Experiment 4

Elastic and Inelastic Collisions

Reading and problems (1 point for each problem):

Taylor Chapters 5 and 6, problems 5.36, 6.2

Overview:

The following experiment explores the conservation of momentum and energy in a closed physical system. As you probably know from the accompanying theoretical course, the conservation of energy and momentum plays an important role in physics and is a fundamental symmetry of nature.

Theoretical introduction

Momentum

For a single particle (or a very small physical object), momentum is defined as the product of the mass of the particle and its velocity:

$$\vec{p} = m\vec{v} \tag{1}$$

Momentum is a vector quantity, making its direction a necessary part of the data. To define the momentum in our three-dimensional space completely, one needs to specify its three components in *x*, *y* and *z* direction. The momentum of a system of more than one particle is the vector sum of the individual momenta:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + ... = m\vec{v}_1 + m\vec{v}_2 + ... \tag{2}$$

The 2^{nd} Newton's law of mechanics can be written in a form which states that the rate of the change of the system's momentum with time is equal to the sum of the external forces acting on this system:

$$\frac{d\vec{p}}{dt} = \sum \vec{F}$$

(3)

From here we can immediately see that when the system is closed (which means that the net external force acting on the system is zero), the total momentum of the system is conserved (constant).

Energy

Another important quantity describing the evolution of the system is its energy. The total energy of a given system is generally the sum of several different forms of energy. Kinetic energy is the form associated with motion, and for a single particle:

$$KE = \frac{mv^2}{2} \tag{4}$$

Here *v* without the vector symbol stands for the absolute value of the velocity,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

In contrast to momentum, kinetic energy is NOT a vector; for a system of more than one particle the total kinetic energy is the algebraic sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + ¼$$

(5)

Another fundamental law of physics is that the total energy of a system is always conserved. However within a given system, one form of energy may be converted to another (such as potential energy converted to kinetic in the experiment entitled Analysis of a Freely Falling Body). Therefore, kinetic energy alone is often not conserved.

Collisions

An important area of application of the conservation laws is the study of the collisions of various physical bodies. In many cases, it is hard to assess how exactly the colliding bodies interact with each other. However, in a closed system, the conservation laws often allow to obtain the information about many important properties of the collision without going into the complicated details of the collision dynamics. In this lab, we will see in practice how the conservation of momentum and total energy relate various parameters (masses, velocities) of the system independently of the nature of the interaction between the colliding bodies.

Elastic and inelastic collisions

Assume we have two particles with masses m_1, m_2 and speeds $\vec{v}_{1i}$ and $\vec{v}_{2i}$ which collide, without any external force, resulting in speeds of $\vec{v}_{1f}$ and $\vec{v}_{2f}$ after the collision (*i* and *f* stand for *initial* and *final*). Conservation of momentum then states that the total momentum before the collision $\vec{P}_i$ is equal to the total momentum after the collision $\vec{P}_f$:

$$\vec{P}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}, \quad \vec{P}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \text{and} \quad \vec{P}_i = \vec{P}_f \tag{6}$$

There are two basic kinds of collisions, elastic and inelastic:

In an elastic collision, two or more bodies come together, collide, and then move apart again with no loss in total kinetic energy. An example would be two identical "superballs", colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad \text{or} \quad KE_i = KE_f \tag{7}$$

In an inelastic collision, the bodies collide and come apart again, but now some kinetic energy is lost (converted to some other form of energy). An example would be the collision between a baseball and a bat. If the bodies collide and stick together, the collision is called completely inelastic. In this case, all of the kinetic energy relative to the center of mass of the whole system is lost in the collision (converted to other forms).

In this experiment you will be dealing with

a) a completely inelastic collision in which all kinetic energy relative to the center of mass of the system is lost, but momentum is still conserved, and
b) a nearly elastic collision in which both momentum and kinetic energy are conserved to within a few percent.

