

# Experiment 1

## Simple Measurements and Error Estimation

### Reading and problems (1 point for each problem):

Read Taylor sections 3.6-3.10  
Do problems 3.18, 3.22, 3.23, 3.28

### Experiment 1 Goals

1. To perform simple measurements as accurately as possible and to estimate uncertainties in these measurements.
2. To gain practice in computing propagation of errors.
3. To investigate the distribution of random errors.

### Theoretical introduction

The main purpose of this experiment is to introduce you to methods of dealing with the uncertainties of the experiment (for more background, see the [Appendix](#)). The basic procedures to correctly estimate the uncertainty in the knowledge of the measured value (*the error of the measurement*) include:

- Estimation of the uncertainty in the values directly measured by, or read from, the measurement device (*directly measured quantities, Taylor, Chapter 1*);
- correct treatment of the random errors of the experiment (*Chapters 4 and 5*);
- calculation of the errors of the quantities which are not measured directly (*the propagation of errors, Chapter 3*);
- rounding off the insignificant digits in the directly measured and calculated quantities (Chapter 2 and the [Appendix](#) to this lab).

### Preliminary discussion (15-30 minutes).

Before the lab, you are asked to read and understand the theoretical material for this lab (Exp1 and *Taylor*). Before the experiment starts, your group needs to decide which information will be relevant to your experiment. Discuss what

you will do in the lab and what preliminary knowledge is required for successful completion of each step.

Think about organizing your work in an efficient way. How should you use *Kgraph* to expedite your calculations and unit conversions (when necessary)?

### **Questions for the preliminary discussion.**

1. Suppose during each of several measurements we find a value, which lies in the same interval of the scale of the measuring device. For example, each time we measure the length to be between 176 and 177 mm, with the length between the ticks on the ruler equal to 1 mm. How do we estimate an uncertainty in our the measured length in this case?
2. Now suppose we use a much more precise device (say the laser micrometer). Due to a higher precision, this device can resolve the miniscule changes in the length due to the random mechanical deformations of the object, and in each measurement we will see the slight unsystematic changes in the observed length. From these data, how can we find the most likely value of the length? How can we estimate the spread in the measurements? Which formulas will we use?
3. Next, we are going to find out if the two independent measurements from 1 and 2 are consistent with each other. Which procedure will we use? Is there a quantitative method to find out if two measured lengths are in agreement? Is there a quantitative method to estimate how certain our conclusion about the agreement or the disagreement of these measured values is?
4. If we are going to use the results of our measurements to calculate some other quantities (e.g., calculate the density of the rod using the measurements of its dimensions and the mass), which formulas will we use to calculate the mean values and the uncertainties of these quantities?
5. In our calculation, the calculator (computer) will typically return the results with as many digits as possible. However, some of these digits will be produced from the part of the input number, which we do not know, and, correspondingly, they will have no real meaning. Which procedure will you follow to systematically get rid of these insignificant digits?

## **Density Measurements**

### **Introduction**

Your text (Sec. 1.3, p. 5) describes how Archimedes was able to determine the composition of a king's crown by measuring its density. We will attempt to perform a similar exercise, but we shall use copper instead of gold. Copper

has a density of  $8.94 \text{ g/cm}^3$  at 20 degrees Celsius (C). We will consider later what to do if the temperature is not exactly 20 degrees C. Your task today is to measure the density, calculate the appropriate uncertainties and decide whether your measurement agrees with the given value.

## Part A

In our lab, we will use rulers, vernier calipers and micrometers. Discuss in your group the following questions:

- a) Which of these instruments is the most precise; the least precise? How do you know? In particular, is the caliper more precise than the ruler? **Hint:** try to understand if the moving scale on the caliper (the vernier) can be used to increase the precision of the length measurement compared to a simple ruler. If you don't see how to use it, you may refer to the Appendix.
- b) Write down the uncertainties of the length measurements with each of these instruments.

Use the ruler to measure the three dimensions of the block. Repeat these measurements with the vernier caliper and the micrometer. Assign uncertainties to your measurements. For each instrument, measure the length with the highest precision possible.

Measure the mass of your block and estimate its uncertainty.

