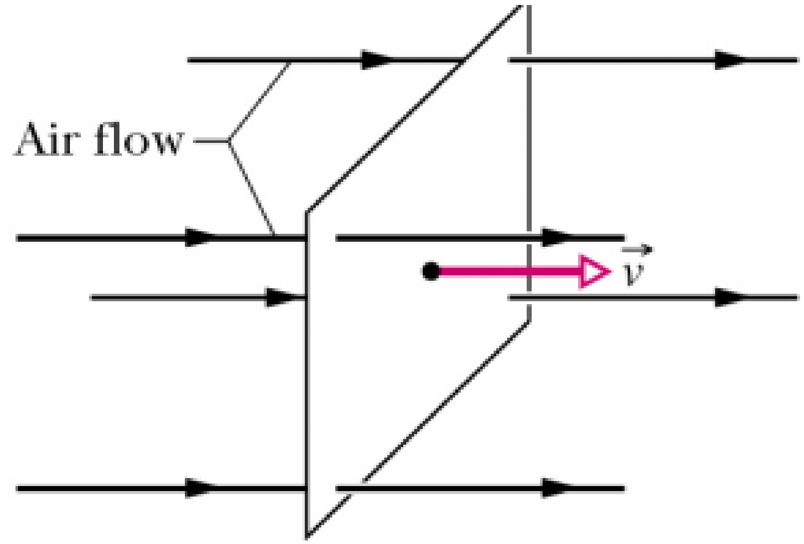
  Gauss' Law

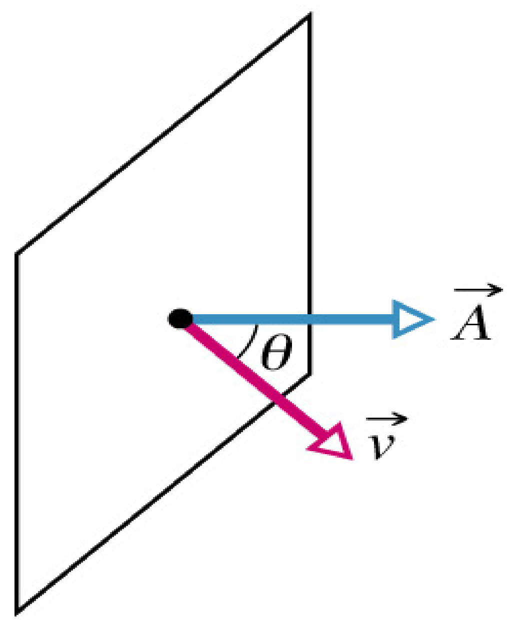
- Equivalent to Coulomb's law
- Use in symmetrical situations
- Gaussian surfaces – hypothetical closed surface

Flux

- Flux, Φ, is rate of flow through an area
- Create area vector, $\vec{A}$
  - magnitude is A,
  - direction is normal (⊥) to area
- Flux of a velocity field through an area
- Relate velocity and area by