Conservation laws for macroscopic bodies

So far we were talking about the system of point-like particles, however, the conservation of the momentum is also valid for macroscopic objects. This is because the motion of any macroscopic object can be decomposed into the motion of its center of mass (which is a point in the space) with a given linear momentum, and a rotation of the object around this center of mass. Then, the conservation of the linear momentum is again valid for the motion of the centers of the masses of the objects. However, some of the linear kinetic energy can be transformed into the rotational energy of the objects, which should be accounted for in a real experiment.

Experimental setup

We will study the momentum and energy conservation in the following simplified situation:

a) we will look on the collision of only 2 objects;
b) the motion of these objects will be linear and one-dimensional, so that we can choose the reference frame in such a way that only x-components of the objects' momenta are non-zero; the sign of these components depends on the direction of the motion;
c) the experimental apparatus can be set up in a way to almost completely eliminate the net external force on the system.

Our objects will be two carts of different masses, with one initially at rest. The carts move on an air track, which ensures that the motion is one-dimensional and reduces the friction between the carts and the surface. The velocities of the carts can be measured with the help of the photogates, which are described in more details below. Also, it is possible to attach various bumpers (magnets, rubber bands, etc) to the carts, which will change the nature of the interaction forces between the carts.

Before the beginning of the measurements, spend at least 15 minutes to figure out which external factors can disturb the motion of the carts on the track, and what you should do to reduce or eliminate these factors. Remember, the successful completion of this lab strongly depends on your ability to create an almost closed system. Make a few practice trials to see if you can achieve an unperturbed one-dimensional collision of the carts. Adjust the level of the air track and the power of the air supply if necessary.

Questions for preliminary discussion

1. In a collision in a closed system, can some of the total momentum be lost? Some of the kinetic energy?

2. In an elastic (inelastic) collision in a closed system, can the total momentum be lost? The kinetic energy?
3. If the collision is two-dimensional, can the momentum be conserved in *x*-direction but lost in *y*-direction?
4. In our experiment, can we achieve a completely elastic collision? Completely inelastic collision?
5. Draw a diagram of forces acting on each cart. Which forces will influence the total *P* and *KE* most?

Measurements

Inelastic collision

1. In the first part of the lab we make sure that after the collision the carts stick together and move with some velocity common to both masses. Thus, we have to measure the velocity of cart 1 before the collision and the common velocity of the carts 1 and 2 after the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows measuring the time it takes the cart to go through it. The velocities are calculated by dividing the length of the plate on the cart 1 by the measured time (speed = length/time).

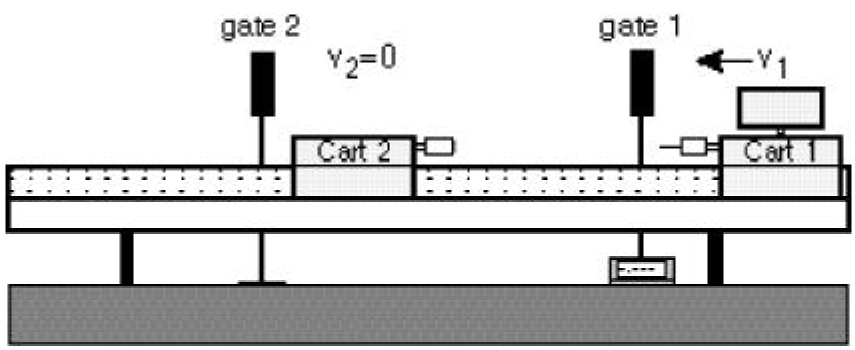

Figure 1: The initial state of the carts before inelastic collision (note that the fin on cart 2 can be removed).

Position cart 2 close to the gate 2 and set the photogate timer to "GATE" mode and the memory switch in “ON” position. In this mode the photogate will display the first time interval measured. Subsequent measurements will not be displayed (only the first one is), but the times are added in the memory. By pushing the “READ” switch you can display the memory contents, which is the sum of all measurements. Example: the initial reading for cart 1 (the time that it took to pass through the gate 1) is 0.300 seconds. Cart 1 collides with cart2 and they go together through the photogate 2 (Figure 2). Suppose it now takes 0.513 seconds. The display will remain at 0.300, but the memory will contain 0.300 + 0.513=0.813 seconds. To find the second time, you have to subtract the first time from the contents of the memory. Try this out by moving the cart through the gate by hand a few times.