For the ruler measurement, compute the volume of the block and its uncertainty. Make two estimates of your uncertainty using alternatively Eqs. 3.18 and 3.19 on p. 61 of the text. Which estimate is more appropriate for this calculation? Why?

Pay attention to the units. In the calculation, follow the rules for rounding off the insignificant figures. If the calculation is done correctly, the smallest significant figure in the final mean value will be of the same order as the final uncertainty.

Compute the density and its uncertainty. Use Eq. 3.18 on p. 61 of Taylor.

Calculate the density and its uncertainty using the data obtained with the help of the caliper and micrometer.

## Part B

Answer the following questions:

1. Is your value consistent with the density of pure copper? Could it, instead, be a copper alloy? (Copper alloys (e.g. bronze and brass) have densities which can range from  $7.5 \text{ g/cm}^3$  to  $10 \text{ g/cm}^3$ ). For more information, see the appendix on [Alloy Densities](#). Try to figure out how to justify your conclusion quantitatively. (Hint: compare the discrepancy between the theoretical and experimental values with some other number).
2. In the computation of density, what were the greatest sources of uncertainty? Which were the smallest?
3. If you had made only one set of measurements with a ruler, would you arrive to the same answer for the Question 1? Why?
4. What systematic errors might we be overlooking? Are any of these big enough to affect your estimate of uncertainty? Consider, for instance, the temperature dependence of the density of metal, irregularities in the shape of the block, and any other errors you can think of.

## Useful Information

If a metal is heated, its length increases by an amount  $\Delta L$  given by:

$$\Delta L = \alpha \cdot L \cdot \Delta T$$

where  $L$  is the original length,  $\Delta T$  is the increase in temperature, and  $\alpha$  is the thermal coefficient of linear expansion. For aluminum,  $\alpha = 23 \times 10^{-6}$  (per degree C). For steel,  $\alpha = 11 \times 10^{-6}$  (per degree C). For copper,  $\alpha = 17 \times 10^{-6}$  (per degree C).

## Part C

Use the density measurement to determine the material of 3 other unknown objects. Again, present the numeric arguments supporting your conclusions.

In your report, include 3 measured densities of the rectangular block and the densities of 3 unknown objects, as well as the relevant uncertainties.

## Random Uncertainties

In this part of the experiment we ask you to perform one of the simplest of repetitive measurements in order to investigate random and systematic errors.

### Part A: Time Measurements

At the front of the room is a large digital clock that is supposed to count at a constant rate of one count per second. Assume that it does count at a constant rate, but do not assume that the stated rate of one count per second is correct.

The clock will count from 1 to 20, blank out for some unspecified length of time, and then begin counting again. To measure the clock's count rate, you will be given a timer whose systematic error is less than 0.001 seconds for time intervals of ten seconds or less. However, you will be relying on hand-eye coordination, which means your measurements will have random uncertainties. Your reaction time is unavoidably variable. You may also be systematically underestimating or overestimating the total time.

1. Choose a counting interval at least 10 counts long and time 25 of them. One person should time while the other records the data on the data sheet belonging to the person timing. To avoid an unconscious skewing of data, the person timing should not look at the data sheet until all 25 measurements have been recorded. This is essential; otherwise, you will introduce a bias into your measuring procedure! Make a few practice runs before taking data. Also, avoid starting a timing interval on the first count. Use the first of three counts to develop a tempo with which to synchronize your start.
2. Exchange places with your partner, and time 25 more counting intervals. Thus, each person will have a data sheet with 25 timings recorded on it.

### Part B: Data Analysis

1. From your data set of 25 measurements compute the average or "mean"

time per count  $\bar{T} = \frac{1}{N} \sum_{i=1}^N T_i$ , where the  $T_i$  represents single measurements of the time per count and  $N = 25$ , the number of measurements. Using this value of  $\bar{T}$  to compute the standard deviation,  $\sigma$ , defined as:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (T_i - \bar{T})^2}$$

and from that, the standard deviation of the mean defined as:

$$\sigma_m = \frac{\sigma}{\sqrt{N}}$$

Show the calculations explicitly for the first 5 measurements, but you may use a calculator or the computers for the full 25. Read the statistics section in the Kgraph manual.

