Model Write-Up

The following is an example of a very thorough write-up. In particular, the introduction beautifully summarizes the important optics and equations. Even more detail than necessary is given in every section. (In other words, your write-ups do not have to be quite this long.)

Here are some other important comments: Since uncertainty was a major topic in this lab (L1), the uncertainty equations are given explicitly. In future labs, it is not necessary to show the details of the uncertainty estimates. In general, please remember to either make a table or graph showing all your measured data, or give examples and the range of values. Lastly, to make it easier to grade, it would be helpful if the specific answers to the questions were labeled **A1**, ect.

→ S. Tessmer

(*Partner:*)
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Thin Converging Lens

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Abstract

This experiment had two goals: to investigate the properties of a Thin Converging Lens, and to provide exercises in uncertainty propagation.

In this experiment, the lens studied was found to have the following properties:

Radii of Curvature:
$R_1 = 5.6 \pm 0.3$ cm and $R_2 = -5.6 \pm 0.3$ cm ✓

Focal Length:
$f = 4.98 \pm 0.07$ cm ✓

Index of Refraction:
$n = 1.56 \pm 0.03$ ✓

The exercises in uncertainty propagation proved to be laborious, but ultimately satisfying. The experiment served as a reminder to be liberal in making uncertainties for measurements, so that final measurements do not claim more accuracy than actually obtained.

Introduction

In this experiment, the properties of a Thin Converging Lens were studied. There are three main properties of any Thin Converging lens. They are:

- R = Radius of Curvature
- f = Focal Length
- n = Index of Refraction

Great Details

These three properties are related to each other via the Lensmaker equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad R_2 < 0$$

The *radius of curvature* (or in fact, radii of curvature, for each lens has two sides of possibly differing radii) is the most easily altered property of a lens – with lenses of all sorts of various sizes available. Flatter lenses have larger radii of curvatures, while rounder lenses have smaller radii of curvatures.

The *index of refraction* is a property dependant on the material of the lens. It is inversely proportional to the speed of light through that particular material, according to the following equation:

$$n = \frac{c}{v}$$

In this case, c is the speed of light in a vacuum, and v is the speed of light through the particular material. Air has an index of refraction of 1.00 with other materials having values greater than 1.00. Glass typically has an index of refraction of 1.50, although certain impurities may possibly change that value.

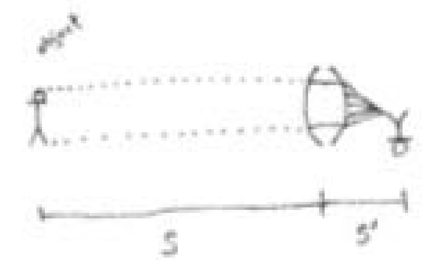
The final property – *focal length*, is the dependent variable in the Lensmaker equation. It depends upon the radii of curvature and the index of refraction. The focal length is the minimum distance required to focus an object that is infinitely far away.

As objects come closer to the lens, they require more distance to focus, according to the Thin Lens Equation, shown below:

$$\frac{1}{f}=\left(\frac{1}{s}+\frac{1}{s'}\right)$$

In the equation, s is the *object distance* – the distance between the object and the lens – and s' is the *image distance* – the distance required to focus the object on a screen. Figure 1 below is a drawing showing the distances s and s' and the general set up of this experiment. Interestingly, the image of the object always appears upside down – a fact discussed later. By altering the object distance s and measuring the new image distance s', a relationship can be seen which determines the focal length. This experiment explores that relationship, and the association it has with the properties listed above.

Figure 1



The radii of curvature can easily be measured by using a spherometer. The focal length can be estimated by attempting to focus an object that is far away (although infinite object distance is of course impossible to achieve). However, focal length is much more accurately determined by measuring the image distances (s') of objects of varying distance (s) away and using the Thin Lens Equation to make the calculations. Having found the focal length, the index of refraction can be determined using the Lensmaker Equation, and compared to the known values per material. This was the procedure taken during this particular experiment, done for one particular given lens.

