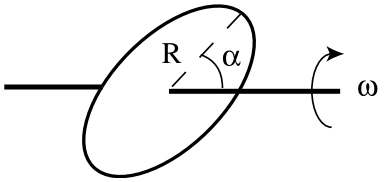
F03 PHY820: Midterm II

November 24, 2003

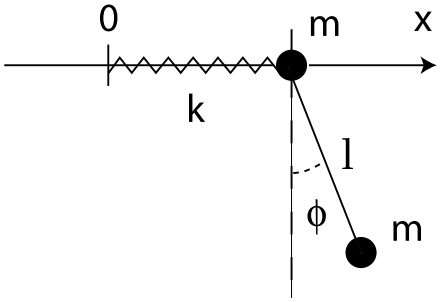
NOTE: Sign your exam. Show all work. Justify your answers.

1. Deduce elementary properties of transformation matrices for infinitesimal rotations.

2. (a) Find the tensor of inertia of a uniform disk of radius R and mass m, about the disk center, using the disk symmetry axis as one of the coordinate axes for the tensor. (b) The disk is welded to an axle passing through the center, so that its plane is inclined at an angle α relative to the axle. What is the moment of inertia of the disk about the axle? (c) The axle is rotated uniformly about its axis at an angular frequency ω. Using Euler's equations, find the torque, magnitude and direction, delivered by the axle to the disk.



3. A bead of mass m is threaded onto a horizontal wire along the x axis and attached to an end of a massless spring, placed along the wire, characterized by the spring constant k. The other end of the spring is nailed at the origin of the x-axis. A second bead of mass m is hung from the first bead by a thread of length ℓ. Find the frequencies of free oscillations for the system of the two beads, assuming motion within one vertical plane.



4. The carthesian coordinate system $\mathcal{O}'$ rotates about an inertial system $\mathcal{O}$ at a constant angular velocity $\vec{\omega}$. (a) Consider a particle of mass m moving in a potential $V(\vec{r})$. How is the particle velocity $\vec{v}$ in $\mathcal{O}$ related to the velocity $\vec{v}'$ perceived in $\mathcal{O}'$? Obtain a Lagrangian for the particle in terms of the coordinates $\vec{r}'$ in $\mathcal{O}'$. (b) Obtain the Hamilton's function and the canonical equations of motion for the particle in $\mathcal{O}'$, relying on the coordinates $\vec{r}'$.