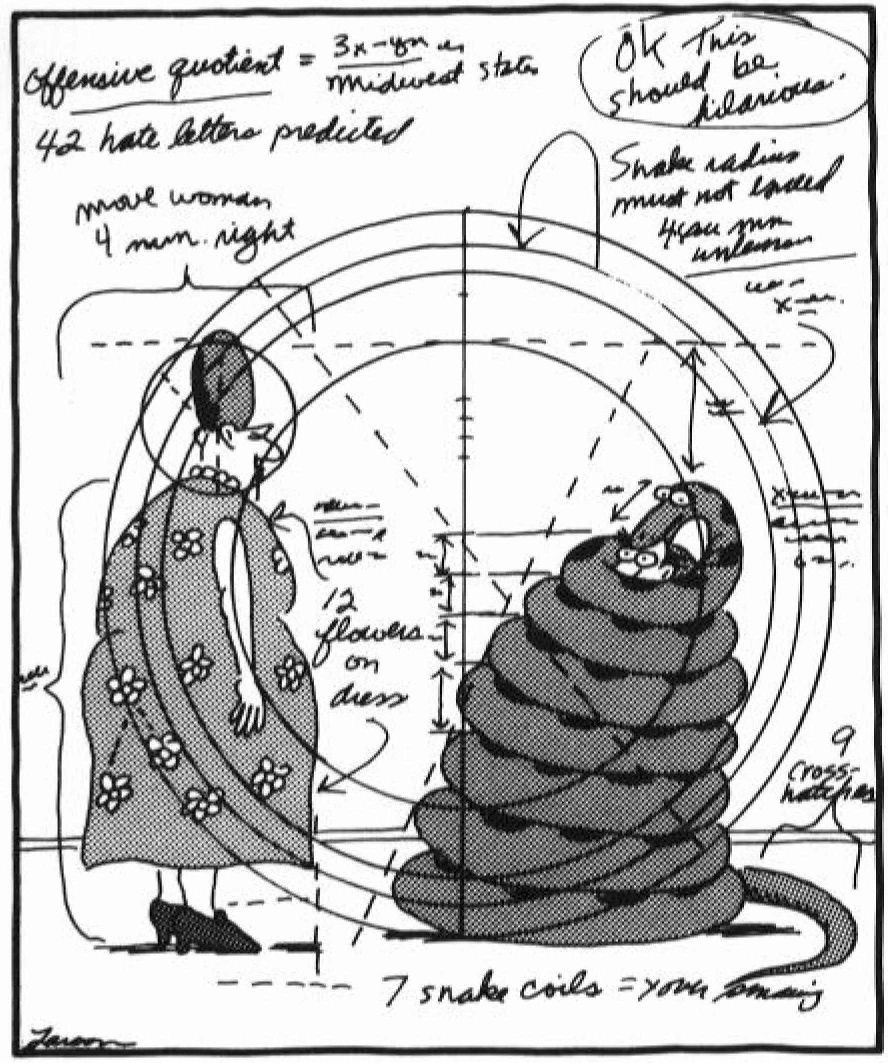
November 5th

Chapter 33
RLC Circuits



Revealing some of the mathematical computations every cartoonist must know.

What's up for the rest of the term

- (A9) C33 – RLC circuits
- (A10) C34 – Electromagnetic (EM) waves
- (A11) C35 – Optics and images with EM waves
- (A12) C36 – Interference of EM waves

  C37 – Diffraction of EM waves

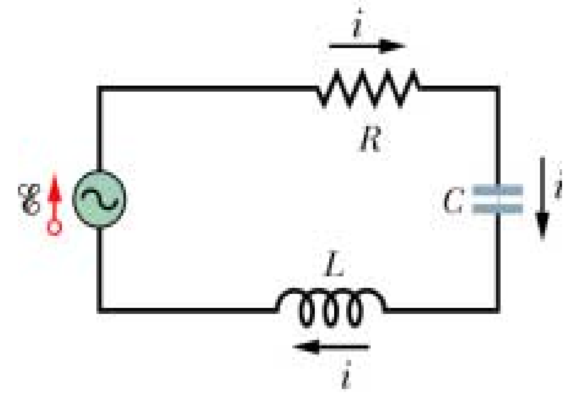
Summary of Forced Oscillations

Element	Reactance/ Resistance	Phase of Current	Phase angle ϕ	Amplitude Relation
Resistor	R	In phase	0°	$V_R = I_R R$
Capacitor	$X_C = 1/\omega_d C$	Leads v_C (ICE)	-90°	$V_C = I_C X_C$
Inductor	$X_L = \omega_d L$	Lags v_L (ELI)	+90°	$V_L = I_L X_L$