$$\Phi = (v\cos\theta)A = \vec{v} \bullet \vec{A}$$



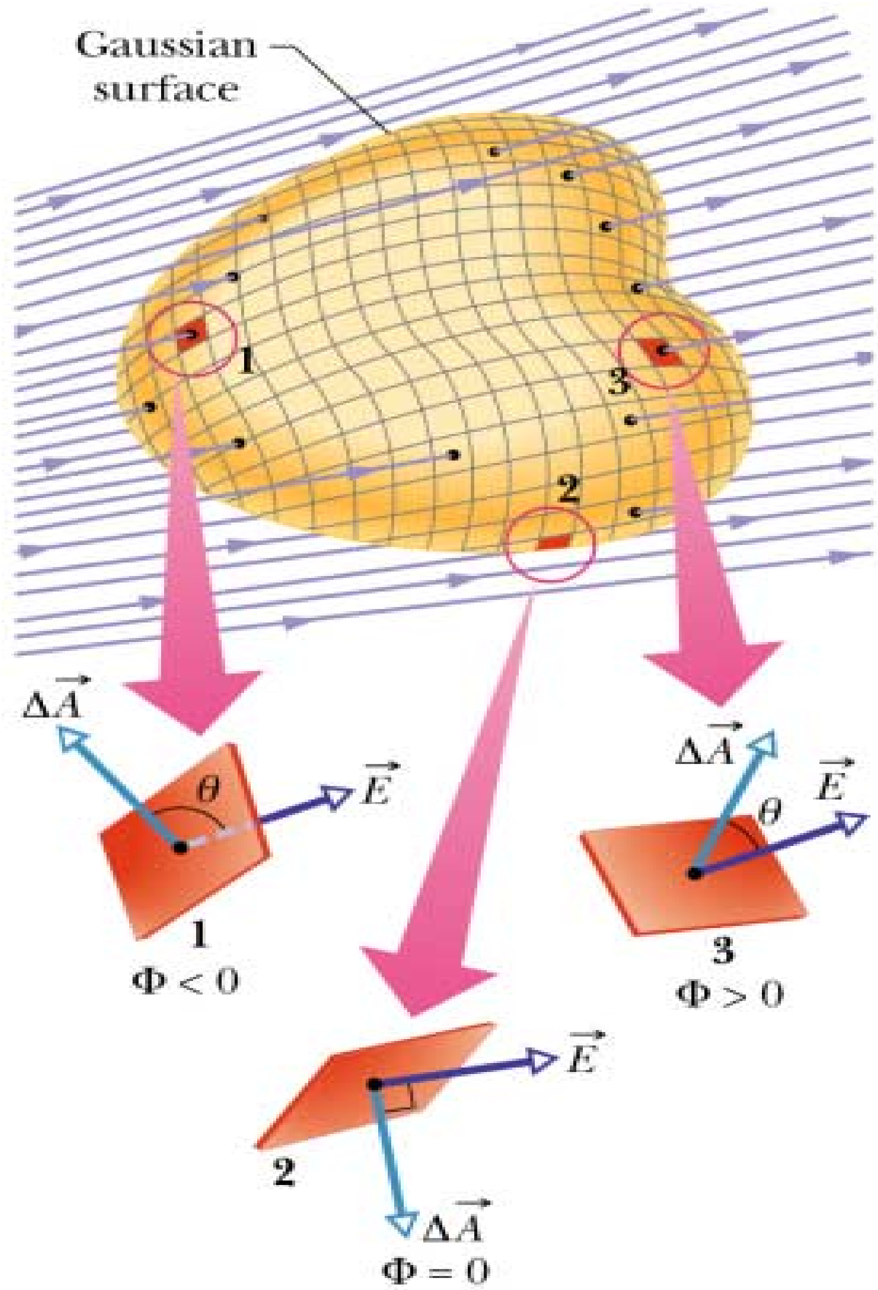


Flux

- Gaussian surface in non-uniform *E* field
- Divide Gaussian surface into squares of area ΔA

