Experiment 2

F r e e F a l l

Suggested Reading for this Lab

Taylor, Section 2.6, Sections 3.10 - 3.11, Sections 8.1 - 8.5

Goals

1. To study the time dependence of the velocity and position of a body falling freely under the influence of gravity.
2. To measure the value of the gravitational constant in East Lansing and compare it to the accepted value g = 9.804 m/s^2.
3. To use least squares fitting methods to obtain best values for unknown parameters and their uncertainties.

Theoretical Introduction

An object falling freely near the surface of the Earth experiences a constant downward acceleration caused by the pull of the Earth's gravity, g. If we choose the upward direction as positive, the sign of the body's acceleration is negative, a = -g. We now ask the question: "If the acceleration a(t) is given, how do we find the velocity v(t) and the distance y(t) that the body has traveled in a time t?" To derive the equations of motion we apply integral calculus. Thus, choosing the direction of motion along the y-axis only, we can write

$$a(t) = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2} = -g \; . \qquad (1)$$

We integrate this equation with respect to time to get the instantaneous downward velocity v(t):

$$\int_{v_0}^{v} dv = -\int_{0}^{t} g dt$$

$$v(t) = v_0 - gt \qquad (2)$$

where v_0 is the velocity at time t = 0. We can integrate Eq. (2) once more to find the distance that the object has fallen in a time t:

$$v(t) \equiv \frac{dy}{dt} = v_0 - gt$$
$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2 \qquad (3)$$

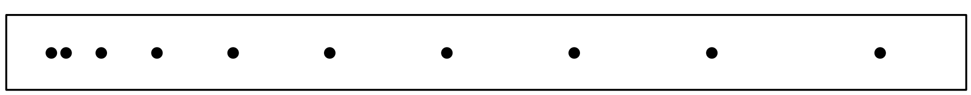
where y_0 is the object's position at time t = 0.

You may recall from your study of linear motion in kinematics, that we could have arrived at the same expressions, if we just substituted a = -g (and y = x) in the equations of linear motion:

$$v(t) = v_0 + at\,; \quad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 .$$

Experimental Procedure

In Exp2, you will perform measurements of g with the Behr free fall apparatus. A cylinder is dropped and a record of its fall is made. Before measurement, the cylinder is suspended at the top of the stand with an electromagnet. When the electromagnet is turned off, the cylinder begins to fall. Simultaneously, the spark timer starts to send high-voltage pulses between two wires. Your instructor will demonstrate how to operate the Behr free fall apparatus for you. As the cylinder falls, it closes the gap between the two wires and a spark will jump from one wire to the other at the point where the cylinder passes. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the paper tape. The time interval between the two adjacent sparks is about 1/60th of a second. You can measure the actual interval using the period counter. The appearance of the beginning portion of such a tape is indicated below, with time increasing to the right.



You will use two methods for determining the kinematical trajectory of the cylinder. On your paper tape, you should have about 30 burn marks. (1) Take the points in order and measure the differences, Δy_i, between adjacent points, using the most precise measuring instrument available. (2) Measure the position, $y_i(t_i)$, starting with the first point and making your measurements using a metric tape measure or ruler.

Assign the first usable point as y = 0, t = 0. Assign uncertainties to the measurements, stating in your report how you arrived at these values. From the intervals Δy_i, the average velocity for each interval is calculated as:

$$v_i = \frac{\Delta y_i}{\Delta t} . \qquad (4)$$

Compute v_i for each i and, using your estimated uncertainties in Δy_i, compute an uncertainty for each velocity v_i. Show your method of calculating the uncertainty for v_i in your notebook; this should also be included in your report.

(1) Make a graph of v(t) vs. time by plotting v_i vs. $i\Delta t$.

(2) Make a graph of y(t) vs. time.

On both types of plots, label the axes using SI units. Your notebook should have graphs of both experiments with preliminary calculations shown. Note that in (1) you determine the average velocity over each time interval so that the appropriate time is at the midpoint of the interval.

Questions for Preliminary Discussion

1. Using Eq. (3) show that v_i, as defined in Eq. (4) is the instantaneous velocity at the middle of the time interval.
2. How does v depend on t?
3. How does y depend on t?
4. What kind of systematic errors might influence your experiment?
5. What is the reason for using 1/60 s time intervals?
6. How do you calculate parameters and uncertainties in least squares fits?

Data and Graphical Analysis

Prepare a data sheet in *Kgraph* which includes Δy_i, error in Δy_i, $y_i(t)$, error in $y_i(t)$, $i\Delta t$, v_i, error in v_i, etc. Be sure to label each column correctly with appropriate units

As pointed out in Section 2.6 of your text, a graph of instantaneous velocity versus time can be used to test the linear dependence of v(t) and the quadratic dependence of y(t) on Δt. The data that you have taken give you the average velocity in each interval. Make a graph of v(t) vs. $i\Delta t$ by plotting v (in m/s) vs. $i\Delta t$ using *Kgraph*. Also, plot y(t) vs. time. Be sure to label the axes and include units for each variable.

Refer to the *Kgraph* manual to find out how to put error bars on your graph; the discussion of this may be found in Sec. 9.8, p. 249. You will also find useful the section on creating a series in Sec. 18.4, p. 470.

For *Kgraph* to calculate the errors in the curve fit parameters you *cannot* use the polynomial curve fit routines in the menu. You need to create a user-defined function as shown in Sec. 10.4, p. 282.

Questions to be discussed

These questions should be addressed in your report in a narrative form. They can be included in the Results or Conclusions sections. Answers such as "no" or "yes" are not useful.

1. Does v depend linearly on t? Does y depend quadratically on t? Using Kgraph, use least squares fits to address quantitatively these questions.
2. Does your straight line pass within all error bars? If not, suggest reasons why this is the case. For instance, your estimates of error bars could be faulty or there may be systematic errors.
3. What is the y-axis intercept as determined from your 1st graph and what does it mean?
4. Determine the slope from your graph of v(t) vs. t and the value of g in SI units. Show the calculation in your notebook as well as in your report.
5. Compute the constants v_0 and g from your data points using Eqs. 8.10 - 8.12 in your text. Then compute uncertainties σ_v, σ_{vo}, σ_g for a line of the form $v = v_0 - gt$; see Taylor Eqs. 8.15 - 8.17. Chapter 8 discusses the procedure for calculating these constants and uncertainties. In your report show how to calculate the uncertainties in the curve fit parameters.
6. Calculate y_0, v_0, g and their uncertainties using *Kgraph*. Do the values for g and its uncertainty agree with your determinations in 4?
7. Are your results reproducible? That is, when you repeat your measurements do you find g values that differ by less than one standard deviation from one another?
8. Using your most reliable results for g, compute the percentage deviation of your result from the accepted value and address the size of the deviation.
9. If you replaced the cylinder by one with different mass and then performed the experiment again, how would your results differ?
10. What does "terminal velocity" of a falling object mean? What are the implications for your experiment?

Appendix

Significant Figures in Your Report

When reporting results, for example g, you must always include value and uncertainty in the form $g \pm \delta g$. Refer to the text or the Appendix in Exp1 for guidance on assigning a valid number of significant figures.