- ELI (positively) is the ICE man
  - Voltage or emf (E) before current (I) in an inductor (L)
  - Phase constant ϕ is positive for an inductor
  - Current (I) before voltage or emf (E) in capacitor (C)

RLC circuits

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$



- Current is largest when

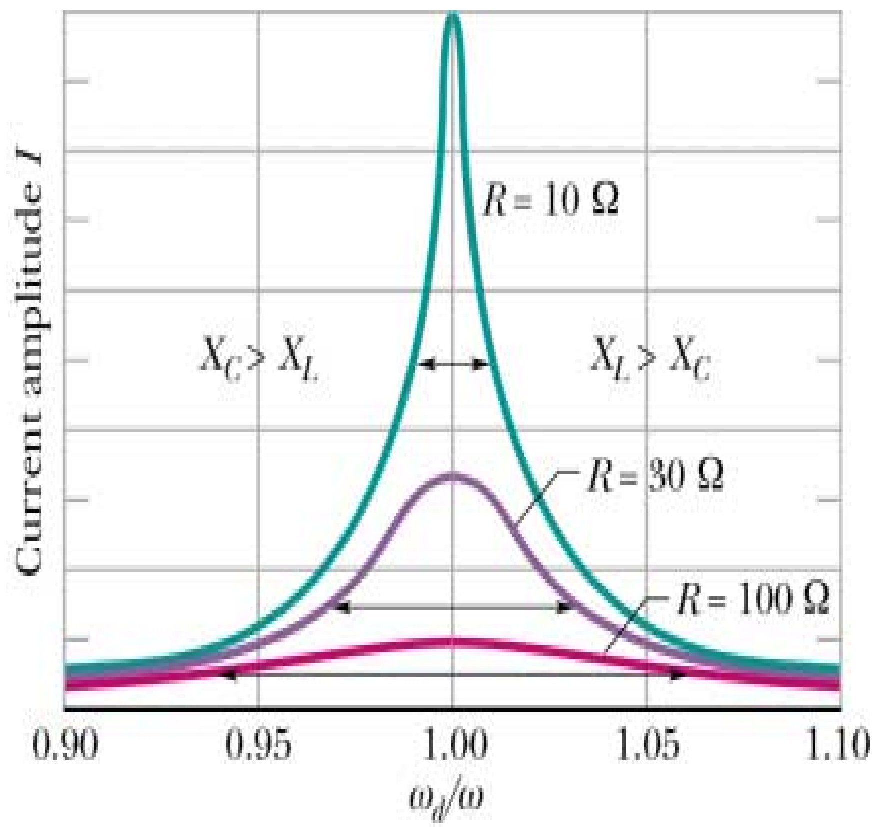
$$\omega_d L - \frac{1}{\omega_d C} = 0$$

- Or

$$\omega_d = \sqrt{\frac{1}{LC}} = \omega$$

ω is also called the resonance frequency (in the homework)