- Flux of *E* field is

$$\Phi = \sum \vec{E} \bullet \Delta\vec{A}$$



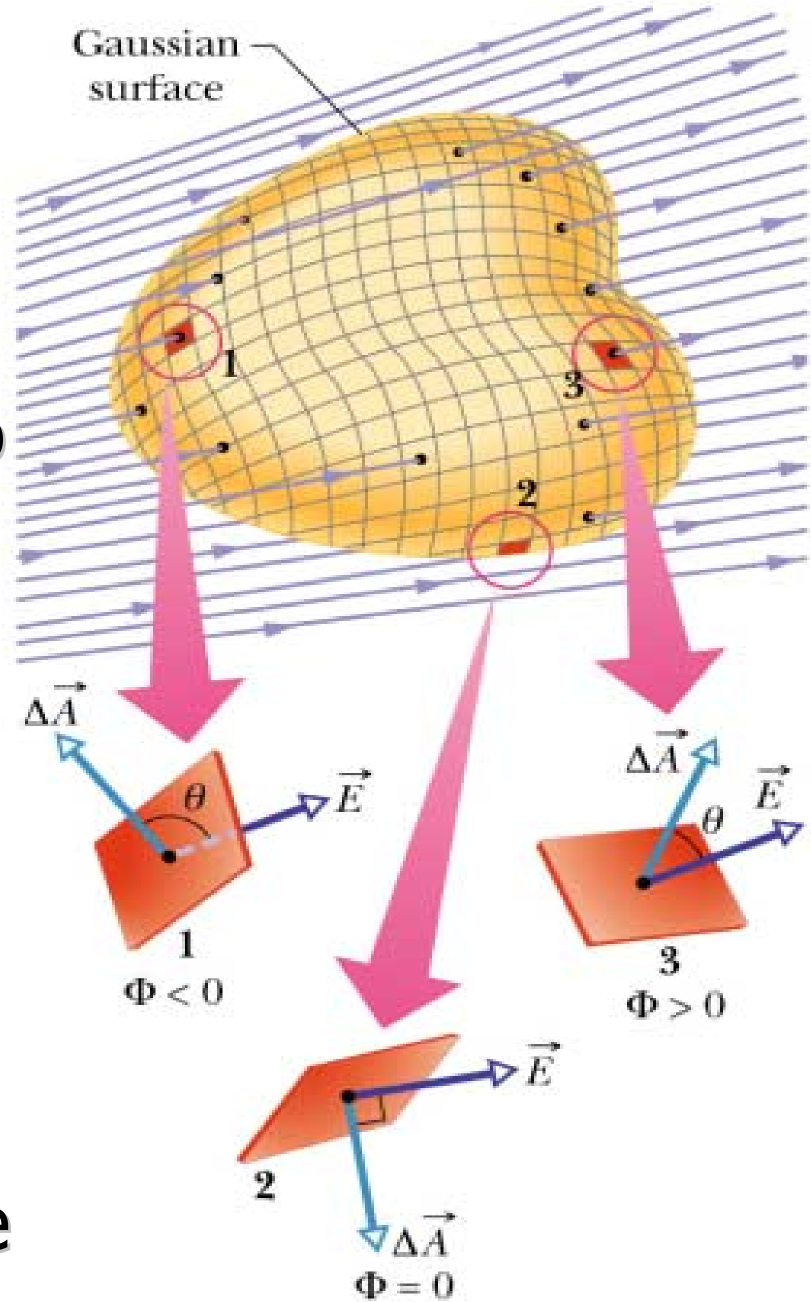
Flux

$$\Phi = \sum \vec{E} \bullet \Delta\vec{A}$$

- Let ΔA become small so flux becomes integral over Gaussian surface

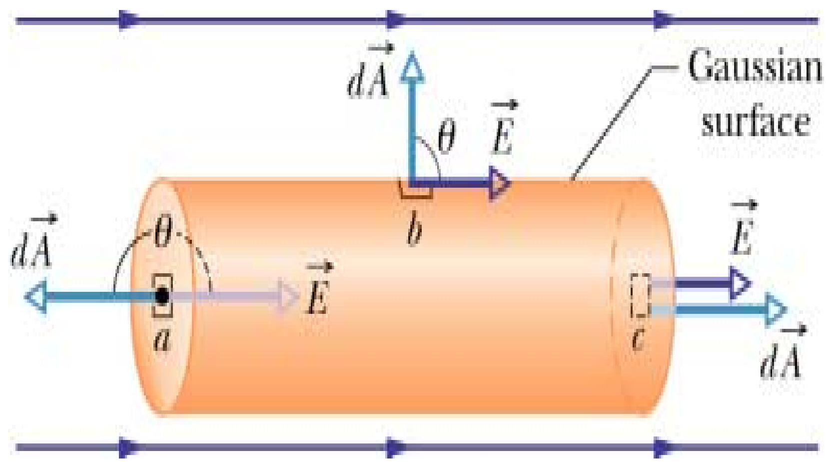
$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

- Flux is proportional to net # of E field lines passing through surface



Flux

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$
$$= \oint E \cos\theta dA$$