Make a bin histogram from your twenty-five measurements. The x axis should represent the time  $T$ . It should be divided into bins, each with a width of about  $\Delta = 0.4\sigma$ . The y axis should be  $n_k$ , which is defined to be the number of

measurements which fall in the  $k$ th time bin. Data might fall right on the border of two bins. Make a convention for placing these points such that they are always placed in either the higher or the lower bin. Remain consistent with your handling of these points. Clearly mark the points of  $\bar{T}$  and  $\pm\sigma$  for your measurements. The quantity  $g$  can be shown to be the best estimate of your measurement and the region included in the range  $\pm\sigma$  should contain about 68% of your data points if your errors are random and consequently the distribution of your measurements is normal or Gaussian. Draw an appropriate Gaussian distribution given by

$$g(T, T_{\text{mean}}, \sigma) = g_{\text{max}} \exp(-(T - T_{\text{mean}})^2 / (2\sigma^2)),$$

on your stack histogram, where  $T_{\text{mean}}$  and  $\sigma$  are your best estimates for the mean and standard deviation of your time measurements. (Here, choose an appropriate estimate for  $g_{\text{max}}$ . Calculate  $g(T, T_{\text{mean}}, \sigma)$  at five points, plot them by hand on your histogram plot and connect these points with a smooth curve.) With a finite number of measurements such as 25, your distribution may not resemble the expected "bell" shape to a great degree. The standard deviation of the mean  $\sigma_m$ , is the best estimate of the uncertainty in the measurement of the mean. Note that, unlike the standard deviation, this uncertainty can be made arbitrarily small by taking a sufficiently large number of measurements.

### Part C: Conclusions

1. Does a significant systematic error exist? In other words, is there a significant discrepancy in the time measured by the large clock? Note that we have already made the hypothesis that the large digital clock is running correctly, and now we want to check whether this hypothesis is in agreement with our statistical analysis.
2. Using an equation similar to Eq. 5.67 on p. 150 of your text, compute the following quantity for your 25 measurements:

$$t = \frac{|\bar{T} - T_{\text{exp}}|}{\sigma_m}$$

where  $T_{\text{exp}}$  is the expected time if the clock were running correctly.  $t$  has the meaning of the number of the standard deviations of the mean needed to cover the difference between the mean and expected times.

3. Assuming that your measurements follow a normal distribution, calculate the probability  $P$  of obtaining a result that differs from  $T_{\text{exp}}$  by  $t$  or more. In other words, what is the probability  $1-P$  that  $|\bar{T} - T_{\text{exp}}| > t\sigma_m$ ? You may need

to use the table on p. 286 of your textbook, which gives the probability  $P$  that  $|\bar{T} - T_{\text{exp}}| < t\sigma_m$  for the Gaussian distribution with the standard deviation  $\sigma_m$ , and for any given  $t$ .

4. Is the discrepancy significant or not? This is somewhat a matter of opinion since we must decide what is significant. If the probability is less than 5%, the probability is "unreasonably" small and discrepancy is significant. On the other hand, if the probability is greater than approximately 32%, the probability is "rather large", and the discrepancy is not significant. In this context, state your conclusions clearly.

5. How many  $\sigma_m$  is the discrepancy  $|\bar{T} - T_{\text{exp}}|$  when the probability  $P=5\%$ ; when  $P=32\%$ ? From this and the previous answer, formulate a "rule of thumb" allowing you to determine if the discrepancy is significant or not. (Hint: a discrepancy wouldn't be that *significant* if a third of the time you observe one.)

6. Suppose you had recorded only your first 5 measurements. What would you have concluded about the existence of a significant discrepancy? Were the remaining 20 measurements necessary in your opinion? Explain.

## Structure of the report

A good report should include:

- a) A short explanation of the goals of this lab (not just a copy from the beginning of this write-up);
- b) A description of what you did, including the representative samples of the measured data and calculations, the discussion of the most important results you obtained, etc.
- c) Answers to all the questions;
- d) A brief conclusion or summary.
- e) Please also list at least three things that your group did well during this lab, and at least one thing that could be improved.

## Appendices

### Appendix 1. Theory of Uncertainties

Contrary to the naïve expectation, the experiments in physics typically involve not only the measurements of various quantitative parameters of nature. In almost all the situations the experimentalist has also to present an argument showing how confident she is about the numeric values obtained. Among other things, this confidence in the validity of the presented numeric data strongly depends on the accuracy of the measurement procedure. As a simple example, it is impractical to measure a mass of a feather using the scale from the truck weigh station, which is hardly sensitive to the weight less than a few pounds.