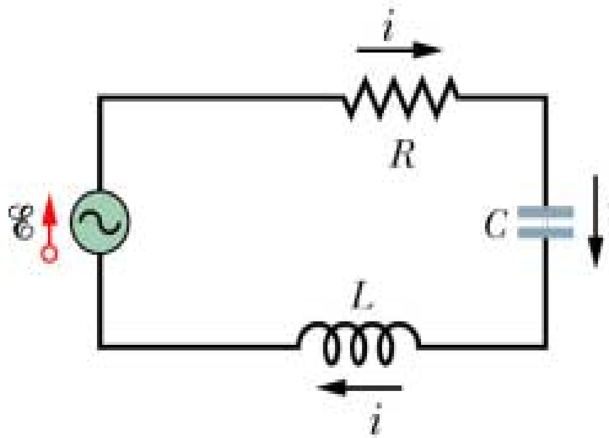
$$\omega = \omega_d = 2\pi f_d$$



AC circuits

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf

$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$



- Elements are in series so same current is driven through each

$$i = I \sin(\omega_d t - \phi)$$

- From the loop rule, at any time *t*, the sum of the voltages across the elements must equal the applied emf

$$\mathcal{E} = v_R + v_C + v_L$$

AC circuits - Equations

$$\mathcal{E}_m = IZ$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

- Define impedance, Z to be

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where

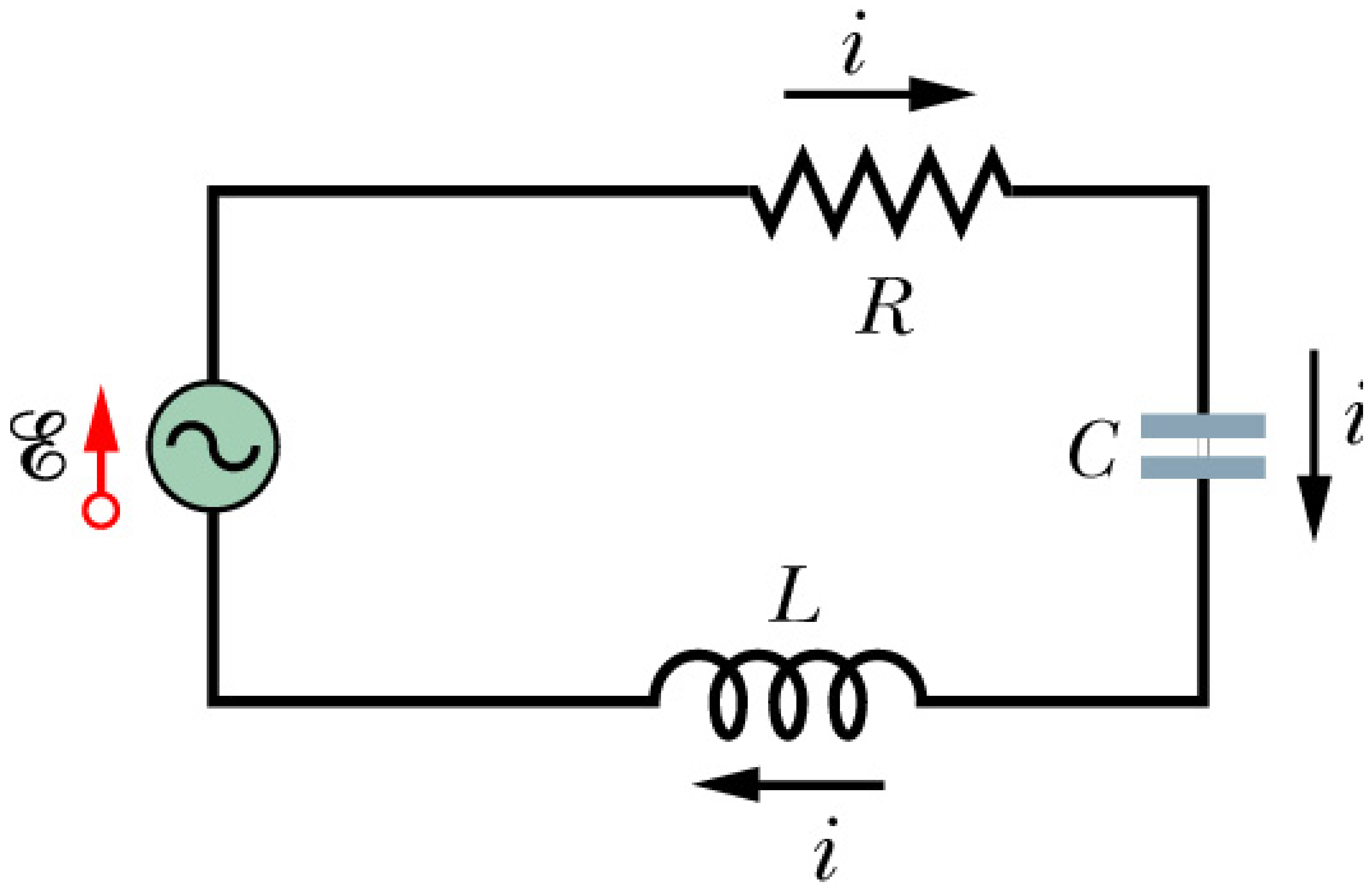
$$X_L = \omega_d L$$

$$X_C = \frac{1}{\omega_d C}$$