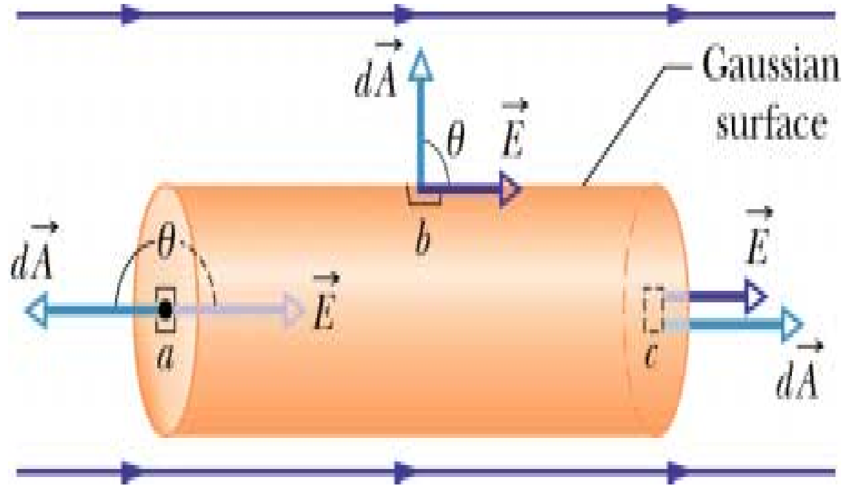


- If E field points inward at surface, Φ is –
- If E field points outward at surface, Φ is +
- If E field is along surface, Φ is zero
- If equal # of field lines enter as leave closed surface the net Φ is zero

Flux

- Calculate flux of uniform *E* through cylinder

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$



- 3 surfaces - a, b, and c

$$\Phi = \int_a \vec{E} \bullet d\vec{A} + \int_b \vec{E} \bullet d\vec{A} + \int_c \vec{E} \bullet d\vec{A}$$