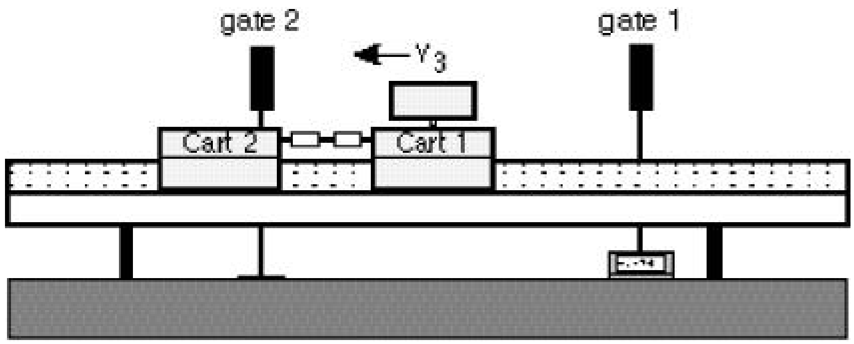

Figure 2: The final state of the carts after inelastic collision.

2. Do 3 sets of inelastic collisions consisting of 2 trials each. Vary the masses of the carts by adding the masses (small metal disks) to them. In these measurements, use the needle and putty bumpers and measure the initial and final velocities for the following sets of masses:

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2;
Trial 3+4: 2 mass disks on cart 1, 2 mass disks on cart 2;
Trail 5+6: 2 mass disks on cart1, no mass disks on cart2.

3. In each measurement, you need to find all the initial and final masses and velocities, and use them to calculate the initial and final total momentum and kinetic energy. Tables attached to the end of this manual will help you to organize your recordings.

In your directory, you will find 2 preset datasheets, *Coll1* and *Coll2*. You will use *Coll1* to calculate the momenta and kinetic energies, and *Coll2* to find the percent change in the total momentum and the kinetic energy.

Elastic collisions

In an almost elastic collision, the main difference from the previous part of the lab is that after the collision the carts move separately. The elastic bumper allows them to bounce off with almost no conversion of the kinetic energy into the other forms of energy.

As before, cart 2 initially stays at rest, and before the collision we have to measure only the velocity of the cart 1 v_{1i} (Figure 3). However, after the collision we have to measure the velocities of both carts, v_{1f} and v_{2f} (Figure 4). Thus, all in all we have to measure three times (t_{1i}, t_{1f}, t_{2f}), while the photogate system allows to simultaneously measure only two of them.

We can get out of this situation if, after the measurement of the initial time t_{1i}, but before the collision, we reset the timer. You have to make several practice trials to quickly remember and reset the contents of the timer before the carts collide. Then, we can again use the contents of the timer display and the memory to find t_{1f} and t_{2f}.

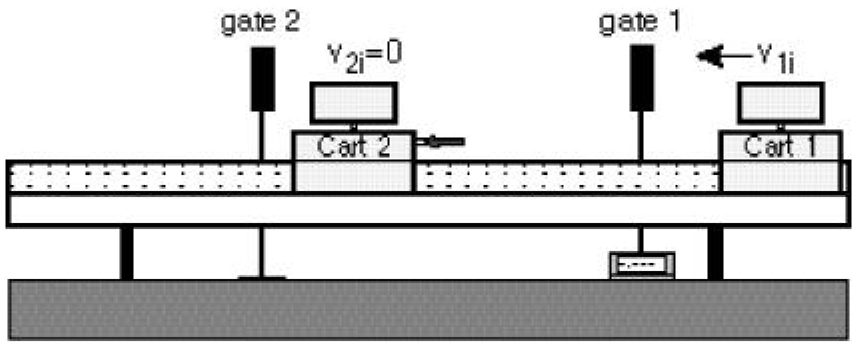

Figure 3: The initial state of the carts before elastic collision.