- Resonant frequency – natural freq = driving freq

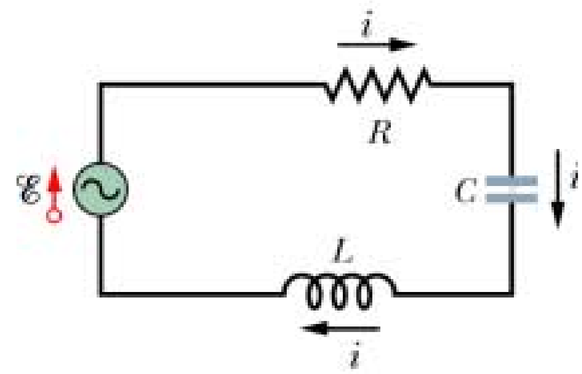
$$\omega = \omega_d = 2\pi f_d$$

AC circuit Examples

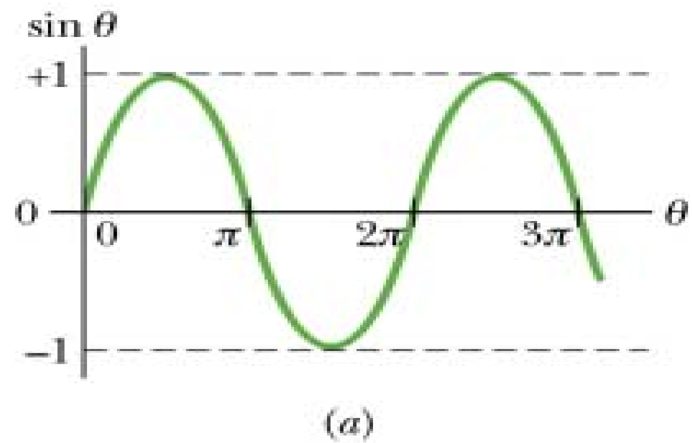


AC circuits

- Instantaneous rate which energy is dissipated (power) in a resistor is $P = i^2 R$

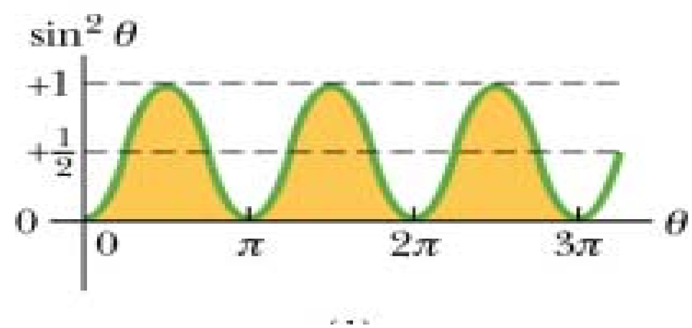


- But $i = I\sin(\omega_d t - \phi)$



$$P = I^2 R \sin^2(\omega_d t - \phi)$$

- Want average (rms) rate
  - Average over complete cycle T



$$\langle \sin^2\theta \rangle = 1/2$$

AC circuits

- For alternating current circuits define root-mean-square or rms values for *i*, *V* and emf

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}} \qquad \mathcal{E}_{rms} = \frac{\mathcal{E}}{\sqrt{2}}$$