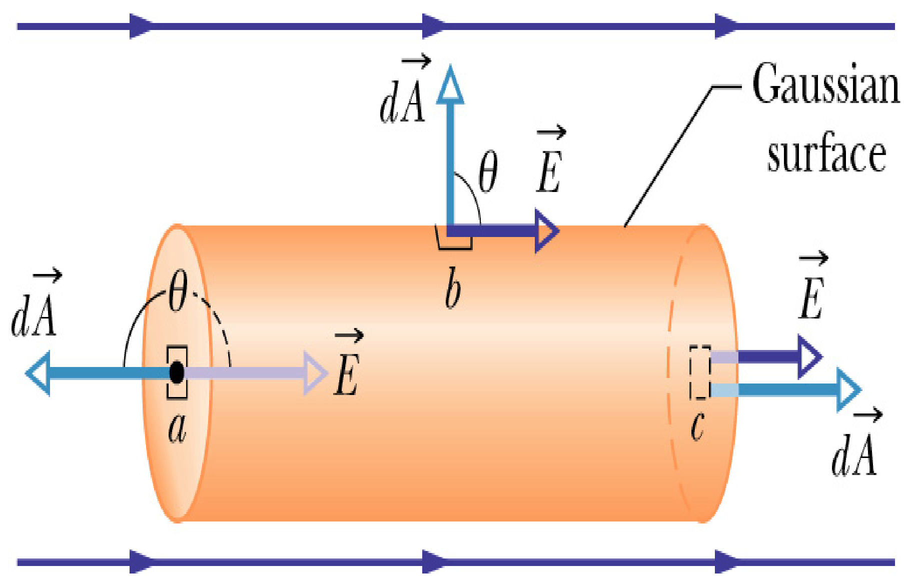
- Flux is $\Phi = 0$

Flux

$$\int_a E(\cos 180)dA = -EA$$

$$\int_b E(\cos 90)dA = 0$$

$$\int_c E(\cos 0)dA = EA$$



$$\Phi = \oint \vec{E} \bullet d\vec{A} = -EA + 0 + EA = 0$$

Gauss' Law

- Gauss' Law

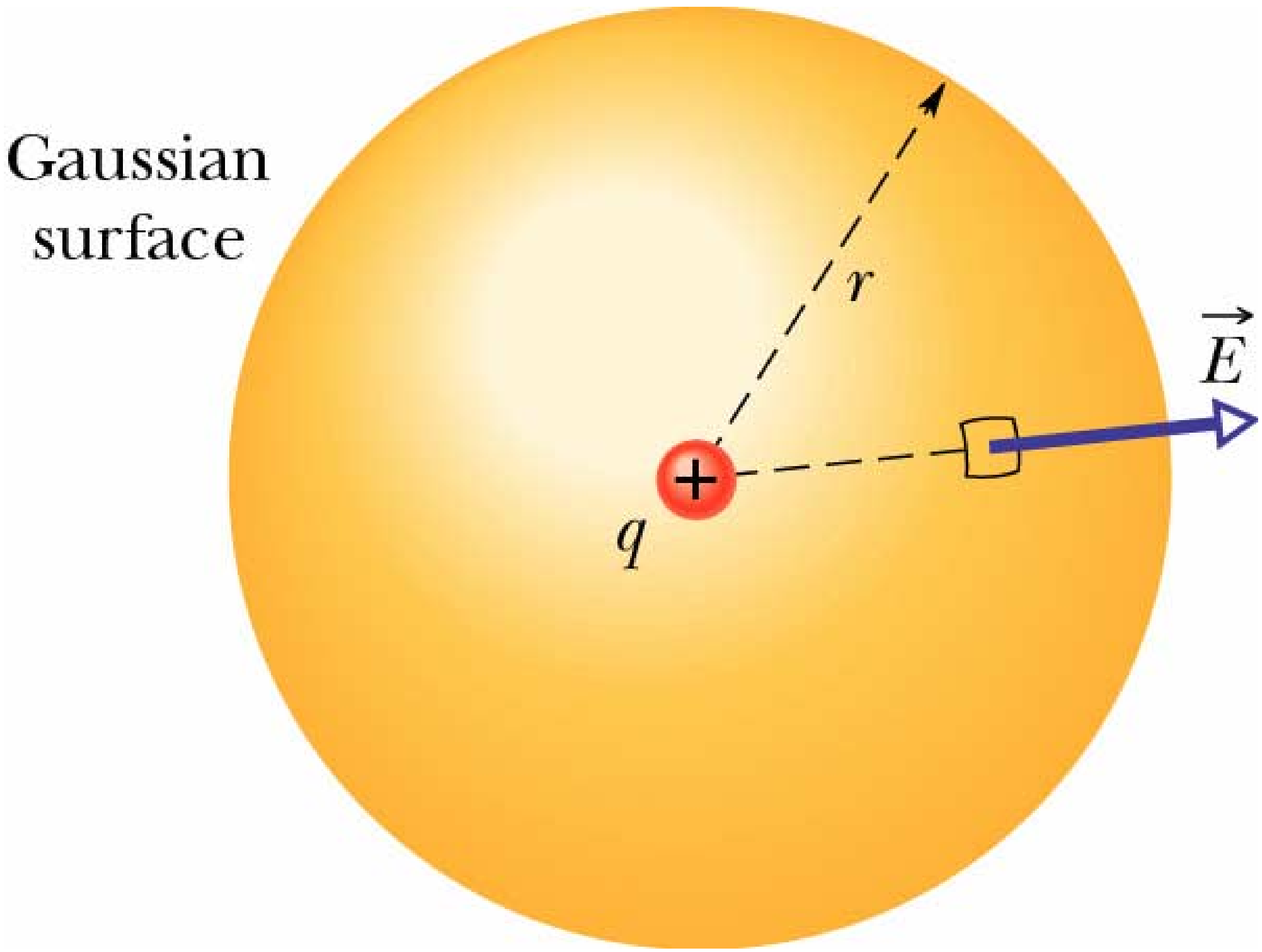
$$\varepsilon_0 \Phi = q_{enc}$$

- Also write it as

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