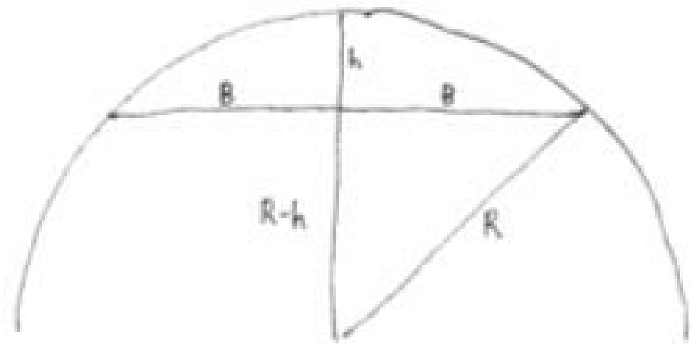
Analysis and Discussion

The first step in this experiment was to estimate the focal length of the given lens. To do this, one partner stood on the far side of the room, approximating an object of infinite distance away. The second partner tried to focus the first partner on the wall and then measure the distance between the wall and the lens. This distance, an approximate focal length, was found to be 4.0 ± 0.5 cm. This was a difficult measurement to make, and the original error estimate of 0.5 cm was probably too bold. Holding the lens so that the image stayed focused while measuring the distance was tough. Poor lighting, and an object distance of far less than infinite distance made for what was probably a poor estimate deserving a larger uncertainty.

An interesting discovery was that the partner appeared upside-down. As the light enters the lens, rays from the top of the object are directed downward, while rays from the bottom of the object are directed upward. Thus, at the focal length, the image appears in focus, but upside-down.

Next, a spherometer was used to measure the radii of curvature for the two sides of the lens. This measurement wasn't as straightforward as desired. A zero-position needed to be determined on the spherometer, and measurements made relative to it. The actual length measured was a height h, shown in Figure 2, from which the radius could be determined using a formula based on Pythagoras' Theorem.

Figure 2



$$R^2 = (R-h)^2 + B^2$$
$$R^2 = R^2 - 2RH + H^2 + B^2$$
$$\ldots$$
$$R = \frac{B^2}{2H} + \frac{H}{2}$$

Details removed, the final resulting measurements were:

$$R_1 = 5.6 \pm 0.3 \text{ cm}$$
$$R_2 = -5.6 \pm 0.3 \text{ cm}$$

The value for R_2 is negative because it is the back edge of the lens, having a curve in the opposite direction as the front edge.

The error estimates of R_1 and R_2 were propagated. Uncertainties in the zero-position x_0 and the measured distance x_1 were estimated as σ_{x0} and σ_{x1} respectively. They resulted in a propagated uncertainty for h of:

$$\sigma_h = \sqrt{(\sigma_{x0})^2 + (\sigma_{x1})^2}$$ ✓

An uncertainty in the spherometer radius b was also estimated as σ_b. Together, these uncertainties were used to calculate the overall uncertainty of R (σ_R):

$$\sigma_R = \sqrt{\left(\frac{dR}{db}\right)^2 (\sigma_b)^2 + \left(\frac{dR}{dh}\right)^2 (\sigma_h)^2}$$ ✓

These errors depended on the measured quantities of R_1 and R_2, but since they were essentially equal they were approximated to be exactly equal and only one uncertainty was made for both measurements.

Now a more accurate measurement of focal length was desired. To obtain it, a lamp was set up on an optical rail, shining through the lens onto a screen as pictured in Figure 1 above. A crosshair shaped like a T served as the object in this experiment, and was located on the end of the lamp. The screen was placed an arbitrary distance away from the lamp and the lens was moved until the image on the screen was brought into focus. Once everything was in focus, measurements were made of s and s' and the process began again. Four different trials were done with four different lamp-to-screen distances.

The results of the measurements of s and s' are summarized in Figure 3 below.