- Ammeters, voltmeters - give rms values
- The average (rms) power dissipated by resistor in an ac circuit is

$$P_{rms} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R \qquad P_{rms} = I_{rms}^2 R$$

- (Called P_{ave} in the book)

AC circuits

- If ac circuit has only resistive load *Z=R* (e.g. at the resonance frequency)

$$P_{rms} = \mathcal{E}_{rms} I_{rms}$$

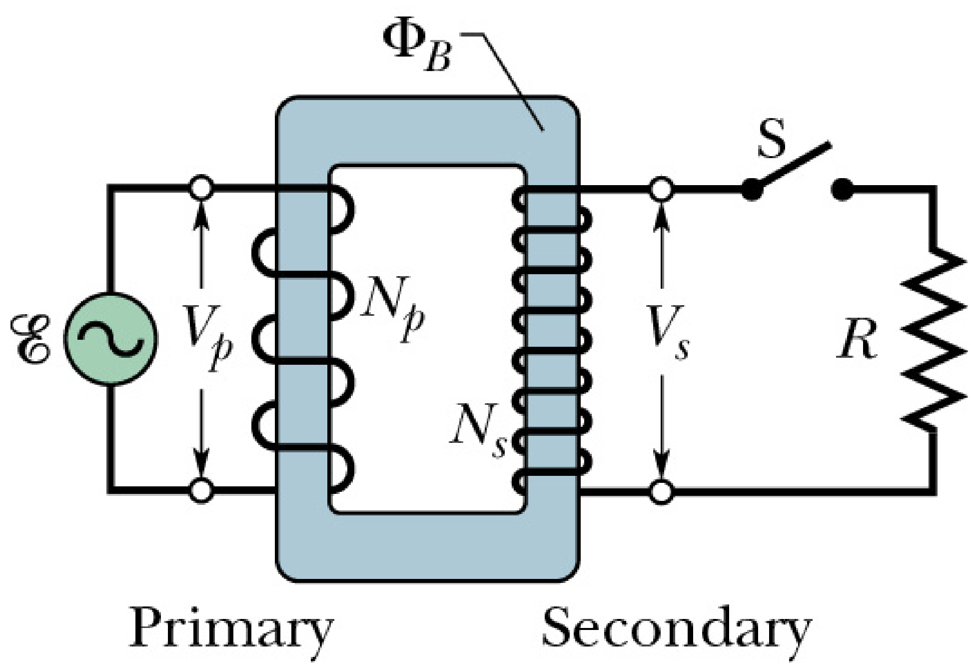
- Trade-off between current and voltage
  - For general use want low voltage
  - Means high current but

$$P_{rms} = I_{rms}^{2} R$$

- General energy transmission rule: Transmit at the highest possible voltage and the lowest possible current

AC circuits

- Transformer – device used to raise (for transmission) and lower (for use) the ac voltage in a circuit, keeping *iV* constant
  - Has 2 coils (primary and secondary) wound on same iron core with different #s of turns

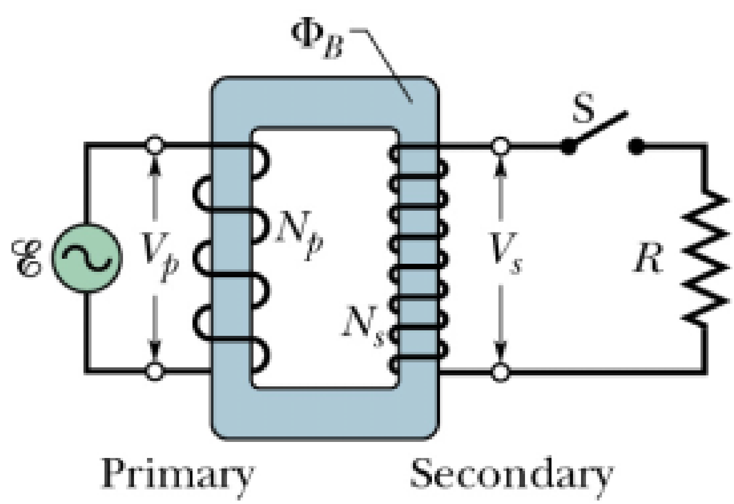


AC circuits

- Alternating primary current induces alternating magnetic flux in iron core
- Same core in both coils so induced flux also goes through the secondary coil
- Using Faraday's law