- Net charge q_{enc} is sum of all enclosed charges and may be +, -, or zero

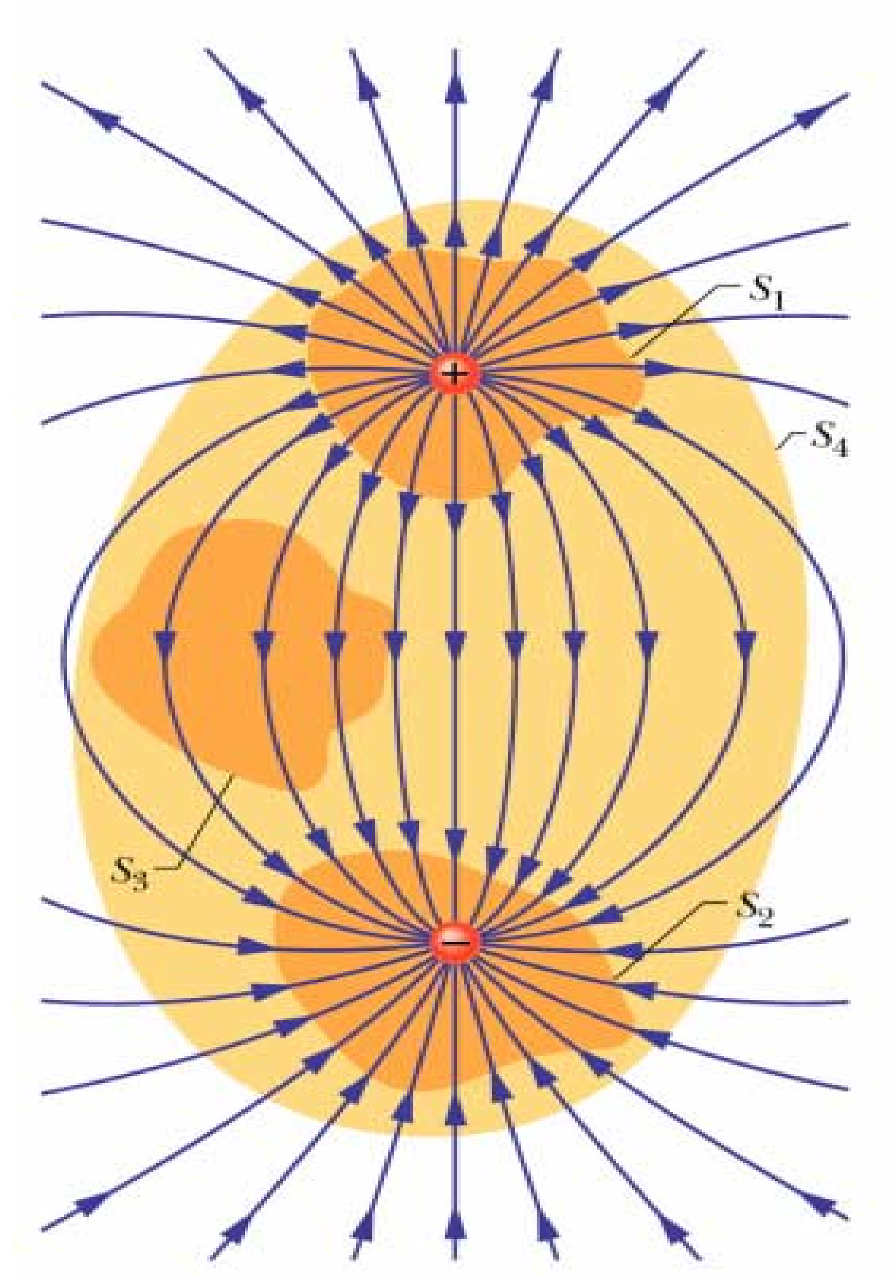
Gauss' Law = Coulomb's Law



- What is the flux for each surface?

$$\varepsilon_0 \Phi = q_{enc}$$

- net S_1 - q_{enc} is +
  Φ is outward and +
- S_2 - q_{enc} is -
  Φ is inward and –
- S_3 – q_{enc} is 0
  Φ is 0
- S_4 – total q_{enc} is 0
  Φ is 0