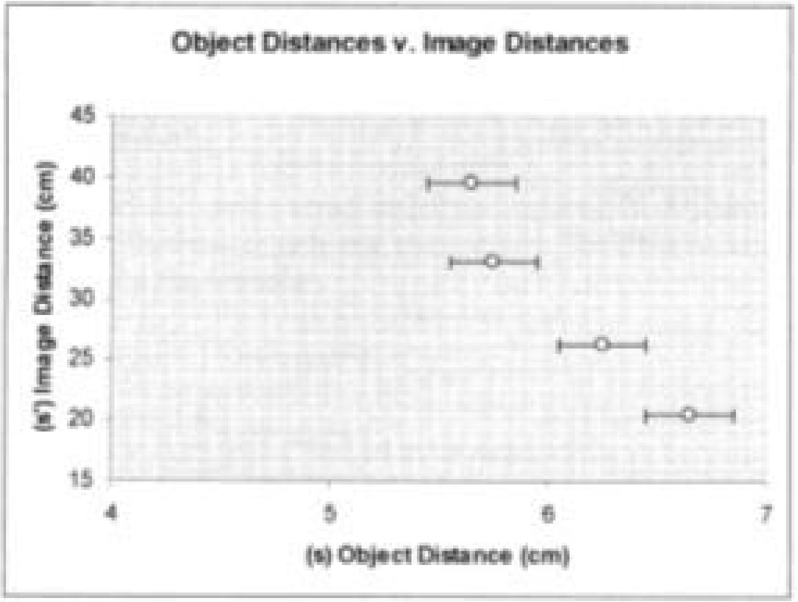


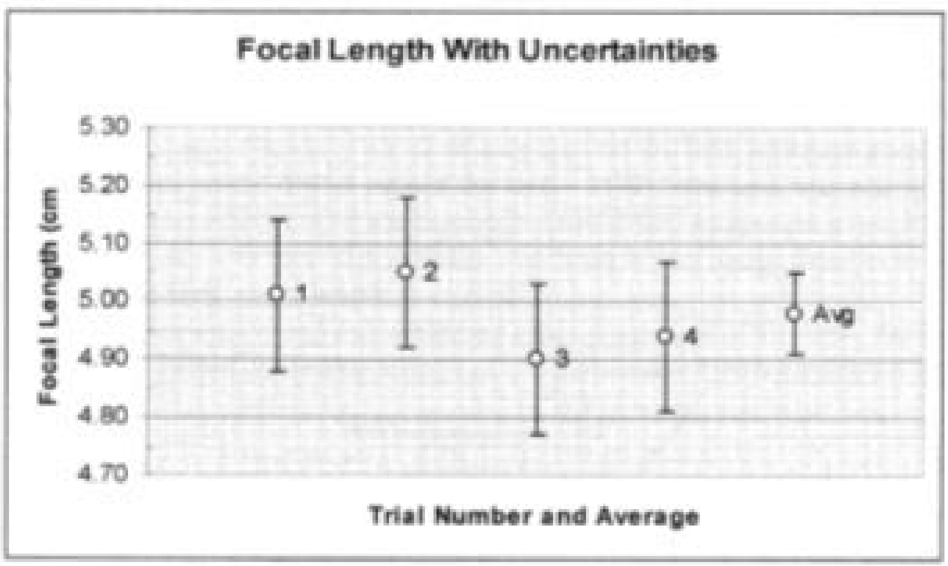
Figure 3

Uncertainties for s and s' (σ_s and $\sigma_{s'}$ respectively) were estimated, and are shown on the graph. Due to scale, the relatively small uncertainty for s' cannot be seen.

Having measured s and s', the focal length could now be calculated for each of the trials according to the following equation, derived from the Thin Lens Equation:

$$f = \frac{1}{\left(\frac{1}{s} + \frac{1}{s'}\right)}$$

The resulting values are shown in Figure 4 below. Having found these four values, the average was found and that value taken to be the best estimate of the focal length. For this experiment's particular lens, the focal length was found to be $f = 4.98 \pm 0.07$ cm. This value is also pictured in Figure 4 below.



← Very Nice

Figure 4

The uncertainties for each f_i shown above were propagated from the uncertainties σ_s and $\sigma_{s'}$ according to the following formula:

$$\sigma_{f_i} = \sqrt{\left(\frac{df}{ds}\right)^2 (\sigma_s)^2 + \left(\frac{df}{ds'}\right)^2 (\sigma_{s'})^2}$$

Since the values of σ_{fi} depended on the actual calculated values of f_i an approximation was made allowing one f_i to be used, resulting in only one uncertainty. The final uncertainty for the average of f was found using the formula:

$$\sigma_f = \frac{\sigma_{f_i}}{\sqrt{n}}$$

Since there were four trials, n = 4.