Another challenge has to be met when the scientist tries to compare the results of her experiment with the data from another experiments, or with the theoretical predictions. Since the conditions of the measurement almost always vary from an experiment to an experiment, and since they are also different from the idealized situation of the theoretical model, the compared values most likely will not match each other exactly. The task is then to figure out how important the factors creating this discrepancy are. If these factors are stable (do not change from measurement to measurement) and well noticeable, they are called systematic errors. If, on the contrary, these factors are more or less random and on average compensate each other, they are called random, or statistic errors. An important fact is that the uncertainty due to the random errors can be reduced by increasing the number of measurements.

## Appendix 2. Significant figures

In the calculations, it is always important to distinguish significant figures in the presented numbers from insignificant. The following simple rules will help you in this task.

1. Since the error of the measurement is only an approximate estimate of the uncertainty of the measurement, we do not need to keep more than one or two largest digits in it. The smallest digit in the mean value should be of the same order as the smallest significant digit of the uncertainty. Examples:

Incorrect	Correct
$517.436 \pm 0.1234$	$517.4 \pm 0.1$ or $517.44 \pm 0.12$
$24.3441364 \pm 0.002$	$24.344 \pm 0.002$
$12385 \pm 341$	$12400 \pm 300$ or $12390 \pm 340$

2. When adding or subtracting two numbers, the result should have the same number of the significant digits after the decimal point as the least precise summand. Example:



$$517.45 + 34.7824 = 552.23$$

3. When multiplying or dividing, the result should have the same total number of the significant digits as the least precise multiplier. Example:

$$1234.3 \times 23.45 \approx 28940$$

4. For other operations (raising to power, square root, exponent) the rule is similar to the one for the multiplication and division: you should keep as many significant digits in the final result as you had in the input. Example:

$$\sqrt{3.567} \approx 1.889$$

### Appendix 3. Alloy Densities

Below is a table of typical values of density (specific gravity) for various commercial metals and alloys. Units are  $\text{g cm}^{-3}$  or  $\text{kg m}^{-3}$ .

Common name and classification	Thermal conductivity			Specific gravity
	J/sec cm °K	Btu/hr ft °F	kcal/sec cm °C	
Ingot iron (included for comparison)	1.3	77	0.32	7.86
Plain carbon steel AISI-SAE 1020	1.0	56	0.23	7.86
Stainless steel type 304	0.3	19	0.08	8.02
Cast gray iron ASTM A48-48, Class 25	0.8	43	0.20	7.2
Malleable iron ASTM A47				7.32
Ductile cast iron ASTM A 239, A236	0.6	34	0.14	7.2
Nirodcast iron, type 2	0.7	41	0.17	7.3
Cast 18-7 alloy (11E) ASTM A297-63T	0.04	2	0.01	7.6
Hastelloy C	0.2	10	0.04	3.94
Inconel X, annealed	0.3	17	0.07	8.25
Haynes Stellite alloy 25 (L605)	0.2	10	0.04	9.15
Aluminum alloy 3003, rolled ASTM B 221	2.8	164	0.68	2.73
Aluminum alloy 2017, annealed ASTM B 221	3.0	174	0.72	2.8
Aluminum alloy 380 ASTM 3C54B	1.8	102	0.42	2.7
Copper ASTM B 152, B124, B133, B1, B2, B3	7.1	411	1.70	8.91
Yellow brass (high brass) ASTM B 36, B 134, B135	2.2	126	0.52	8.47
Aluminum bronze ASTM B149, alloy A; ASTM B124, B150	1.3	75	0.31	7.8
Beryllium copper 25 ASTM B 194	0.2	11	0.05	8.25

## Appendix 4. THE VERNIER CALIPER

A vernier caliper consists of a high quality metal ruler with a special vernier scale attached which allows the ruler to be read with greater precision than would otherwise be possible. The vernier scale provides a means of making measurements of distance (or length) to an accuracy of a tenth of a millimeter or better. Although this section will be devoted to the use of the vernier caliper, it should be noted that vernier scales can be used to make accurate measurements of many different quantities. In the future, you will also use an instrument with a vernier scale to make precise readings of angular displacements.