Gauss' Law

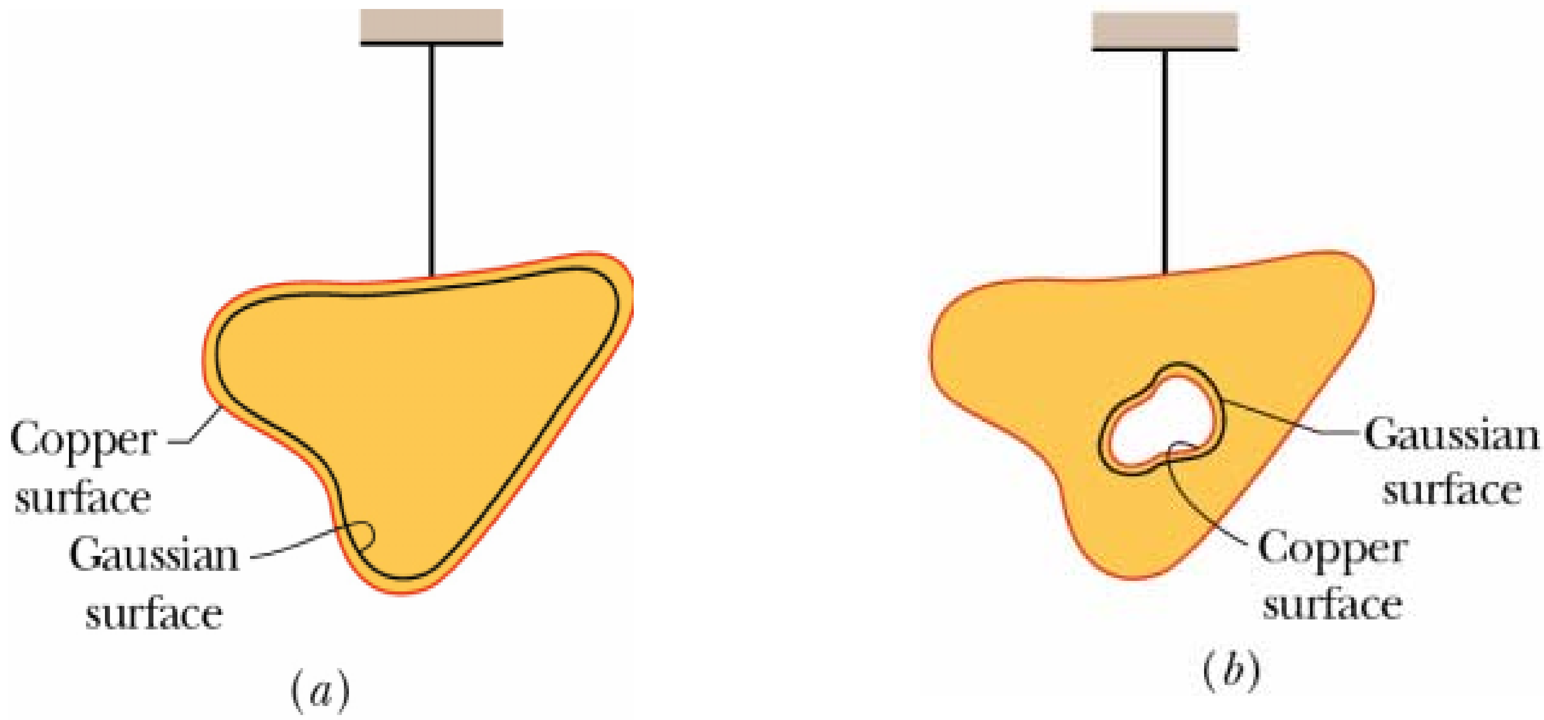
- What happens to the flux if I had a charge, Q, outside a Gaussian surface?

$$\varepsilon_0 \Phi = q_{enc}$$

- *Nothing* - q_{enc} does not change
- E field does change but charge outside the surface contributes zero net Φ through surface

Conductors

- Theorem for charged isolated conductor with a net charge Q
  - Charge is always on the surface
  - No charge inside the conductor
  - $E = 0$ inside the conductor
- At the surface of a charged conductor the E field is $\perp$ to the surface



Copper
surface
Gaussian
surface
(a)
Gaussian
surface
Copper
surface
(b)