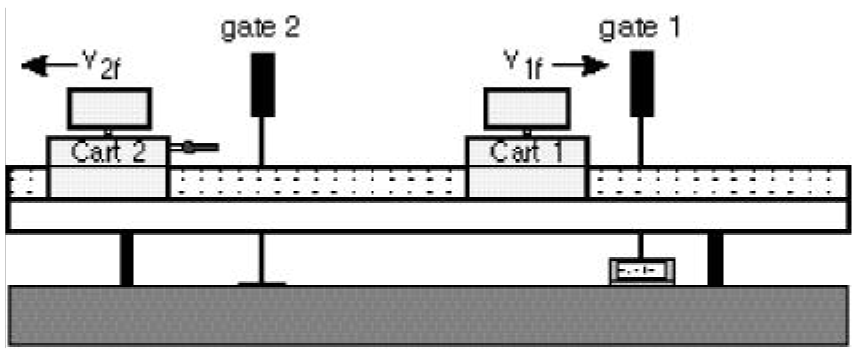

Figure 4: The final state of the carts after elastic collision.

The plates on two carts have the same length (check this!). The experiment will be done with cart 2 initially at rest. You will do 6 trials with the following choices of m_1 and m_2:

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2
Trial 3+4: 2 mass disk on cart 1, 2 mass disks on cart 2
Trail 5+6: 4 mass disks on cart 1 no mass disk on cart 2

Pay attention to the sign of the velocities, which depends on the direction of tmotion of the cart. If the percentage change in momentum or kinetic energy before and after the collision is greater than 10%, repeat the measurement more carefully (collide slower/faster, etc.). Since the datasheet is set up it is easy to see whether momentum/energy is better conserved with every trial you do.

Questions to be discussed

Conservation of momentum

1. For both inelastic and elastic collisions, create the graphs for the relative change of the total momentum $d_P = (P_f - P_i)/P_i$ versus the number of the measurement. Show the error of d_P with the help of error bars. Use different symbols to show the data obtained with different types of bumpers. Also, show the theoretical prediction for d_P.
2. According to the graphs, does the change in the momentum depend on the type of interaction? In other words, does d_P show a systematic dependence on the type of the bumper that you have used?
3. According to the graphs, was the total momentum conserved in the collisions? Use the “two standard deviations” rule to justify your answer. If the momentum was not conserved, discuss the reason why. Also, explain what you have done to improve the situation and whether you succeeded.

Conservation of energy

4. It is possible to calculate the percentage of the kinetic energy lost in a completely inelastic collision; you will find that this percentage depends only on the masses of the carts used in the collision, if one of the carts starts from rest.

After the completely inelastic collision, the carts move together, so that

$$v_{1f} = v_{2f} = v_3$$

The initial *KE* is given by:

$$KE_i = \frac{m_1 v_{1i}^2}{2} + \frac{m_1 v_{2i}^2}{2} \quad \text{. But, since } v_{2i} = 0$$

$$KE_i = \frac{m_1 v_{1i}^2}{2} \tag{8}$$

The final *KE* is given by:

$$KE_f = v_3{}^2 \, \frac{m_1 + m_2}{2}, \tag{9}$$

From conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)\, v_3 \qquad \text{or, since } v_{2i} = 0$$

$$m_1 v_{1i} = (m_1 + m_2) v_3 \tag{10}$$

Since the collision is inelastic, the initial *KE* is not equal to the final *KE*. Use equations (8), (9), and (10) to obtain an expression for $d_{KE} = \dfrac{(KE_f - KE_i)}{KE_i}$. Plot d_{KE} vs. $\dfrac{m_1}{m_1 + m_2}$.

Show the error bars for d_{KE} and the line for the theoretical prediction. Discuss the consistency of your data for d_{KE} with the theory.

5. Repeat steps 1,2,3 for d_{KE} in the nearly elastic collision. What was the average loss of the kinetic energy in this part of the experiment? Did you achieve the goal of 10% loss of *KE* in the nearly elastic collisions? Which factors is this loss most sensitive to?