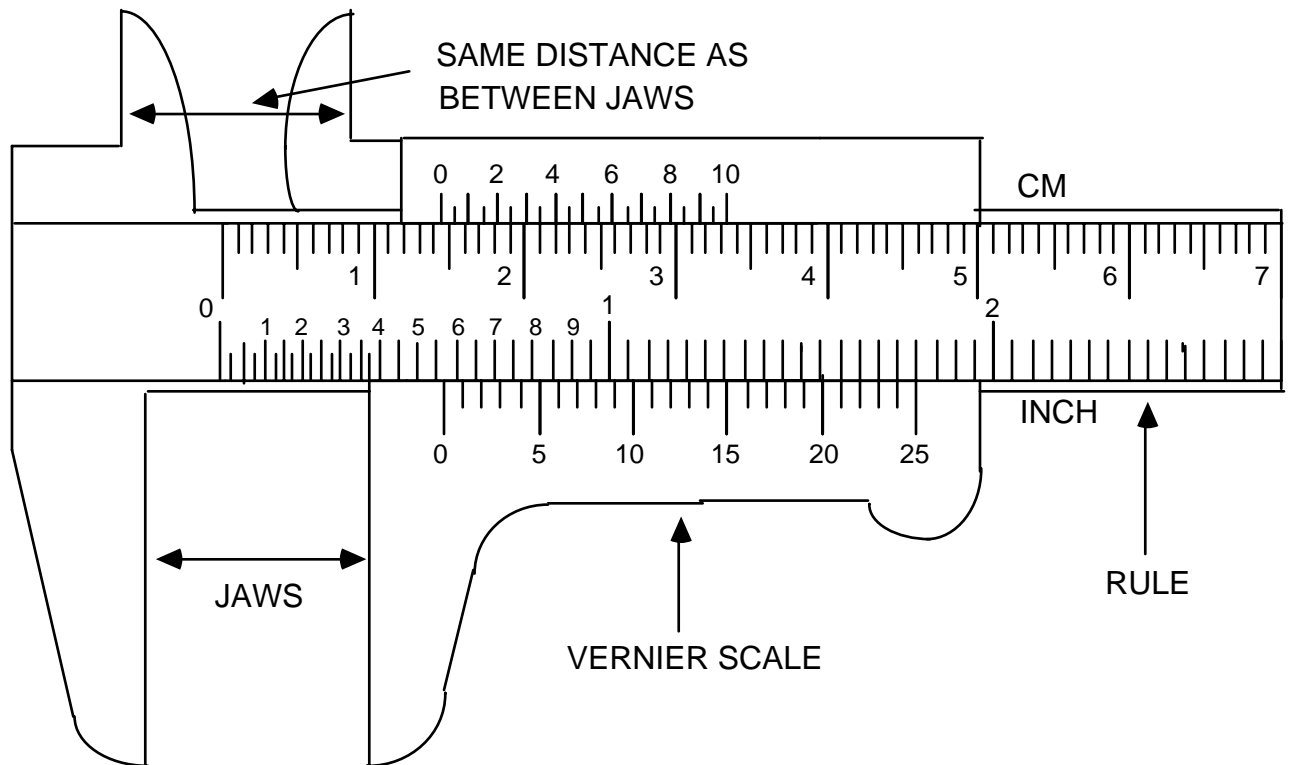


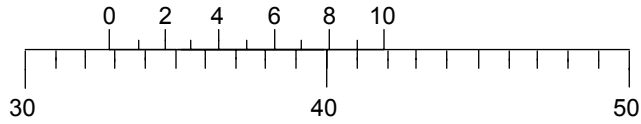
Figure 1: Vernier Caliper

Looking at the vernier caliper in Fig. 1, notice that while the units on the rule portion are similar to those on an ordinary metric ruler, the gradations on the vernier scale are slightly different. The number of vernier gradations is always one more than the number on rule for the same distance. The line on the vernier which is aligned with one on the rule tells us the fraction of the units on the rule. For example, in Fig. 1 the vernier reads 1.440 cm or 0.567 in.