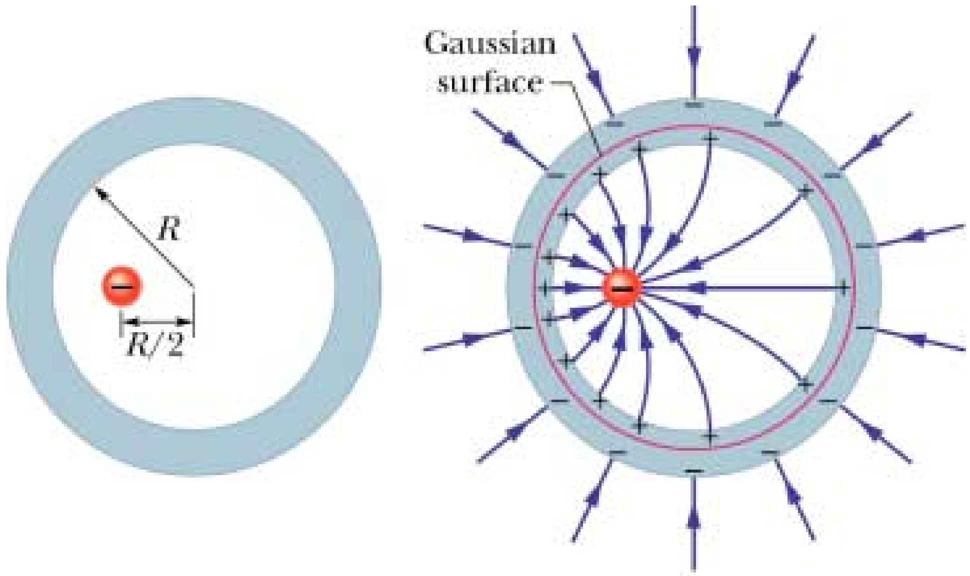
Conductors

- Usually charge on conductor is not uniform (except for a sphere)

- Charge will accumulate more at sharp points on an irregularly shaped conductor

Example 1a

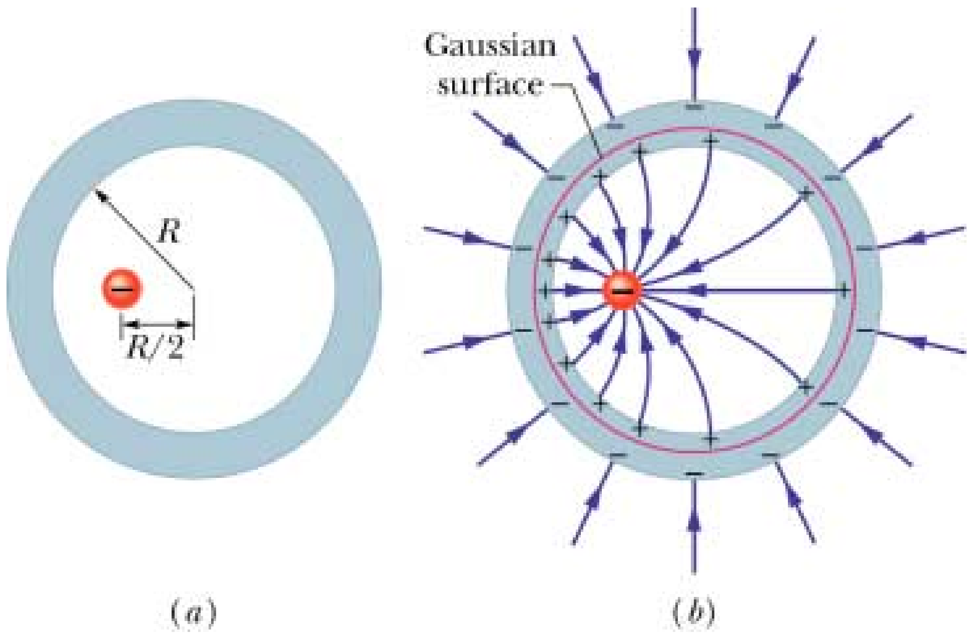
- Have point charge of $-5.0\mu C$ not centered inside an electrically neutral spherical metal shell
- What are the induced charges on the inner and outer surfaces of the shell?



(a) (b)

Example 1b

- E=0 inside conductor
- Thus Φ=0 for Gaussian surface
- So net charge enclosed must be 0
- Induced charge of +5.0μC lies on inner wall of sphere
- Shell is neutral so charge of -5.0μC on outer wall



Example 1c

- Are the charges on the sphere surfaces uniform?
- Charge is off-center so more + charge collects on inner wall nearest point charge
- Outer wall the charge is uniform
  - No E inside shell to affect distribution
  - Spherical shape