Revealing some of the mathematical computations every cartoonist must know.
What’s up for the rest of the term

- (A9) C33 – RLC circuits
- (A10) C34 – Electromagnetic (EM) waves
- (A11) C35 – Optics and images with EM waves
- (A12) C36 – Interference of EM waves
- C37 – Diffraction of EM waves
### Summary of Forced Oscillations

<table>
<thead>
<tr>
<th>Element</th>
<th>Reactance/Resistance</th>
<th>Phase of Current</th>
<th>Phase angle $\phi$</th>
<th>Amplitude Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>In phase</td>
<td>$0^\circ$</td>
<td>$V_R = I_R R$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$X_C = \frac{1}{\omega_d C}$</td>
<td>Leads $v_C$ (ICE)</td>
<td>$-90^\circ$</td>
<td>$V_C = I_C X_C$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$X_L = \omega_d L$</td>
<td>Lags $v_L$ (ELI)</td>
<td>$+90^\circ$</td>
<td>$V_L = I_L X_L$</td>
</tr>
</tbody>
</table>

- **ELI (positively) is the ICE man**
  - Voltage or emf (E) before current (I) in an inductor (L)
  - Phase constant $\phi$ is positive for an inductor
  - Current (I) before voltage or emf (E) in capacitor (C)
RLC circuits

\[ I = \frac{E_m}{\sqrt{R^2 + \left( \omega_d L - \frac{1}{\omega_d C} \right)^2}} \]

- Current is largest when
  \[ \omega_d L - \frac{1}{\omega_d C} = 0 \]
- Or
  \[ \omega_d = \sqrt{\frac{1}{LC}} = \omega \]

\( \omega \) is also called the resonance frequency (in the homework)

\[ \omega = \omega_d = 2\pi f_d \]
AC circuits

- RLC circuit – resistor, capacitor and inductor in series
- Apply alternating emf

\[ \mathcal{E} = \mathcal{E}_m \sin \omega_d t \]

- Elements are in series so same current is driven through each
- From the loop rule, at any time \( t \), the sum of the voltages across the elements must equal the applied emf

\[ \mathcal{E} = v_R + v_C + v_L \]

\[ i = I \sin(\omega_d t - \phi) \]
**AC circuits - Equations**

- Define **impedance, Z** to be
  \[
  Z = \sqrt{R^2 + (X_L - X_C)^2}
  \]

- **Resonant frequency** —
  natural freq = driving freq
  \[
  \omega = \omega_d = 2\pi f_d
  \]

- \[
  \tan \phi = \frac{X_L - X_C}{R}
  \]

\[
X_L = \omega_d L
\]

\[
X_C = \frac{1}{\omega_d C}
\]
AC circuits

- Instantaneous rate which energy is dissipated (power) in a resistor is
  \[ P = i^2 R \]

- But
  \[ i = I \sin(\omega_d t - \phi) \]
  \[ P = I^2 R \sin^2(\omega_d t - \phi) \]

- Want average (rms) rate
  - Average over complete cycle T
  \[ \langle \sin^2 \theta \rangle = 1/2 \]
AC circuits

- For alternating current circuits define **root-mean-square or rms** values for $i$, $V$ and emf

\[ I_{rms} = \frac{I}{\sqrt{2}} \quad V_{rms} = \frac{V}{\sqrt{2}} \quad \mathcal{E}_{rms} = \frac{\mathcal{E}}{\sqrt{2}} \]

- Ammeters, voltmeters - give rms values

- The average (rms) power dissipated by a resistor in an ac circuit is

\[ P_{rms} = \frac{I^2 R}{2} = \left( \frac{I}{\sqrt{2}} \right)^2 R \quad \text{or} \quad P_{rms} = I_{rms}^2 R \]

- (Called $P_{ave}$ in the book)
AC circuits

- If ac circuit has only resistive load $Z=R$ (e.g. at the resonance frequency)

\[ P_{\text{rms}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \]

- Trade-off between current and voltage
  - For general use want low voltage
  - Means high current but

\[ P_{\text{rms}} = I_{\text{rms}}^2 R \]

- General energy transmission rule:
  Transmit at the highest possible voltage and the lowest possible current
AC circuits

- **Transformer** – device used to raise (for transmission) and lower (for use) the ac voltage in a circuit, keeping $iV$ constant
  - Has 2 coils (primary and secondary) wound on same iron core with different #s of turns
AC circuits

- Alternating primary current induces alternating magnetic flux in iron core
- Same core in both coils so induced flux also goes through the secondary coil
- Using Faraday’s law

\[
V_P = -N_P \frac{d\Phi_B}{dt}
\]

\[
V_S = -N_S \frac{d\Phi_B}{dt}
\]

\[
\frac{V_P}{N_P} = \frac{V_S}{N_S}
\]
AC circuits

- Transformation of voltage is
  
  \[ V_s = V_p \frac{N_S}{N_P} \]

- If \( N_S > N_P \) called a step-up transformer
- If \( N_S < N_P \) called a step-down transformer
AC circuits

- Conservation of energy

\[ I_P V_P = I_S V_S \]

\[ I_S = I_P \frac{V_P}{V_S} = I_P \frac{N_P}{N_S} \]
AC circuits

- The current $I_P$ appears in primary circuit due to $R$ in secondary circuit.

$$I_P V_P = I_S V_S \quad I_S = \frac{V_S}{R}$$

$$I_P = \frac{V_S}{R} \frac{V_S}{V_P} = \frac{1}{R} \frac{V_S}{V_P}^2 V_P = \frac{1}{R} \left( \frac{N_S}{N_P} \right)^2 V_P$$

- Has for of $I_P = \frac{V_P}{R_{eq}}$ where

$$R_{eq} = \left( \frac{N_P}{N_S} \right)^2 R$$