September 30th

Circuits - Chapter 28
Review (Fig. 28-3)

- **Kirchhoff’s loop rule** – in traversing a circuit loop the sum of the changes in $V$ is zero, $\Delta V = 0$

- **Resistance rule** – Move through resistor in direction of current $V = -iR$ (+ to -; higher to lower), in opposite direction $V = +iR$ (- to +; up the hill).

- **Emf rule** – Move through emf device $V = +E$ going – to +, in opposite direction $V = -E$. 
Circuits (Fig. 28-5a)

- **Resistors in series** (one path from b to a)
- Have identical currents, $i$, through them
- Use Kirchhoff’s loop rule

\[ E - iR_1 - iR_2 - iR_3 = 0 \]

\[ i = \frac{E}{R_1 + R_2 + R_3} \]
Circuits (Figs. 28-5a, 28-5b)

- **Resistors in series**
- The 3 resistors act the same as an equivalent resistor $R_{eq}$.

\[
i = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{R_{eq}}
\]

\[
R_{eq} = R_1 + R_2 + R_3
\]
Circuits (Figs. 28-5a, 28-5b)

- Resistors in series
- Resistors have identical currents, $i$
- Sum of $V$s across resistors = applied $V$
- $R_{eq}$ is sum of all resistors

$$R_{eq} = \sum_{j=1}^{n} R_j$$
Circuits - Checkpoint #2 (Fig. 28-5a)

- If $R_1 > R_2 > R_3$, rank greatest first
- A) current through resistors
  - $i$ is same for all, tie

\[ V = iR \]

- B) $V$ across them
  - $R_1, R_2, R_3$
Circuits (Fig. 28-8a)

- Resistors in parallel
- Have same V across them
- Arbitrarily choose direction for currents in each branch
- Write down current relation for each resistor

\[ i_1 = \frac{V}{R_1} \quad i_2 = \frac{V}{R_2} \quad i_3 = \frac{V}{R_3} \]
Circuits (Fig. 28-8a)

- Resistors in parallel (more than one path from a to b)
- Apply Kirchhoff’s junction rule at point a
- Substitute current values

\[ i = i_1 + i_2 + i_3 \]

\[ i = V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \]
Circuits (Figs. 28-8a, 28-8b)

- Resistors in parallel
- Replace 3 resistors with equivalent resistor, $R_{eq}$

\[
\frac{1}{R_{eq}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)
\]
Circuits - Checkpoint #4

- Battery with potential $V$ supplies current $i$ to 2 identical resistors

- What is $V$ across and $i$ through either of the resistors if they are connected in

  - A) Series – What is constant?
    - $i$ is same, $V_1 = V/2$

  - B) Parallel – What is constant?
    - $V$ is same, $i_1 = i/2$

$$V = iR$$
Circuits

- **Resistors**
  - Series
    
    $R_{eq} = \sum_{j=1}^{n} R_j$
    
- Parallel
  
    $\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$

- **Capacitors**
  - Series
    
    $\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$
    
- Parallel
  
    $C_{eq} = \sum_{j=1}^{n} C_j$
How to Analyze Complex Circuits

- **Kirchhoff’s junction rule (or current law) –**
  - From conservation of charge
  - Sum of currents entering a junction is equal to sum of currents leaving that junction

- **Kirchhoff’s loop rule (or voltage law) –**
  - From conservation of energy
  - Sum of changes in potential going around a complete circuit loop equals zero
Kirchhoff’s Rule #1: Circuits (Fig. 28-8a)

- Arbitrarily label currents, using different subscript for each branch
- Using conservation of charge at each junction

\[ i_{in} = i_{out} \]

- At point d
  \[ i_1 + i_3 = i_2 \]

- At point b
  \[ i_1 + i_3 = i_2 \]

- At point a
  \[ i_1 = i_1 \]

- At point c
  \[ i_2 = i_2 \]
Circuits (Figs. 28-6a, 28-6b)

- What is $i$ of the circuit?

- Use Kirchhoff’s loop rule
Clockwise from point a gives

\[ -\frac{\mathcal{E}_2}{2} - ir_2 - iR - ir_1 + \frac{\mathcal{E}_1}{1} = 0 \]

Counterclockwise from point a gives

\[ -\frac{\mathcal{E}_1}{1} + ir_1 + iR + ir_2 + \frac{\mathcal{E}_2}{2} = 0 \]
Circuits (Fig. 28-6a)

- Solve for \( i \)

\[
-\frac{v_1}{1} + ir_1 + iR + ir_2 + \frac{v_2}{2} = 0
\]

\[
i = \frac{1 - 2}{R + r_1 + r_2}
\]

\( v_1 = 4.4V \)  \( v_2 = 2.1V \)

\( r_1 = 2.3\Omega \)  \( r_2 = 1.8\Omega \)

\( R = 5.5\Omega \)

\( i = 0.2396A \approx 240mA \)
A real battery has $E = 12\, \text{V}$ and $r = 2\, \Omega$. Is the voltage $V$ across the terminals greater than, less than or equal to 12V if the current in the battery is

- A) from $-$ to $+$ terminal: LESS THAN
  \[ V_a + E - ir = V_b \]

- B) from $+$ to $-$: GREATER THAN
  \[ V_a + E + ir = V_b \]

- C) $i = 0$: EQUAL TO 12V