Measuring the image distances s' served to be rather difficult because the images were sometimes hard to bring into focus. One thing that helped was putting an iris into the system which acted exactly like the iris in a human eye. It reduced the amount of light going through the lens and made the images sharper. As the iris radius decreased, parts of the image became clearer still, having better defined edges, most noticeably along the outside edges of the image. This effect may be due to reducing the effects of interference caused by imperfect reflections of the light from the outside edges of the lens. Since the lens is not perfectly ellipsoidal, it is a Paraxial lens, meaning only the rays passing through near the very center of the lens are without distortion. By reducing the iris size, the outside most rays of light can no longer get through, allowing only the central rays through, resulting in a less distorted image.

The best estimate of f = 4.98 ± 0.07 cm is quite a bit higher than the rough estimate of f = 4.0 ± 0.5 cm made at the beginning of the experiment. As mentioned above, the rough estimate was a very hard measurement to make, and the estimated uncertainty was too small.

One interesting property of the Thin Converging Lens was explored at this point, by trying to bring into focus something that was a distance less than the focal length away. Every object distance tried so far was at least a focal length away. As the object moved closer, the distance required to bring the image into focus increased. According to the equation, to bring a object that is one focal length away into focus, an infinite distance would be required. It seemed that bringing an object that is even *closer* into focus would require *more than infinite* distance – but how was that possible?

In reality, this problem is handled in the Thin Lens Equation by having an image distance that is negative. Since image distances are distances beyond the lens, a negative image distance meant that the image was on the object side of the lens – in fact, the lens was acting more like a mirror! Unfortunately this effect could not be observed with the equipment available. What was observed was the difficulty – in fact impossibility – of bringing an image into focus whose object length was less than the lens' focal length.

Finally, having measured and calculated all other properties of the lens, the index of refraction could be calculated according a form of the Lensmaker Equation shown below:

$$n = \frac{1}{f\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} + 1$$

For the particular lens used in this experiment, the index of refraction was calculated to be n = 1.56 ± 0.03. The error estimate was propagated from the errors σ_f and σ_R:

$$\sigma_n = \sqrt{\left(\frac{dn}{df}\right)^2 (\sigma_f)^2 + \left(\frac{dn}{dR}\right)^2 (\sigma_R)^2}$$

The value obtained seems reasonable, as the given index of refraction for glass is 1.50. The calculation may seem off at first, but the exact material of the lens in this experiment was unknown, only assumed to be perfect glass. Small imperfections and scratches in the lens could exist which could decrease the speed of light through the lens, resulting in a higher index of refraction – so a value of 1.56 certainly seems reasonable enough.

The uncertainty σ_n of 0.03 does seem a little presumptuous. Since the book value of n_{glass} = 1.50, a larger uncertainty which includes that value is desirable. Of course, it may be that the lens in this experiment truly has that index of refraction. It seems more ethical to claim less accuracy (i.e. have a higher uncertainty value). Making too bold a claim may prove to be a fallacy, or even an outright lie. Being more modest on the uncertainties of measurements made (e.g. σ_{s0} and σ_s) would increase the values of each of the resulting propagated uncertainties. This may prove desirable in the future. However, having too modest a claim would result in a claim that is too vague to prove useful. A healthy balance is needed.

I would not go quite that far

Conclusion

The goals of this experiment were to investigate the properties of a Thin Converging Lens, and to exercise using error propagation techniques to find uncertainties for calculated values.

In summary, this experiment revealed that for a particular lens with radii of curvature R_1 = 5.6 ± 0.3 cm and R_2 = -5.6 ± 0.3 cm, the focal length f = 4.98 ± 0.07 cm and the index of refraction n = 1.56 ± 0.03. These values all agree with the associated optical theory. ✓

Efforts in uncertainty propagation and study yielded two results. First, error propagation is tedious and error prone, but does yield convincing and powerful results validating the effort. Second, the propagated errors depend heavily on the original error estimates. Making too bold of an original estimated uncertainty for a particular measurement often results in a final uncertainty that seems too good to be true. In future experiments, a little more modesty in error estimation may be prudent.

Great Job!