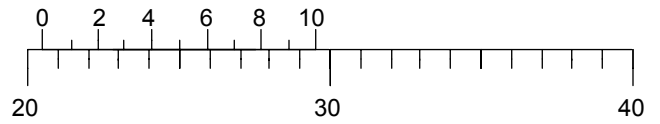
To use the vernier caliper:

- (1) Roll the thumb wheel until the jaws are completely closed (touching each other). Now check whether the caliper is reading exactly zero. If not, record the caliper reading, and subtract this number from each measurement you make with the caliper.
- (2) Use either the inside edges of the jaws, or the outside edges of the two prongs at the top of the caliper to make your measurement. Do not use the tips of the prongs. Roll the thumb wheel until these surfaces line up with the end points of the distance you are measuring.
- (3) To read the caliper:
  - (a) record the numbers which correspond to the last line on the rule which falls before the index line on the vernier scale. On the following page, this would be 32 since the index line falls just after the 32 cm line.
  - (b) count to the right on the vernier scale until you reach a vernier line which lines up with a line on the rule and record the number of this vernier line as your last digit. on the following page it is the ninth vernier line which is aligned with one on the rule, so the whole distance is 32.9 cm.

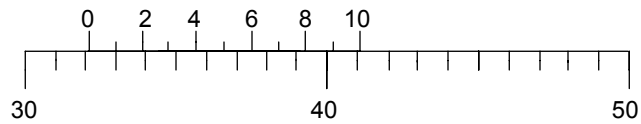
The following pages show six vernier scales, similar to that on the vernier caliper, which will allow you to test your ability to read a vernier caliper.



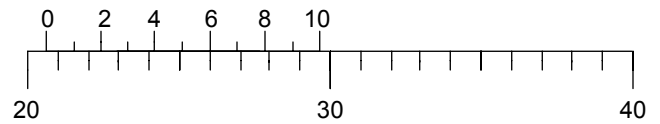
Ans: 32.9



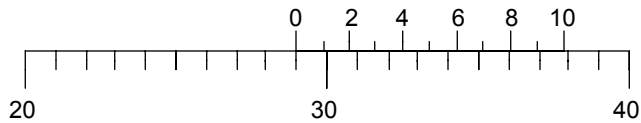
Ans:



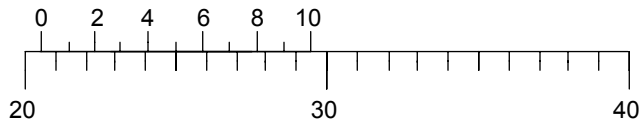
Ans: 32.1



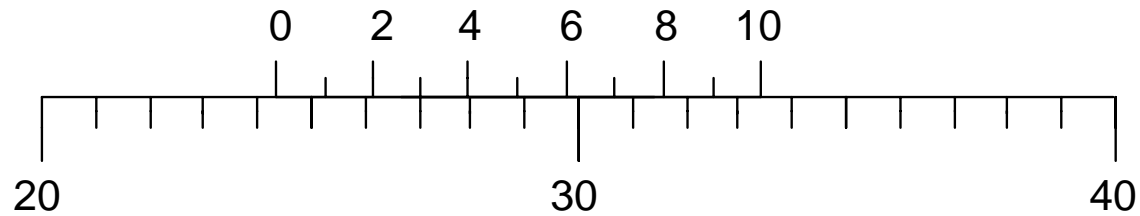
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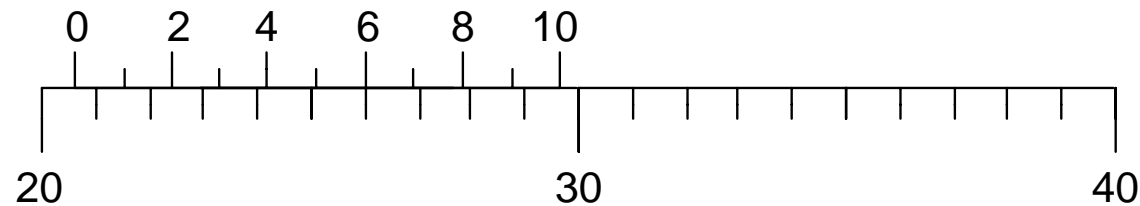
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