December 2nd

Diffraction

Chapters 37
Schedule

• Dec. 3-5 (Wed-Fri) – Review for final

• Dec. 3 (Wed) – HW set #12 due

• Dec. 8 (Mon) – Corrections #3 due

• Dec. 8 (Mon) – Final Exam 5:45-7:45pm
Final Exam

• **Monday December 8th - 5:45 to 7:45 pm**
  - Section 1 – N130 BCC (Business College)
  - Section 2 – 158 NR (Natural Resources)
• If >2 finals on Mon. may take make-up final exam on **Tues. from 8-10am**
  - Email Prof. Tollefson if need make-up
• Allowed 3 sheets of notes (both sides) and calculator
• Covers Chapters 22-37, HW sets 1-12
• Need photo ID
Diffraction

- Waves **diffract** (bend) if pass through an opening whose size is comparable to its wavelength.
- The narrower the slit, the greater the diffraction.
- Previous example of **double-slit interference** assumed slit width $a$ much smaller than $\lambda$ of incident light and we talked about 2 light rays.
Do we still get an interference pattern if we have only one slit?

Yes, see a bright central maximum and then other less bright spots on the sides (side maxima) separated by dark minima.

Caused by interference of wavelets from same wavefront going through slit.
Diffraction

- Location of the first minimum for single slit diffraction pattern
- Divide slit width in half
- Wavelet from bottom half travels further distance to point $P_1$ than from top half
- Wavelets will destructively interfere if

$$\Delta L = \frac{\lambda}{2}$$
**Diffraction**

- Assume screen far away from slit so rays are $\parallel$
- Use trig to find $\Delta L$
  \[ \sin \theta = \frac{\Delta L}{a/2} \]
- For **first minimum** on screen
  \[ \Delta L = \frac{\lambda}{2} = \frac{a}{2} \sin \theta \]
- Rearrange to find
  \[ a \sin \theta = \lambda \]
Minima (dark fringes) from single-slit diffraction

\[ a \sin \theta = m\lambda, \ m = 1, 2, 3, \ldots \]

- \( a \) is the width of slit
- \( m = 0 \) is missing
  - where the bright central maximum is located
- Location on the screen is:

\[ y = \frac{m \lambda D}{a} \]

- Maxima are half-way between minima
Checkpoint #1

- Diffraction pattern from single narrow slit illuminated by blue light (450 nm). Do the minima shift away from or toward the bright central maximum if we switch to yellow light (575 nm)?

\[ a \sin \theta = m \lambda \]

- If wavelength \( \lambda \) increases then \( \sin \theta \) and \( \theta \) increase so fringes move further away from center

- Location of minima from diffraction changes for different wavelengths \( \lambda \)
**Checkpoint #1**

- Diffraction pattern from single narrow slit illuminated by blue light (450 nm). Do the minima shift away from or toward the bright central maximum if we decrease the slit width?

\[ a \sin \theta = m \lambda \quad \text{or} \quad \sin \theta = \frac{m \lambda}{a} \]

- Decrease slit width \( a \), increases \( \sin \theta \) and \( \theta \) so fringes move further away from center.

- If \( a = \lambda \) then \( \theta = 90^\circ \) for first dark fringe, only see interference pattern since bright maxima from diffraction covers entire viewing area.
Diffraction from Single Slit

- Intensity of light as function of angle $\theta$ from central axis

$$I(\theta) = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2$$

- $I_m$ is intensity at central maximum
- $\alpha$ is $\frac{1}{2}$ the phase difference $\phi$ between top and bottom rays from slit with width $a$

$$\alpha = \frac{1}{2} \phi = \frac{\pi a}{\lambda} \sin \theta$$
Diffraction Single Slit

- Width of central diffraction maximum decreases as slit gets bigger ($a$ increases)
  - Light undergoes less bending as slit gets bigger
- Secondary maxima much dimmer than central
- If slit is much wider than wavelength of light ($a >> \lambda$) the secondary maxima disappear, no longer have diffraction from single slit
Diffraction

- **Interference** –
  - Combining waves from small number of coherent sources – double-slit experiment with $a << \lambda$

- **Diffraction** –
  - Combining of large number of wavelets from single wavefront – as in single slit experiment

- **Diffraction and interference are both**
  - the result of combining waves with different phases at a given point
  - Usually present simultaneously

- Example see photo 37-14 p.902
Diffraction

- Previous discussion of double-slit experiment assumed width of slit $a \ll \lambda$
- Central maxima of diffraction pattern from each single slit covered entire screen so only saw interference pattern
- Bright fringes had same intensity and equal size
Diffraction

- Double slit interference pattern is modified by diffraction pattern of light passing through each slit.
- Position of fringes doesn’t change but intensity is affected.

![Double-slit Interference](image1)
![Single-slit Diffraction](image2)

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Double-slit Interference

![Combined Pattern](image3)
Diffraction

- Intensity of combined pattern is given by
  \[ I(\theta) = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta \]

- \( I_m \) is intensity of central maximum

- First term \( (\sin \alpha/\alpha)^2 \) is due to diffraction from slit of width \( a \) where
  \[ \alpha = \frac{\pi a}{\lambda} \sin \theta \]

- Second term of \( \cos^2 \beta \) is due to two-slit interference with slits separated by distance \( d \) where
  \[ \beta = \frac{\pi d}{\lambda} \sin \theta \]
Diffraction

- **Typical situation**
- **Location of diffraction minima** given by
  \[ y_1 = \frac{m_1 \lambda D}{a}, \quad m_1 = 1,2,... \]
- **Location of two-slit maxima** given by
  \[ y_2 = \frac{m_2 \lambda D}{d}, \quad m_2 = 0,1,2,... \]
Diffraction – Circular Aperture

- Circular aperture of diameter \( d \) makes a ring pattern with the first minima at
  - Can treat your eye’s pupil as a circular aperture
- Rayleigh’s criterion for resolvability is angular separation between two point sources must be

\[
\sin \theta = 1.22 \frac{\lambda}{d}
\]

\[
\theta_R = 1.22 \frac{\lambda}{d}
\]