$$V_P = -N_P \frac{d\Phi_B}{dt}$$

$$V_S = -N_S \frac{d\Phi_B}{dt}$$



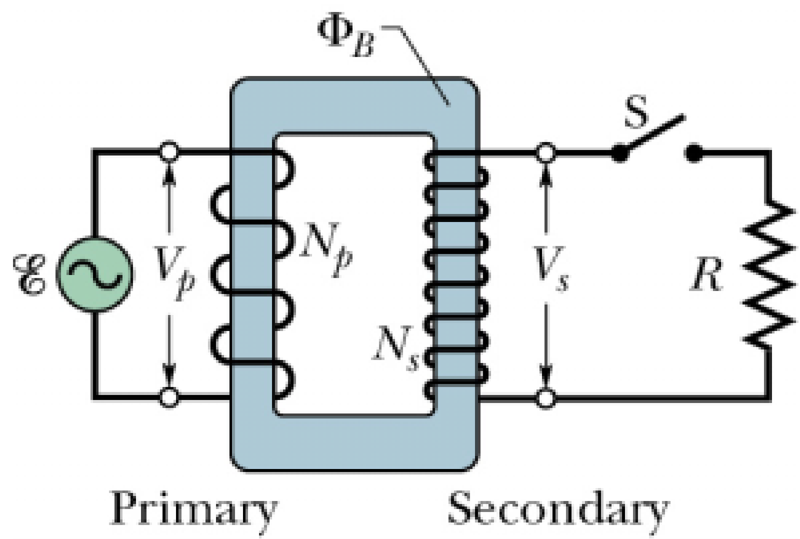
$$\frac{V_P}{N_P} = \frac{V_S}{N_S}$$

AC circuits

- Transformation of voltage is

$$V_S = V_P \frac{N_S}{N_P}$$

- If $N_S > N_P$ called a step-up transformer
- If $N_S < N_P$ called a step-down transformer

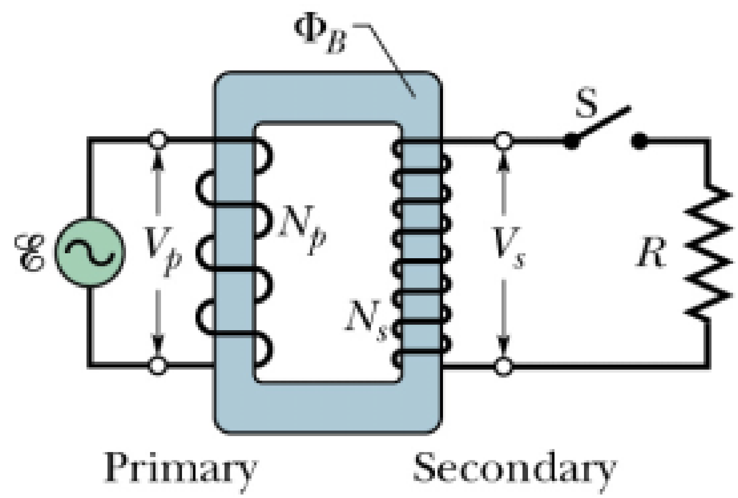


AC circuits

- Conservation of energy

$$I_P V_P = I_S V_S$$

$$I_S = I_P \frac{V_P}{V_S} = I_P \frac{N_P}{N_S}$$



AC circuits

- The current I_P appears in primary circuit due to R in secondary circuit.

$$I_P V_P = I_S V_S \qquad I_S = V_S / R$$

$$I_P = \frac{V_S}{R}\frac{V_S}{V_P} = \frac{1}{R}\frac{V_S^2}{V_P^2}V_P = \frac{1}{R}\left(\frac{N_S}{N_P}\right)^2 V_P$$

- Has for of $I_P = V_P/R_{eq}$ where

$$R_{eq} = \left(\frac{N_P}{N_S}\right)^2 R$$